THERMODYNAMICS AND HEAT POWER ENGINEERING

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Preface

... book has developed out of authors two earlier books on Theory and Problems of Thermodynamics. The good reception received by the previous books prompted the authors to widen the scope and completely rewrite the text so as to include practically entire field of Thermodynamics applied to Heat Power Engineering in one volume. The book is intended to meet the needs of students studying Thermodynamics, Heat Engines, Prime Movers, Internal Combustion Engines and Gas Turbines, Refrigeration and Air Conditioning, and Heat Transfer courses for university and professional examinations. It is hoped that the book will also be of considerable interest as a reference volume to practising engineers who wish to revise or extend their knowledge.

The concept of teaching of thermodynamics has considerably changed in recent years. There is now great emphasis on the First and Second Laws of Thermodynamics and generalised application of these Laws to all heat power machinery. This new trend has been followed in this book.

The theory has been dealt in clear and lucid manner. In each chapter theory has been followed by a list of "Important Points", drawn from the authors' long experience of teaching various classes. These points will help students avoiding common pitfalls. Another notable feature of the text material is the inclusion of answers to theory questions appearing with numerical problems in examinations, which are rather out of way for a general text book.

The importance of numerical problems in thermodynamics cannot be over emphasised. Numerical problems help in providing clear concepts of the knowledge derived from theory and fixing them in the mind. The problems in the book generally follow the style of questions set in university and professional examinations though some departure have been made to illustrate important principles and commercial practice. Solutions to problems are to slide rule accuracy. The answers to problems have been obtained using the 'Steam and Other Tables' arranged by authors and published by Messrs Jain Brothers, New Delhi-5 in 1969.

One of the greatest difficulties of students is the judicious selection of problems from many available. Keeping this in view all problems have been classified with titles and they have been carefully arranged in graded manner to include a wide range and depth. Repetition of similar problems has been avoided to keep the number small, which should induce the student to study the book completely. Many of the solved problems have been illustrated with diagrams, which have been drawn to scale to give a clear conception. For example no amount of description can convince a reader about the impracticability of the Carnot Cycle than its *P-v* diagram drawn to scale. Generally at the end of solution a note has been given about the significance of the problem, mistakes to be avoided and most important of all, checks for the accuracy of the results. In unsolved examples intermediate steps have been given for checking the solutions, as the problem is being worked out. This should help individual study outside the class-room.

The use of different symbols cause lot of confusion in the study of Applied Thermodynamics. In this book generally British Standard symbols have been used as the Indian Standards have not yet been formulated. The units are generally given in algebraic form (kgf/cm^2 instead of kgf per square centimetre), because this is more logical. Unfortunately the word weight is often used for mass. Throughout the text the word 'mass' has been used wherever appropriate.

The authors thank their many colleagues and friends who gave valuable suggestions in writing this text book. The diagrams for the book have been drawn by Shri P. S. Kachhwaha and the authors are thankful for his neat work. The author's thanks are also due to the Publishers M/s Jain Brothers, for help and cooperation in getting the book published in a short time.

Inspite of best efforts errors are likely to creep in any work of this magnitude and therefore the authors request the indulgence of readers to bring the error to their notice and also give their valuable suggestions, which will be gratefully acknowledged and incorporated in future editions.

Lastly, authors are grateful to Vimla Mathur and Meena Mehta who largely contributed to completion of this work by gladly suffering the neglect and lack of attention towards family while their husbands were busy working on the book.

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M. L. Mathur
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Nomenclature

A, a	area
C	velocity
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_p	molar heat at constant pressure
C_v	molar heat at constant volume
D, d	bore; diameter
E	internal energy (general)
e	specific internal energy; base of natural logarithms
F	force; configuration factor
f	some function
Gr	Grashof number
g	gravitational acceleration
H	enthalpy
h	specific enthalpy, heat transfer coefficient
J	Joule's equivalent
K	temperature on the Kelvin scale (*i.e.*, Celsius absolute)
k	thermal conductivity; isentropic index for steam, blade velocity coefficient; nozzle efficiency
L	stroke; fundamental dimension of length
l	length; characteristic linear dimension
M	molecular weight; fundamental dimension of mass
m	mass
N	rotational speed; cycles per unit time, refrigeration effect
Nu	Nusselt number
n	polytropic index; number of moles
P	power
Pr	Prandtl number
p	absolute pressure; pitch
p_m	mean effective pressure
Q	heat; rate of heat transfer; fundamental dimension of heat
q	rate of heat transfer per unit area

R gas constant ; thermal resistance ; radius

Re Reynolds number

R_o universal gas constant

r radius ; expansion ratio ; compression ratio

r_p pressure ratio

S entropy

s specific entropy

T absolute thermodynamic temperature ; torque ; fundamental dimension of time

t temperature ;

U internal energy ; overall heat transfer coefficient ; blade speed

u specific internal energy

V volume ; velocity relative to blade

v specific volume

W work ; rate of work transfer ; brake load

x dryness fraction ; length

z height above datum

Greek Symbols

α angle of absolute velocity ; nozzle angle ; absorptivity, angle of relative velocity

β angle of relative velocity ; coefficient of cubical expansion

γ ratio of specific heat, C_p/C_v

ε emissivity

η efficiency

θ temperature difference, fundamental dimension of temperature

λ wave length

μ dynamic viscosity ; degree of saturation

ν kinetic viscosity (μ/ρ)

ρ density ; reflectivity

σ Stefan-Boltzmann constant

τ transmissivity

ϕ relative humidity

ω specific humidity or humidity ratio ; angular velocity

Λ degree of reaction

Suffixes

a	air ; atmospheric ; axial velocity
b	black body
c	critical value
db	dry bulb
dp	dew point
e	exit
f	saturated liquid ; flow component
g	saturated vapour ; gas
fg	change of phase at constant pressure
i	inlet ; a constituent in a mixture , inside surface
m	mean
o	overall ; outside ; zero or reference condition
s	vapour , swept volume ; stage ; free stream
t	total head or stagnation conditions
w	water ; wall
wb	wet bulb



1

The First Law of Thermodynamics and its Applications

1-1. Thermodynamic Properties. A property is any observable or measurable characteristic of a system. The change of property is fixed by the end states and is independent of the path followed. Properties may be classified as *intensive* and *extensive*. Intensive properties are independent of the mass; for example temperature, pressure and density are intensive properties. Extensive properties are dependent on mass of the system. Examples of extensive properties are total volume, total internal energy, etc.

The condition or state of a pure substance* in liquid or gaseous forms is defined by two independent properties, for example by pressure and temperature. Independent properties are not dependent on one another. The examples of dependent properties are density and volume which are not independent of each other During the boiling of a liquid the pressure and temperature of the liquid and vapour mixture are not independent For a particular state defined by two independent properties the substance has particular values of thermodynamic properties. Examples of thermodynamic properties, besides pressure, volume and temperature, are internal energy, enthalpy and entropy.

1-2. Temperature. It is an intensive thermodynamic property and is sensed directly. A body which feels hotter than another is considered to have higher temperature. The measurement of temperature is made through the measurement of other properties such as volume, pressure, electric resistivity, etc., and the instrument by which this measurment is made is called *thermometer*.

---

*A pure substance is a system which is homogeneous in composition and homogeneous and invariable in chemical aggregation.

Equality of temperature of two or more bodies is defined as that condition in which no change in any observable properties occurs when they are brought into communication with each other.

If two bodies are each equal in temperature to a temperature of the third body they are equal in temperature to each other. This statement is called *Zeroth's Law of Thermodynamics*.

1·3. Work. Every reader is familiar with the concept of work used in mechanics which is defined as follows : "work is done by a force when it acts upon a body moving in the direction of the force". The amount of work is equal to the product of force and the distance moved in the direction of the force. This definition of work, however, is too restrictive for the complex problems of thermodynamics. Before giving the thermodynamic definition of the work it will be important to define the term *system*.

The *system* is any prescribed and identifiable collection of matter whose behaviour is being investigated. The system has a constant mass of matter. This quantity of mass is enclosed by a *boundary* and separates it from the *surroundings*. Fig. 1·1 shows the usual representation of system, boundary and surroundings. The

If neither mass nor energy are allowed to cross the boundary of a system it is called an *isolated system.*

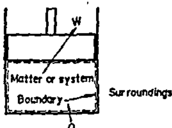


Fig. 1·2. Closed system.

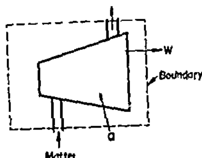


Fig. 1·3. Open system.

In the first two systems work and heat flow *from* or *to* the surroundings crossing the boundary When work is done by the system on the surroundings it is said to be positive.

The following is the thermodynamic definition of work. *Work is energy in transition which flows from a system to the surroundings during a given process if the sole effect external to the system can be reduced to the raising of a weight.*

The above definition of work is restricted to positive work When a system does a positive work its surroundings do an equal amount of negative work and vice-versa (Fig. 1·4).

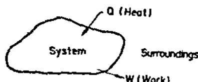


Fig 1·4. Representation of positive work and heat flow.

Work is transient. It is present during the interaction but does not exist either before or after the interaction and thus it is not a property like pressure or volume having finite values in a system at rest.

Besides mechanical work there can be other types of work. For example work is transferred in flow of electricity. The mechanical work, however, is virtually the only kind of work which can be conveniently delivered in any appreciable quantity to gaseous systems.

1·4. Resisted and Unresisted Expansion. A resisted e sion is one in which force exerted by the fluid on a piston is

by an equal and opposite force exerted by the piston on the fluid and the pressure of the fluid at any instant is very nearly uniform throughout the cylinder. In resisted expansion all the effort of the fluid goes into moving the piston *i.e.* the boundary, and none into moving itself. Reciprocating engines are examples of this *fully resisted expansion.* In contrast to this is *unresisted or free expansion* as would occur when a fluid escapes from a hole in a vessel. *Throttling* and *free expansion* (porous plug expansion) are the examples of unresisted expansion. Fully resisted expansion is a reversible process whereas unresisted or free expansion is an irreversible process. Reversibility is discussed in chapter 2.

The work done in resisted expansion is called *displacement work.* It is also known as '*PdV*' work because in resisted expansion the area of *P-V* diagram is equal to the work done as discussed in the next article. Only a reversible process, which passes from one state of equilibrium to another, can be represented on *P-V* diagram. An irreversible process cannot be truely represented on *P-V* diagram and, therefore, may be indicated by dotted lines.

Only in the case of resisted expansion the area of *P-V* diagram represents the work done.

1.5. Pressure-Volume Diagram. The expansion or compression of a gas or vapour in the part of a system can be represented graphically on pressure-volume diagram. In this pressure is plotted as ordinate and volume as abcissae. Fig. 1·5 shows the curve for

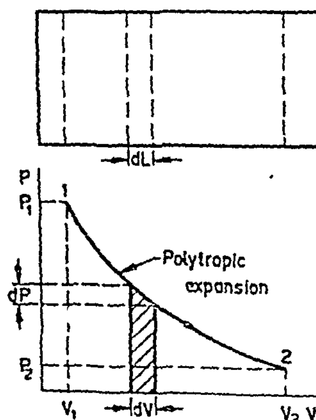


Fig. 1·5. Polytropic expansion on *P-V* diagram.

polytropic expansion. Each point on the curve gives the pressure *P* and the corresponding volume *V*.

Consider the piston at any point in the cylinder and let corresponding pressure and volume at this point be P and V respectively. At this instant let the movement of the piston due to the force be dL. This will cause change in pressure and volume. Let the change in pressure and volume be dP and dV respectively.

Let dW be the work done in small movement of the piston.

Then, dW=(Average force)×(distance travelled)

$$=\left\{\frac{P+(P+dP)}{2}\right\}\times A\times dL$$

$$=PdV+\frac{dPdV}{2} \qquad [\text{as } AdL=dV$$

Both dP and dV are of small order and therefore their product is negligible as compared to PdV.

$\therefore$ $dW=PdV$=area of shaded strip in the P-V diagram.

Hence, total work done during expansion from state 1 to state 2 will be the sum of all such areas

or $$W=\int_1^2 PdV \qquad (1\ 1)$$

Thus, the work done in any fully resisted expansion is given by the area under the curve on P-V diagram In an unresisted expansion or an irreversible process $W\neq\int PdV$.

1·6. Heat Heat is energy in transition which flows by virtue of temperature difference from one system to another at lower temperature when they are brought into communication. Heat, like work, is transitory in nature, it is never contained in a body. If heat is transferred by surroundings to the system it is said to be positive for the system as shown in Fig. 1·2.

Heat is measured by the mass of a prescribed material which can be raised in temperature from one prescribed level to another. The prescribed material is water, the mass 1 kg, from a temperature of 15·5°C to 16·5°C at a pressure of 1 atmosphere, the heat then is a unit and is known as *kilo-calorie* (kcal)

Specific Heats. Specific heat of a substance is defined as the heat required to raise unit mass of substance through unit degree of temperature Gases have two specific heats depending on the manner in which they are heated, namely, the *specific heat at constant pressure* C_p, and the *specific heat at constant volume* C_v. The specific heat at constant pressure is always greater than the specific heat at constant volume by the amount of work done in

constant pressure expansion of the gas. The ratio of the specific heats of any given gas is a constant and is represented by a symbol γ.

$$\frac{C_p}{C_v}=\gamma \tag{1·2}$$

For atmospheric air, $C_p=0{\cdot}24$, $C_v=0{\cdot}171$ and $\gamma=1{\cdot}4$.

Note. For rigorous definition of specific heats, *see* article 1·11.

1·7. The First Law of Thermodynamics. The first law of thermodynamics states that heat and work are mutually convertible. The conversion rate, called the mechanical equivalent of heat, J, was found experimentally by Joule. The value of J depends on the units chosen for heat and work. In metric system of units one kilogram-caloric (kcal) heat unit is equivalent to 427 kgf m of work.

The first law of thermodynamics states that in a closed system undergoing through a cycle (the end state being precisely the same as the initial state) the net work delivered to the surroundings is proportional to the net heat taken from the surroundings,

i.e. $$J \oint dQ = \oint dW \tag{1·3}$$

$\oint dQ$=algebric sum of heat taken from the surroundings during the cyclic process.

where $\oint dW$=algebraic sum of the work delivered to the surroundings during the cyclic process.

and J=mechanical equivalent of heat.

Equation (1·3) makes it possible to use the same units for heat and work. If consistent units are used the mechanical equivalent of heat 'J' can be deleted from Eq. (1·3) and it can be written as

$$\oint dQ = \oint dW \tag{1·3 (a)}$$

1·8. First Law Applied to Change of State of a Closed System Undergoing a Non-cyclic Process. The first law of thermodynamics as applied to a non-cyclic process where transfer of heat and work take place must be accounted for by change in a property called energy E.

It can be proved from the First Law that $(Q-W)$ is a thermodynamic property. $(Q-W)$ is independent of path and its value depends only on the end states. The numerical value of $(Q-W)$ is change in energy ΔE.

$$\Delta E = E_1 - E_2 = Q - W \tag{1·4}$$

Since there is no absolute value of energy it is permissible to assume its value to be zero at any specified state.

$$\therefore \quad E = Q - W \qquad [1·4\ (a)]$$

In an isolated system,

$$Q=0, \quad W=0$$

$$\therefore \quad \Delta E = 0 \tag{1·5}$$

This is the *Law of Conservation of Energy* which may be regarded as an alternative statement of the first law of thermodynamics if coupled with the statement that E is a property.

Internal Energy. The energy E represents the macroscopic and microscopic energy in a given state. In the study of thermodynamics kinetic energy (KE) and the potential energy (PE) (macroscopic energies) are generally considered separately and the energy associated with moleculer motions (microscopic energy) is considered separately and is termed as internal energy U.

$$dE = dU + d\,(KE) + d\,(PE) \tag{1·6}$$

In the case of closed or non-flow thermodynamic system kinetic energy and potential energy terms are zero and hence energy is equal to the internal energy and Eq. (1·4) becomes

$$Q = W + \Delta U \qquad [1·6\ (a)]$$

For unit mass $\quad \Delta q = w + \Delta u \qquad [1·6\ (b)]$

The following statements can be derived from Eq (1·4).

(*i*) The cyclic integral of E is always zero while that of Q or W may be other than zero

(*ii*) The differential dE is an exact differential in terms of the variables (the independent properties) that determine the state. No similar statement can be made concerning the differentials dQ and dW because these values depend on the process by which the state is approached.

1·9. Gas Laws. A gas is a substance which cannot be liquefied by application of pressure at constant temperature, however

large the pressure may be. In contrast, a vapour is a gaseous substance which can be liquefied by applying pressure at constant temperature. From the engineering point of view it would suffice to define a substance as gas when its evaporation from its liquid state is complete and to define it as vapour when its evaporation is incomplete and therefore has liquid particles in suspension. The laws of perfect gases do not apply to vapours.

A *perfect gas* is defined as a gas which strictly obeys the Boyle's and Charles' laws. For engineering purposes air is treated as a perfect gas. The following are the gas laws :

(*i*) *Boyle's law.* Boyle's law states that when any gas is heated at constant temperature its volume multiplied by pressure remains constant.

$$PV = \text{constant, if } T \text{ is constant} \qquad (1\cdot7)$$

(*ii*) *Charles' law.* Charles' law states that when any gas is heated at constant pressure its change in volume varies directly with the temperature range.

$$\frac{V}{T} = \text{constant, if } P \text{ is constant} \qquad (1\cdot8)$$

At higher pressures and temperatures Boyle's law and Charles' law are not strictly followed by any gas.

The combination of equations (1·7) and (1·8) gives the *characteristic equation of gas, i.e.*

$$PV = mRT \qquad [1\cdot9\ (a)]$$

where m = mass of the gas

R = constant for the gas

and T = absolute temperature

For one kg of gas the equation may be written in the form

$$\frac{Pv}{T} = R \qquad [1\cdot9\ (b)]$$

For atmospheric air,

$$R = 29\cdot27 \text{ kgf m per kg per } ^\circ K$$

(*iii*) *Regnault's law.* Regnault's law states that the specific heats remain constant for all temperatures and pressures. This is not strictly true as the specific heats are found to increase with temperature. In problems specific heats are taken as constant unless otherwise mentioned.

(iv) Joule's law. Joule's law states that the change of internal energy is proportional to the change of temperature.

Change of internal energy, $dU \propto dT$

$\therefore \qquad dU = \text{mass} \times \text{constant} \times dT$

This constant in the case of perfect gases is the specific heat at constant volume and hence,

$$U_2 - U_1 = m\, C_v (T_2 - T_1) \qquad (1\cdot10)$$

1·10. Mol and Molar Heat. Mol and Kg Mol. According to Avogadro's law equal volumes of different perfect gases at the same pressure and temperature contain an equal number of molecules. Therefore, equal volume of different perfect gases have a mass in direct proportion to mass of the molecules of which they consist. It has been found that the mass in kg of 22·41 m^3 of any perfect gas at N.T P. is equal to its molecular weight.

22·41 m^3 of O_2 at N.T.P. has a mass of 32 kg and its molecular weight is 32.

Similarly, 22 41 m^3 of N_2 at N.T.P. has a mass of 28 kg and its molecular weight is 28.

and, 22 41 m^3 of H_2 at N.T.P. has a mass of 2 kg and its molecular weight is 2.

This 22 41 m^3 *of volume is known as a Mol.* Mol, therefore, is a bigger unit of volume.

One mol of gas (22 41 m^3) at N.T.P. has a mass equal to its molecular weight. This mass is known as one *kg mol.*

Molar heats. Molar heat is defined as the quantity of heat required to raise the temperature of one mol (22·41 m^3) of substance through one degree. The mass of one mol is M kg, where M is the molecular weight.

Molar heat, $\quad C_{v_{mol}} = M\, C_v, \quad$ and $C_{p_{mol}} = M\, C_p \qquad (1\cdot11)$

and
$$\gamma = \frac{C_{p_{mol}}}{C_{v_{mol}}} = \frac{C_p}{C_v}$$

$$\frac{R}{J} = C_p - C_v$$

Multiplying by M

$$\frac{MR}{J} = M\, C_p - M\, C_v$$

large the pressure may be. In contrast, a vapour is a gaseous substance which can be liquefied by applying pressure at constant temperature. From the engineering point of view it would suffice to define a substance as gas when its evaporation from its liquid state is complete and to define it as vapour when its evaporation is incomplete and therefore has liquid particles in suspension. The laws of perfect gases do not apply to vapours.

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where m = mass of the gas
R = constant for the gas
and T = absolute temperature

For one kg of gas the equation may be written in the form

$$\frac{Pv}{T} = R \quad [1\cdot9\ (b)]$$

For atmospheric air,

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$$U_2 - U_1 = m\,C_v(T_2 - T_1) \quad (1{\cdot}10)$$

1·10. Mol and Molar Heat. Mol and Kg Mol. According to Avogadro's law equal volumes of different perfect gases at the same pressure and temperature contain an equal number of molecules. Therefore, equal volume of different perfect gases have a mass in direct proportion to mass of the molecules of which they consist. It has been found that the mass in kg of 22·41 m^3 of any perfect gas at N.T.P. is equal to its molecular weight.

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and, 22 41 m^3 of H_2 at N.T.P. has a mass of 2 kg and its molecular weight is 2.

This 22·41 m^3 of volume is known as a *Mol* Mol, therefore, is a bigger unit of volume.

One mol of gas (22·41 m^3) at N.T.P. has a mass equal to its molecular weight. This mass is known as one *kg mol.*

Molar heats. Molar heat is defined as the quantity of heat required to raise the temperature of one mol (22·41 m^3) of substance through one degree. The mass of one mol is M kg, where M is the molecular weight

Molar heat, $\quad C_{v_{mol}} = M\,C_v, \quad$ and $C_{p_{mol}} = M\,C_p \quad (1{\cdot}11)$

and
$$\gamma = \frac{C_{p_{mol}}}{C_{v_{mol}}} = \frac{C_p}{C_v}$$

$$\frac{R}{J} = C_p - C_v$$

Multiplying by M

$$\frac{MR}{J} = M\,C_p - M\,C_v$$

MR is known as *universal gas constant.*

Universal gas constant in heat units,

$$\frac{MR}{J} = C_{p_{mol}} - C_{v_{mol}} \tag{1·12}$$

Universal Gas Constant. The characteristic equation of a gas is $PV = mRT$. Substituting volume equal to one mol at N.T.P.

$$M \times R = \frac{PV_{mol}}{T}$$

$$= \frac{1·033 \times 10^4 \times 22·41}{273} = 848 \text{ kgf m/kg/mol°}K \quad [1·13(a)]$$

MR is a constant for all gases and is known as *Universal gas constant.* It is given by the symbol R_{mol}.

In heat units $R_{mol} = \frac{MR}{J} = \frac{848}{427} = 1·986$ kcal/kg mol°K [1·13(b)]

1·11. Dalton's Law of Partial Pressures. In gases molecules are so apart and the empty space is so large that the volume occupied by the molecules is negligible in comparison to empty space between them.

Volume occupied by a gas

=volume of molecules+volume of empty space of the vessel.

If a vessel contains some gas it fills the whole space of the vessel. If another gas is introduced in the same vessel it also occupies the whole space. *On this assumption Dalton's law of partial pressure states that the pressure of a mixture of gases is the sum of the pressure which each gas would exert if it existed alone in the space occupied by the mixture at the same temperature.* The latter pressures are known as partial pressures of the gases.

$$P = p_1 + p_2 + p_3 + \ldots\ldots \tag{1·14}$$

where P is the total pressure of mixture and p_1, p_2, p_3, etc., are the partial pressures of the individual gases.

1·12. Heating and Expansion in Non-flow Processes. In a non-flow process the substance remains in the vessel before and after the process and does not leave the system. It occurs when the system is confined in a closed boundary. In this case the work is obtained by the displacement of the boundary and is called *displacement work.* Displacement work is equal to $\int PdV$

during a process as shown in article 1·5. The following are the different non-flow processes.

(i) Constant Volume Process (Isometric). When a gas or vapour is heated in a fixed enclosed space the heating is at constant volume. On P-V diagram the constant volume process is represented by a vertical line and the area under the line is zero as

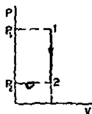


Fig. 1·6. Constant volume process.

shown in Fig. 1 6. There is rise in pressure but there is no work done as there is no change in volume. Mathematically, from the first law of thermodynamics,

$$dQ = dW + dU$$

$$dQ = O + dU \tag{1·15}$$

The effect of addition of heat in constant volume process is to increase the temperature and consequently the internal energy of the system. *The ratio of the increase in internal energy to the corresponding increase in temperature is expressed as derivative and is called the specific heat at constant volume C_v.*

$$C_v = \left(\frac{\partial u}{\partial t}\right)_v \tag{1·16 (a)}$$

It represents the rate of change of a property with temperature rather than a quantity of heat

Considering Eq. [1·16 (a)] for mass m and on integrating we get,

$$U_2 - U_1 = m\, C_v\, (T_2 - T_1) \tag{1 16 (b)}$$

From Eqs. (1·15) and (1·16)

$$Q = U_2 - U_1 = mC_v\,(T_2 - T_1) \tag{1·17}$$

(ii) Constant Pressure Process (Isopiestic). A gas or vapour may be heated under constant pressure as in a cylinder containing a sliding piston. On P-V diagram the constan ure

process is represented by a horizontal line as shown in Fig. 1·7. The heating causes increase in volume and temperature and external work is done. Mathematically, from the first law of thermodynamics,

$$dQ = dW + dU$$
$$= PdV + dU$$
$$= d(PV + U) \qquad [P = \text{constant} \quad (1\cdot18)$$

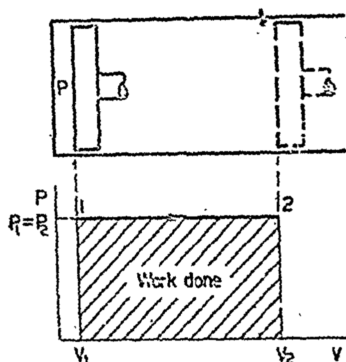


Fig. 1·7. Constant pressure process.

Since $(PV+U)$ is made up entirely of properties, it is also a property and is termed *enthalpy* H.

$$H = PV + U \qquad (1\cdot19)$$

The effect of addition of heat in constant pressure process is to increase the temperature and consequently the enthalpy of the system. *The ratio of the increase in enthalpy to the corresponding increase in temperature is expressed as a derivative and is called the specific heat at constant pressure* C_p.

$$C_p = \left(\frac{\partial h}{\partial t}\right)_P \qquad [1\cdot20(a)]$$

It represents the rate of change of a property with temperature rather than a quantity of heat.

Considering Eq. [1·20 (a)] for mass m and integrating we get,

$$H_2 - H_1 = m\, C_p\, (T_2 - T_1) \qquad [1\cdot20\ (b)]$$

From Eqs. (1·18), (1·19) and (1·20)

$$Q = H_2 - H_1 = m\, C_p\, (T_2 - T_1) \qquad (1\cdot21)$$

Again

Work done, $$W = \int_{V_1}^{V_2} PdV = \frac{P}{J}(V_2 - V_1) \text{ heat units} \qquad (1\cdot22)$$

Now $$dQ = dW + dU$$

$$\therefore \quad Q = \frac{P}{J}(V_2 - V_1) + m\,C_v\,(T_2 - T_1)$$

$$= \frac{mR}{J}(T_2 - T_1) + m\,C_v\,(T_2 - T_1)$$

$$= m(T_2 - T_1)\left[\frac{R}{J} + C_v\right]$$

Substituting the value of Q from Eq. (1·21)

$$mC_p\,(T_2 - T_1) = m\,(T_2 - T_1)\left[\frac{R}{J} + C_v\right]$$

or $$C_p - C_v = \frac{R}{J} \qquad [1\cdot23(a)]$$

Above equation can also be written as

$$C_v = \frac{R}{J(\gamma - 1)} \qquad [1\cdot23(b)]$$

and $$C_p = \frac{\gamma R}{J(\gamma - 1)} \qquad [1\cdot23(c)$$

(iii) Constant Temperature (Isothermal). A gas or vapour may be heated at constant temperature. In this case as there is no change in internal energy the work done in expansion will be equal to the amount of heat supplied through the cylinder

$$dQ = dW + dU$$

$$Q = W + m\,C_v\,(T_2 - T_1)$$

$$= W + 0 \qquad [T_1 = T_2 \qquad (1\cdot24)$$

In the case of a perfect gas constant temperature expansion is same as hyperbolic expansion as shown below.

(iv) Hyperbolic (PV=constant). The hyperbolic expansion is one in which pressure multiplied by volume remains constant

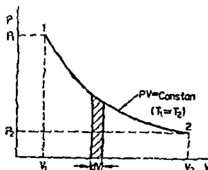


Fig. 1·8. Hyperbolic expansion.

process is represented by a horizontal line as shown in Fig. 1·7. The heating causes increase in volume and temperature and external work is done. Mathematically, from the first law of thermodynamics,

$$\begin{aligned} dQ &= dW + dU \\ &= PdV + dU \\ &= d(PV+U) \end{aligned} \qquad [P = \text{constant} \quad (1{\cdot}18)$$

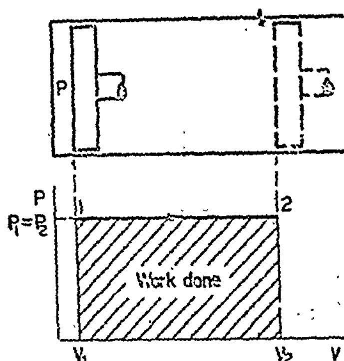


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$$H = PV + U \qquad (1{\cdot}19)$$

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$$C_p = \left(\frac{\partial h}{\partial t}\right)_P \qquad [1{\cdot}20(a)]$$

It represents the rate of change of a property with temperature rather than a quantity of heat.

Considering Eq. [1·20 (a)] for mass m and integrating we get,

$$H_2 - H_1 = m\, C_p\, (T_2 - T_1) \qquad [1{\cdot}20\ (b)]$$

From Eqs. (1·18), (1·19) and (1·20)

$$Q = H_2 - H_1 = m\, C_p\, (T_2 - T_1) \qquad (1{\cdot}21)$$

Again

Work done, $$W = \int_{V_1}^{V_2} PdV = \frac{P}{J}(V_2 - V_1) \text{ heat units} \quad (1{\cdot}22)$$

Now $$dQ = dW + dU$$

$\therefore$ $$Q = \frac{P}{J}(V_2 - V_1) + m\,C_v\,(T_2 - T_1)$$

$$= \frac{mR}{J}(T_2 - T_1) + m\,C_v\,(T_2 - T_1)$$

$$= m(T_2 - T_1)\left[\frac{R}{J} + C_v\right]$$

Substituting the value of Q from Eq. (1·21)

$$mC_p\,(T_2 - T_1) = m\,(T_2 - T_1)\left[\frac{R}{J} + C_v\right]$$

or $$C_p - C_v = \frac{R}{J} \qquad [1\ 23(a)]$$

Above equation can also be written as

$$C_v = \frac{R}{J(\gamma - 1)} \qquad [1·23(b)]$$

and $$C_p = \frac{\gamma R}{J(\gamma - 1)} \qquad [1\ 23(c)$$

(iii) Constant Temperature (Isothermal). A gas or vapour may be heated at constant temperature. In this case as there is no change in internal energy the work done in expansion will be equal to the amount of heat supplied through the cylinder.

$$dQ = dW + dU$$
$$Q = W + m\,C_v\,(T_2 - T_1)$$
$$= W + 0 \qquad [T_1 = T_2 \qquad (1\ 24)$$

In the case of a perfect gas constant temperature expansion is same as hyperbolic expansion as shown below.

(iv) Hyperbolic (PV=constant). The hyperbolic expansion is on e in which pressure multiplied by volume remains constant

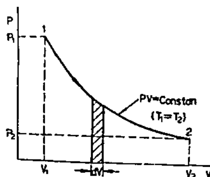


Fig. 1 8. Hyperbolic expansion.

during the whole of expansion. The curve for this expansion is a rectangular hyperbola and hence this is known as hyperbolic expansion.

For a perfect gas, $\frac{PV}{T}$ =constant. Therefore, where PV=constant, T is also a constant. Thus for a perfect gas hyperbolic expansion is also a constant temperature expansion.

In Fig. (1·8) curve 1-2 represents the hyperbolic expansion from initial volume V_1 and pressure P_1 to final volume V_2 and pressure P_2. Consider any point on the expansion curve. Let P and V represent the pressure and volume at this point. Consider a small change of volume dV to occur at this pressure P.

Thus work done during the small change in volume

$$=\text{area of shaded vertical strip}=PdV$$

$\therefore$ Total work done, W=area under the curve 1-2

$$=\int_{V_1}^{V_2} PdV$$

But, $\quad PV=P_1V_1=P_2V_2=\text{constant}, \therefore P=\frac{P_1V_1}{V}$

Hence,

$$W=\int_{V_1}^{V_2} \frac{P_1V_1}{V}\,dV$$

$$=P_1V_1 \log_e \frac{V_2}{V_1}$$

$$=\frac{P_1V_1}{J} \log_e r \quad \text{heat units.} \quad [1{\cdot}25(a)]$$

where $\quad r$=ratio of expansion

and $\quad Q=W+(U_2-U_1)$

$$=\frac{P_1V_1}{J} \log_e r+O \quad [1{\cdot}25(b)]$$

(v) **Polytropic expansion (PV^n=C).** This is the expansion commonly met in practice. The curve represented by this expansion on P-V diagram follows the law PV^n=constant, where n is called the *index of expansion*. The expansion curve on P-V diagram is shown in Fig. 1·9. The work done in this case may be found in the same way as in hyperbolic expansion.

Work done, $\quad W$=area under curve 1-2

$$=\int_{V_1}^{V_2} PdV$$

But, $PV^n = P_1V_1^n = P_2V_2^n = \text{constant}$

$\therefore \quad P = \dfrac{P_1V_1^n}{V^n}$

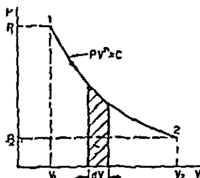


Fig. 1·9. Polytropic expansion.

Hence,
$$W = \int_{V_1}^{V_2} \frac{P_1V_1^n}{V^n} dV = P_1V_1^n \left[\frac{V^{1-n}}{1-n}\right]_{V_1}^{V_2}$$
$$= \frac{P_1V_1}{1-n}\left[V_2^{1-n} - V_1^{1-n}\right]$$

As $P_1V_1^n = P_2V_2^n$, multiplying first term inside the bracket by $P_2V_2^n$ and the second term by $P_1V_1^n$ we get

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$
$$= \frac{P_1V_1 - P_2V_2}{J(n-1)} \quad \text{heat units} \qquad [1\cdot26(a)]$$

For a perfect gas, $PV = mRT$

$$W = \frac{mR\,(T_1 - T_2)}{J(n-1)} \qquad [1\cdot26(b)]$$

and
$$Q = W + (U_2 - U_1)$$
$$= \frac{mR\,(T_1 - T_2)}{J(n-1)} + mC_v(T_2 - T_1) \qquad (1\cdot27)$$

Relations Between P, V and T for Polytropic Process. For any gas, relation between P, V and T for polytropic process can be found from equation

$PV^n = \text{constant}$ and $\dfrac{PV}{T} = \text{constant.}$

Let suffix 1 and 2 represent the initial and final conditions respectively.

(a) *Relation between V and T*

$P_1V_1^n = P_2V_2^n$ or $\dfrac{P_1}{P_2} = \left(\dfrac{V_2}{V_1}\right)^n$

Also $$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

or $$\frac{P_1}{P_2}=\frac{V_2}{V_1}\times\frac{T_1}{T_2}$$

Equating the two values of $\left(\frac{P_1}{P_2}\right)$

$$\frac{V_2}{V_1}\times\frac{T_1}{T_2}=\left(\frac{V_2}{V_1}\right)^n$$

Hence, $$\frac{T_1}{T_2}=\left(\frac{V_2}{V_1}\right)^{n-1}=r^{n-1} \qquad (1\cdot28)$$

(*b*) *Relation between P and V*

$$P_1V_1^n=P_2V_2^n \quad \text{or} \quad \frac{V_1}{V_2}=\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

Also $$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \quad \text{or} \quad \frac{V_1}{V_2}=\frac{P_2}{P_1}\times\frac{T_1}{T_2}$$

Equating the two values of $\left(\frac{V_1}{V_2}\right)$

$$\frac{P_2}{P_1}\times\frac{T_1}{T_2}=\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

hence, $$\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}} \qquad (1\cdot29)$$

In words Eqs. (1·28) and (1·29) can be put as

$$\frac{\text{Initial temperature}}{\text{Final temperature}}=\left(\frac{\text{Final volume}}{\text{Initial volume}}\right)^{n-1}=\left(\frac{\text{Initial pressure}}{\text{Final pressure}}\right)^{\frac{n-1}{n}}$$

(vi) Adiabatic Expansion (PV^γ=Constant). An adiabatic expansion is one in which while doing work no heat is supplied or rejected to the surroundings *i.e.* $Q=0$. In this type of expansion the work is done at the expense of its own internal energy (*see* Fig. 1·10). Therefore the temperature will fall during an adiabatic expansion and will rise during an adiabatic compression. The adiabatic expansion follows the law PV^γ=constant, with the value of index n equal to γ, the ratio of two specific heats. This can be proved as follows.

$$Q = W + (U_2 - U_1)$$

$Q = O \quad \therefore \qquad O = \frac{mR\,(T_1 - T_2)}{J\,(n-1)} + m\,C_v\,(T_2 - T_1)$

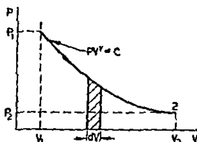


Fig. 1·10. Adiabatic expansion.

Substituting $\frac{R}{J} = C_p - C_v$, and rearranging we get,

$$\frac{C_p - C_v}{n-1} = C_v$$

or $\qquad C_p - C_v = n\,C_v - C_v$

$\therefore \qquad n = \frac{C_p}{C_v} = \gamma$

Thus the equation for adiabatic expansion becomes PV^γ = constant. Fig. 1·10 shows the adiabatic expansion on $P.V$ diagram. Work done in adiabatic expansion is obtained by putting $n = \gamma$ in case of polytropic expansion.

$$\therefore \qquad W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{J(\gamma - 1)}$$

$$= m\,C_v\,(T_2 - T_1) \text{ heat units} \qquad (1\cdot30)$$

Note. In proving $n = \gamma$ assumption was made that the expansion follows the relation PV^n = constant, but this involves empirical assumption and hence the proof cannot be called a rigorous one. The rigorous proof of $n = \gamma$ is as follows :

$$Q = W + (U_2 - U_1)$$

In an adiabatic expansion let a small change in temperature dT cause a small change in volume dV. As no heat interchange takes place, $\qquad Q = O$

$$O = \frac{PdV}{J} + mC_v\,dT \qquad (i)$$

Differentiating the characteristic gas equation, $PV=mRT$

we get, $PdV+VdP=mR\,dT$ (*ii*)

From Eqs. (*i*) and (*ii*)

$$O=\frac{PdV}{J}+\frac{C_v\,(PdV+VdP)}{R}$$

$$=(\gamma-1)PdV+PdV+VdP$$

$$=\gamma PdV+VdP$$

Integrating, $\log_e P+\gamma\log_e V=C_1$

where C_1=constant of integration

Hence $PV^\gamma=C$, where C is another constant.

(*vii*) **Free Expansion.** A free expansion is one in which fluid expands suddenly into a vacuum chamber through an orifice of large

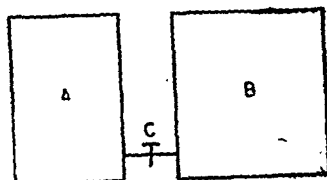


Fig. 1·11. Free expansion.

size. As an example, let there be two chambers A and B connected by a valve C, the chamber B having vacuum. The arrangement is shown in Fig. 1·11. When the valve is opened the gas rushes from A to B with a large velocity and finally is brought to rest by striking the walls of the vessels and by the friction of eddy currents thus formed. The final temperature of the gas is same as the initial temperature in A. Though the expansion of gas through valve C causes fall in temperature, the friction converts the kinetic energy into heat which increases the temperature to the original temperature. In this process as no external heat has been supplied and no external work has been done there is no change in internal energy. The enthalpy of the fluid remains constant during the process.

$$Q=W+(U_2-U_1)$$

$\therefore$ $$O=O+mC_v\,(T_2-T_1)$$

or $$T_2=T_1 \qquad (1\cdot31)$$

The free expansion differs from isothermal expansion in having no external work. For the same reason it is not a true adiabatic process. The free expansion is an irreverisble process and cannot be truely represented on P-V diagram. The work done in this process is not equal to $\int PdV$.

1·12. Comparison of Work Done During Various Non-flow Processes. Fig. 1·12 shows the various non-flow processes on *P-V* diagram. Constant pressure process ($PV^0=C$) is a horizontal

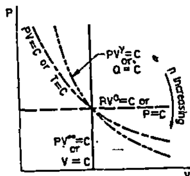


Fig. 1·12. *P-V* diagram for various non-flow processes (Gaseous region).

line and constant volume process ($PV^{\infty}=C$) is a vertical line. All other expansion curves lie between these two lines. It is seen that during expansion, for the same increase in volume pressure drop is greater as the value of n increases. Similarly during compression, for the same decrease in volume pressure rise is greater as the value of n increases. As the area under the curves represent the work done it is seen that for the same expansion ratio work done by the gas diminishes as n increases from 0 to γ.

Table 1·1 gives the types of expansion and their properties. All these are *fully resisted* non-flow expansions.

1·13. Other Expression for Work, Heat and Enthalpy for Polytropic Process.

(*i*) *Work done* $=\dfrac{P_1V_1}{n-1}\left[1-\dfrac{1}{r^{n-1}}\right]$

Proof:

From Eq. 1·26 (*a*), $W=\dfrac{P_1V_1-P_2V_2}{n-1}$

$$=\frac{P_1V_1}{n-1}\left[1-\frac{P_2V_2}{P_1V_1}\right]$$

Now, $P_1V_1^n=P_2V_2^n$

$$\therefore \quad \frac{P_2}{P_1}\times\frac{V_2}{V_1}=\left(\frac{V_1}{V_2}\right)^n\times\frac{V_2}{V_1}=\frac{1}{r^{n-1}}$$

Substituting,

$$W=\frac{P_1V_1}{n-1}\left[1-\frac{1}{r^{n-1}}\right] \qquad (1\cdot32)$$

(ii) *Heat supplied* $= \dfrac{\gamma - n}{\gamma - 1} \times$ work done

Proof :
$$Q = \frac{P_1V_1 - P_2V_2}{J(n-1)} + mC_v(T_2 - T_1)$$
$$= \frac{P_1V_1 - P_2V_2}{J(n-1)} + \frac{mR(T_2 - T_1)}{J(\gamma - 1)}$$

As
$$C_v = \frac{R}{J(\gamma - 1)} \quad PV = mRT$$

$\therefore$
$$Q = \frac{P_1V_1 - P_2V_2}{J(n-1)} + \frac{P_2V_2 - P_1V_1}{J(\gamma - 1)}$$
$$= \frac{P_1V_1 - P_2V_2}{J(n-1)}\left[1 - \frac{n-1}{\gamma - 1}\right]$$
$$= \frac{\gamma - n}{\gamma - 1} \times \text{work done} \qquad [1{\cdot}33\,(a)]$$

Differentiating,
$$dQ = \frac{\gamma - n}{\gamma - 1} \times PdV$$

$\therefore$ Rate of heat interchange per unit change of volume
$$\frac{dQ}{dV} = \frac{\gamma - n}{\gamma - 1} \times P \qquad [1{\cdot}33(b)]$$

(iii) *Change in enthalpy* $= \dfrac{\gamma(P_2V_2 - P_1V_1)}{J(\gamma - 1)}$

Proof :
$$U_2 - U_1 = mC_v(T_2 - T_1)$$

Also
$$PV = mRT$$

$\therefore$
$$U_2 - U_1 = C_v\left(\frac{P_2V_2}{R} - \frac{P_1V_1}{R}\right) \qquad (i)$$

Now
$$C_p - C_v = \frac{R}{J}$$

Dividing by C_v and rearranging,
$$\frac{C_v}{R} = \frac{1}{J(\gamma - 1)} \qquad (ii)$$

From Eqs. (i) and (ii)
$$U_2 - U_1 = \frac{1}{J(\gamma - 1)}(P_2V_2 - P_1V_1)$$

Enthalpy,
$$H = \frac{PV}{J} + U$$

$\therefore$ Change in enthalpy
$$H_2 - H_1 = \frac{P_2V_2}{J} - \frac{P_1V_1}{J} + (U_2 - U_1)$$

$$= \frac{P_2V_2 - P_1V_1}{J} + \frac{P_2V_2 - P_1V_1}{J(\gamma - 1)}$$

$$= \frac{(P_2V_2 - P_1V_1)(\gamma - 1) + (P_2V_2 - P_1V_1)}{J(\gamma - 1)}$$

$$= \frac{\gamma(P_2V_2 - P_1V_1)}{J(\gamma - 1)} \qquad (1\cdot34)$$

1-15. The Steady-flow Process. In a flow process the fluid enters the system and leaves after doing work. This implies open boundary which permits the flow of the matter to and from the system In steady-flow processes the analysis is made by referring to an imaginery fixed region of space called a *control volume* through which the working substance flows. Typical engineering examples of steady-flow are in steam, gas and water turbine plants and air-conditioning plants. Fig. 1·13 shows the steady-flow process in a system.

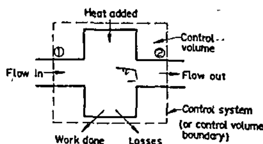


Fig. 1·13. Steady-flow system.

The following are the conditions for the steady-flow process;

1. The composition, state and velocity of the streams crossing the control volume boundaries do not change with time at entrance and exit.

2. The state of the fluid at any point within the control volume is same at all times.

3. The mass rate of flow into the control volume is constant and is equal to the mass rate of flow out of it.

4. The rate at which heat and work cross the boundary is constant.

Referring to Fig. 1·13 let the fresh fluid of mass dm_1 enter the system at 1 and fluid of mass dm_2 leaves the system at 2. Applying the first law of thermodynamics with proper signs in the process

$$Q = W + (E_2 - E_1)$$

Now $$W = W_x - P_1 v_1 dm_1 + P_2 v_2 dm_2$$

where W_x is the external work done and $P_1 v_1 dm_1$ and $P_2 v_2 dm_2$ are displacement work at entry and exit respectively. Change of energy $= e_2 dm_2 - e_1 dm_1$, the changes in the entry and exit pipes (as the changes within the control volume are nil being cyclic).

$$\therefore \quad Q = W_x - P_1 v_1 dm_1 + P_2 v_2 dm_2 + e_2 dm_2 - e_1 dm_1$$

Now, by conservation of mass principle $dm_2 = dm_1 = dm$ (say) and substituting the value of e equal to $u + \dfrac{V^2}{2g} + Z$, as already defined,

$$Q - W_x = dm\left(u_2 + P_2 v_2 + \frac{V_2^2}{2g} + Z_2 - u_1 - P_1 v_1 - \frac{V_1^2}{2g} - Z_1 \right)$$

$$= dm\left[(u_2 - u_1) + (P_2 v_2 + P_1 v_1) + (Z_2 - Z_1) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) \right] \tag{1·34}$$

$$= dm\left(h_2 + \frac{V_2^2}{2g} + Z_2 - h_1 - \frac{V_1^2}{2g} - Z_1 \right)$$

$\therefore$ For unit mass,

$$q - w_x = \Delta\left(h + \frac{V^2}{2g} + Z \right) \tag{1·35}$$

The above equation is called the *steady-flow energy equation* and is abbreviated as *SFEE*.

If the effect of gravity is neglected, (*i.e.* $Z_1 = Z_2$) which is common in thermodynamics, Eq. (1·35) reduces to

$$q - w_x = (h_2 - h_1) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) \tag{1·36}$$

If $V_1 = V_2$, Eq. (1·36), reduces to

$$q - w_x = h_2 - h_1 \tag{1·37}$$

In a non-flow process work of introduction and rejection is zero, $h_2 - h_1 = u_2 - u_1$ and net work done is w

$$\therefore \quad q - w = u_2 - u_1$$

which is same as Eq. (1·6).

(i) Throttling Process. (No heat and work transfer). A throttling expansion is one in which a gas expands through a minute aperture such as a narrow throat, slightly opened valve or a crack. Due to fall in pressure the gas should come out with large velocity but due to friction between the gas and the valve the kinetic energy is converted into heat which warms up the gas to its initial temperature. The net enthalpy of the gas remains constant. Neglecting changes in V and Z

$$q-w_x=\Delta\left(h+\frac{\bar{V}^2}{2g}+Z\right)$$

Substituting, $O-O=h_2-h_1 \quad \therefore \quad h_2=h_1$ and $T_2=T_1$ (1·38)

Throttling process is not a reversible adiabatic process.

As in free expansion, in throttling the enthalpy remains constant. The difference in the free expansion and throttling is that in the former case the gas leaves with large velocity whereas in the latter case the gas has negligible velocity.

(ii) Adiabatic Process (No heat transfer). As already stated an adiabatic expansion is one in which no heat is supplied or rejected *i.e.* $Q=O$. Adiabatic expansion as applied to a non-flow process is dealt in article 1·12.

(*a*) After neglecting the effect of gravity the steady flow energy equation for an adiabatic process reduces to

$$w_x=(h_1-h_2)+\left(\frac{\bar{V}_1^2}{2g}-\frac{\bar{V}_2^2}{2g}\right) \qquad (1·39)$$

The above equation is applicable to *rotary machines* like turbine and compressor plants in which the amount of heat flow is negligible compared to the other quantities.

(*b*) In *reciprocating machines* the flow velocities are small and kinetic energy terms can be neglected and the *SFEE* equation reduces to

$$w_x=h_1-h_2 \qquad (1·40)$$

(*c*) In case of flow of fluid in *nozzles* the heat flow is assumed to be zero and also the external work done is zero and Eq (1·39), therefore, reduces to

$$h_1-h_2=\frac{\bar{V}_2^2}{2g}-\frac{\bar{V}_1^2}{2g} \qquad (1·41)$$

Thus considering an adiabatic flow of fluid in nozzles it can be stated that the increase in kinetic energy of the fluid is equal to the

enthalpy drop between any two points. The converse of the statement is also true and takes place in *diffusers*.

(iii) Processes Involving Heat Transfer But No Work Transfer. In equipment like boilers, superheaters, condensers and other heat exchanger equipment, work transfer is zero. The SFEE equation for such equipment, neglecting kinetic and potential energy terms, reduces to

$$q=h_2-h_1. \tag{1·42}$$

IMPORTANT POINTS

1. The beginner generally commits mistakes in substituting the values of temperature and pressures. All problems should be worked with absolute units.

Absolute temperature in $°K=273+t$ (*see* Fig. 1·14).

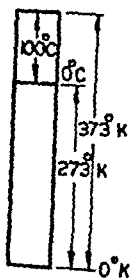


Fig. 1·14.

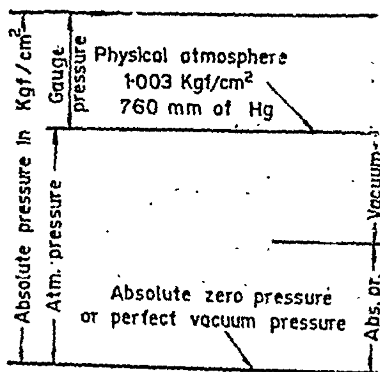


Fig. 1·15.

2. Absolute pressure=gauge pressure+atmospheric pressure (Fig. 1·15). In case of pressures below atmospheric generally vacuum is given.

Absolute pressure=atmospheric pressure—vacuum.

In problems given pressure is to be taken as absolute unless otherwise mentioned.

3. In the characteristic gas equation $PV=mRT$, P must be substituted in kgf/m².

4. It should be carefully noted whether the problem specifies "one kg of gas". If not, mass should be invariably calculated and all the quantities *i.e.*, work done, change in internal energy, heat interchange, enthalpy, etc., should be found for the calculated mass.

If the expression for work done or any other quantity is in terms of PV, like $\frac{P_1V_1-P_2V_2}{n-1}$ or $P_1V_1 \log_e r$, mass is automatically included in it because $PV=mRT$. But the expressions which are in term of T do not include mass and should, therefore, be multiplied by m, such as $U=mC_v(T_2-T_1)$.

5. It should be remembered that heat supplied *to* the gas, work done *by* the gas and *increase* in internal energy are taken as positive quantities, whereas heat rejected *by* the gas, work done *on* the gas and *decrease* in internal energy are taken as negative quantities.

Expressions for work done, internal energy and heat interchange should be used correctly. The signs should not be changed simply to get "positive" results.

6. In the equation $Q=W+\Delta E$, Q, W and ΔE should all be in heat units.

7. In isothermal process change in internal energy is zero In adiabatic process heat interchange is zero. With these exceptions, in all other expansions work is done by the gas (+*ve*), heat flows in the gas (+*ve*) and internal energy is reduced (−*ve*) and in compression work is done on the gas (−*ve*), heat flows out of the gas (−*ve*) and internal energy is increased (+*ve*) provided n is less than γ.

8. Wherever work done is calculated the result can be checked by

$$Q=W+\Delta E$$

provided it is a non-flow process.

9. It should be clearly understood that change in enthalpy dH and heat interchange Q are altogether different quantities. Only in one solitary case of constant pressure process dH is equal to Q.

10. *In problems where simply 'adiabatic' is mentioned, it is to be taken as true or reversible adiabatic.*

11. Table 1·1 should be studied thoroughly before proceeding further.

ILLUSTRATIVE EXAMPLES

1·1. Heat interchange for different laws of compression.

A perfect gas for which the ratio of specific heats is 1·4 occupies a volume of 0·3 m³ at 1·05 kgf/cm² and 27°C. The gas undergoes a compression to 0·06 m³. Find the heat absorbed or rejected by the gas for

each of the following methods of compression, (i) constant pressure, (ii) isothermal, (iii) hyperbolic, (iv) adiabatic, and (v) according to the law $PV^{1.1}$=constant. For gas $R=29{\cdot}02$ kgf m/kg/°K.

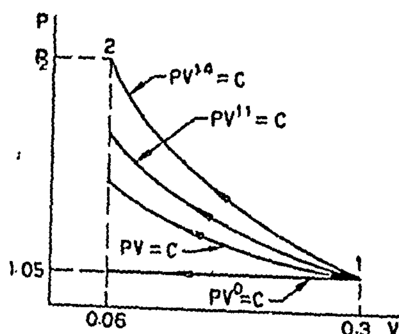


Fig. 1·16.

$$C_p - C_v = \frac{R}{J}$$

$$\therefore \quad C_v = \frac{R}{J(\gamma-1)} = \frac{29{\cdot}02}{427(1{\cdot}4-1)} = 0{\cdot}17$$

$PV=mRT$, $1{\cdot}05\times10^4\times0{\cdot}3=m\times29{\cdot}02\times300$

$$\therefore \quad m=0{\cdot}3618 \text{ kg}$$

(i) *Constant pressure*

$PV=mRT$, $\quad 1{\cdot}05\times10^4\times0{\cdot}06=0{\cdot}3618\times29{\cdot}02\times T_2$

$$\therefore \quad T_2=60°K$$

$$W=\frac{P(V_2-V_1)}{J}=\frac{1{\cdot}05\times10^4(0{\cdot}06-0{\cdot}3)}{427}$$

$$=-5{\cdot}9 \text{ kcal}$$

$$U_2-U_1=mC_v(T_2-T_1)=0{\cdot}3618\times0{\cdot}17(60-300)$$

$$=-14{\cdot}76 \text{ kcal}$$

$$\therefore \quad Q=W+(U_2-U_1)$$

$$=-5{\cdot}9-14{\cdot}76=-20{\cdot}66 \text{ k cal}$$

Ans.

[check $Q=m\,C_p(T_2-T_1)=0{\cdot}3618\times0{\cdot}17(60-300)=-20{\cdot}66$ kcal]

(ii) *Isothermal*

$$W=\frac{PV}{J}\log_e r=\frac{1{\cdot}05\times10^4\times0{\cdot}3}{427}\log_e\frac{0{\cdot}06}{0{\cdot}3}$$

$$=-11{\cdot}88 \text{ kcal.}$$

$$U_2-U_1=mC_v(T_2-T_1)=0$$

$[T_2=T_1$

$\therefore \quad Q = W + (U_2 - U_1) \quad = -11{\cdot}88 + 0 = \underline{-11{\cdot}88 \text{ kcal}}$ Ans.

(*ii*) *Hyperbolic.*

As the gas is a perfect gas, hyperbolic process will yield the same results as in isothermal process.

(*iv*) *Adiabatic*

$$P_1 V_1{}^{\gamma} = P_2 V_2{}^{\gamma}$$

$\therefore \quad 1{\cdot}05 \times 0{\cdot}3^{1\cdot4} = P_2 \times 0{\cdot}06^{1\cdot4} \quad P_2 = 10 \text{ kgf/cm}^2$

$PV = mRT,\ 10 \times 10^4 \times 0{\cdot}06 = 0{\cdot}3618 \times 29{\cdot}02 \times T_2$

$\therefore \quad T_2 = 571{\cdot}4°K$

$$W = \frac{P_1V_1 - P_2V_2}{J(\gamma - 1)} = \frac{10^4(1{\cdot}05 \times 0{\cdot}3 - 10 \times 0{\cdot}06)}{427(1{\cdot}4 - 1)}$$

$$= -16{\cdot}69 \text{ kcal}$$

$$U_2 - U_1 = mC_v(T_2 - T_1)$$

$$= 0{\cdot}3618 \times 0{\cdot}17(571{\cdot}4 - 300)$$

$$= +16{\cdot}69 \text{ kcal}$$

$\underline{Q} = W + (U_2 - U_1) = -16{\cdot}69 + 16{\cdot}69 = 0$ Ans.

[In adiabatic process heat interchange $Q = 0$]

(*v*) *According to the law* $PV^{1\cdot1} = C$

$P_1 V_1{}^n = P_2 V_2{}^n$

$\therefore \quad 1{\cdot}05 \times 0{\cdot}3^{1\cdot1} = P_2 \times 0{\cdot}06^{1\cdot1} \quad P_2 = 6{\cdot}167 \text{ kgf/cm}^2$

$$T_2 = T_1 \times r^{n-1} = 300 \times \left(\frac{0{\cdot}3}{0{\cdot}06}\right)^{1\cdot1-1} = 352{\cdot}5°K$$

$$W = \frac{P_1V_1 - P_2V_2}{J(n-1)}$$

$$= \frac{10^4(1{\cdot}05 \times 0{\cdot}3 - 6{\cdot}167 \times 0{\cdot}06)}{427 \times (1{\cdot}1 - 1)}$$

$$= -12{\cdot}9 \text{ kcal}$$

$U_2 - U_1 = mC_v(T_2 - T_1)$

$= 0{\cdot}3618 \times 0{\cdot}17(352{\cdot}5 - 300) = +3{\cdot}23 \text{ kcal}$

$\underline{Q} = W + (U_2 - U_1) = -12{\cdot}9 + 3{\cdot}23 = \underline{-9{\cdot}67 \text{ kcal}}$ Ans.

Note. (*i*) −ve sign indicates loss or rejection of heat

(*ii*) The work required for compression increases with the increase in index of compression 'n'.

1·2. Polytropic expansion : change in internal energy and enthalpy.

Prove that for any heat engine process when a gas is the working substance the change in internal energy is given by

$\frac{1}{J(\gamma-1)}(P_2V_2-P_1V_1)$ *and the change in enthalpy by*

$$\frac{\gamma}{J(\gamma-1)}(P_2V_2-P_1V_1)$$

Calculate these quantities for 0·5 kg of air expanding according to the law $PV^{1.2}=C$ from 10 kgf/cm², [300°C to 1 kgf/cm². What will be work done by air during the expansion ? $C_p=0.24$, $C_v=0.171$.

For theory—see text.

$$\gamma=\frac{C_p}{C_v}=\frac{0.24}{0.171}=1.4$$

$$C_p-C_v=\frac{R}{J}, \quad 0.24-0.171=\frac{R}{427} \quad \therefore \quad R=29.45$$

$$PV=mRT, \quad 10\times10^4\times V_1=0.5\times29.45\times573$$

$$\therefore \quad V_1=0.08435 \ m^3$$

$$P_1V_1^n=P_2V_2^n, \quad 10\times0.08435^{1.2}=1\times V_2^{1.2}$$

$$\therefore \quad V_2=0.5746 \ m^3$$

Change in internal energy

$$\underline{U_2-U_1}=\frac{1}{J(\gamma-1)}(P_2V_2-P_1V_1)$$

$$=\frac{1\times10^4}{427(1.4-1)}(1\times0.5746-10\times0.08435)$$

$$=\underline{-15.75 \text{ kcal}} \qquad \text{Ans.}$$

(—ve sign indicates decrease in internal energy)

Change in enthalpy

$$\underline{H_2-H_1}=\frac{\gamma(P_2V_2-P_1V_1)}{J(\gamma-1)}$$

$$=1.4\times(-15.75)$$

$$=\underline{-22.05 \text{ kcal}} \qquad \text{Ans.}$$

$$\text{Work done, } \underline{W}=\frac{P_1V_1-P_2V_2}{J(n-1)}$$

$$=\frac{10^4(10\times0.08435-1\times0.5746)}{427(1.2-1)}$$

$$=\underline{31.5 \text{ kcal}} \qquad \text{Ans.}$$

Work is done by the gas and is therefore +ve.

Note. Change in total heat, H, is not equal to heat supplied or rejected, Q, except in constant pressure process. The difference between H and Q should be carefully noted. In this question heat supplied is equal to

$$Q=W+(U_2-U_1)$$

$$=31.5-15.75=+15.75.$$

1-3. Polytropic expansion : Q ; change in enthalpy.

State the first law of thermodynamics.

A volume of 0 2 m³ of gas at 200°C and 10 kgf/cm² is expanded according to the law PV^n=constant, until the volume becomes 0 8 m³ at 20°C. Determine the amount of heat received or rejected and change in enthalpy, given C_p=0 24 and C_v=0·171.

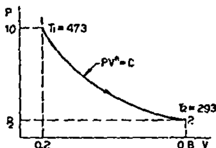


Fig. 1·17.

$$C_p - C_v = \frac{R}{J}$$

$$0{\cdot}24 - 0{\cdot}171 = \frac{R}{427} \quad \therefore \quad R = 29\ 45$$

$$PV = mRT,$$

$$10 \times 10^4 \times 0{\cdot}2 = m \times 29\ 45 \times 473$$

$$\therefore \quad m = 1{\cdot}436 \text{ kg}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}, \quad \frac{473}{293} = \left(\frac{0\ 8}{0{\cdot}2}\right)^{n-1} \quad \therefore \quad n = 1{\cdot}345$$

$$PV = mRT,$$

$$P_2 \times 10^4 \times 0\ 8 = 1{\cdot}436 \times 29{\cdot}46 \times 293$$

$$\therefore \quad P_2 = 1{\cdot}549 \text{ kgf/cm}^2$$

$$W = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{10^4(10 \times 0{\cdot}2 - 1\ 549 \times 0\ 8)}{1{\cdot}345 - 1}$$

$$= 22{,}060 \text{ kgfm or } 51{\cdot}7 \text{ kcal}$$

$$U_2 - U_1 = m\ C_v\ (T_2 - T_1)$$

$$= 1{\cdot}436 \times 0{\cdot}171\ (293 - 473) = -43{\cdot}7 \text{ kcal}$$

$$Q = W + (U_2 - U_1) = 51\ 7 - 43\ 7$$

$$= \underline{8 \text{ kcal}} \qquad \text{(received)} \quad \text{Ans.}$$

$$\underline{H_2 - H_1} = \left(U_2 + \frac{P_2V_2}{J}\right) - \left(U_1 - \frac{P_1V_1}{J}\right)$$

$$= (U_2 - U_1) + \frac{1}{J}\ (P_2V_2 - P_1V_1)$$

$$=-43\cdot7+\frac{10^4}{427}(1\cdot549\times0\cdot8-10\times0\cdot2)$$

$$=\underline{-61\cdot5 \text{ kcal}}$$ Ans.

[Check : $H_2-H_1=mC_p(T_2-T_1)$

$$=1\cdot436\times0\cdot24(293-473)=-61\cdot5 \text{ kcal}].$$

1-4. Combination of flow and non-flow processes : ΔU ; ΔH ; T_3.

Distinguish between internal energy and enthalpy of a fluid.

A volume of 115 litres of air at 1·05 kgf/cm² and 90°C is compressed adiabatically (PV^γ =constant) until the volume is reduced to 11·5 litres. Find the change in internal energy and change in enthalpy. If 0·091 kg of air from a source kept at 180°C is allowed to flow into this space of 11·5 litres, find the temperature and pressure at the end of the operation. Given C_v=0·171 and R/J=0·0685.

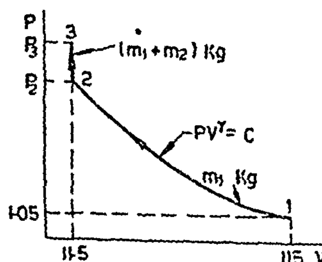


Fig. 1·18.

For theory—see text.

$$C_p-C_v=\frac{R}{J}$$

$$\therefore \quad C_p-0\cdot171=0\cdot0685 \qquad \therefore \; C_p=0\cdot2395$$

$$\gamma=\frac{C_p}{C_v}=\frac{0\cdot2395}{0\cdot171}=1\cdot4$$

$$PV=mRT,\; 1\cdot05\times10^4\times115\times10^{-3}=m\times0\cdot0685\times427\times363$$

$$\therefore \qquad m=0\cdot1138 \text{ kg}$$

$$\frac{T_2}{T_1}=r^{\gamma-1},\; \frac{T_2}{363}=\left(\frac{115}{11\cdot5}\right)^{1\cdot4-1} \qquad \therefore \; T_2=912°K$$

$$\therefore \qquad \underline{U_2-U_1}=mC_v(T_2-T_1)$$

$$=0\cdot1138\times0\cdot171(912-363)$$

$$=\underline{10\cdot69 \text{ kcal.}}$$ Ans.

and $\underline{H_2 - H_1} = mC_p(T_2 - T_1)$

$= 0{\cdot}1138 \times 0{\cdot}2395\,(912 - 363)$

$= \underline{14{\cdot}97 \text{ kcal}}$ Ans.

The last part involves combination of non-flow and flow processes. As volume is constant no external work is done. It may be noted that the mass at the points 2 and 3 are different.

Internal energy at 2+enthalpy of new air=Internal energy at 3.

Taking absolute zero as datum for internal energy,

$$m_1C_vT_2 + m_2C_pT = (m_1 + m_2)C_vT_3$$

$0{\cdot}1138 \times 0{\cdot}171 \times 912 + 0{\cdot}091 \times 0{\cdot}2395 \times 453$

$= (0{\cdot}1138 + 0{\cdot}091) \times 0{\cdot}171 \times T_3$

$\therefore$ $\underline{T_3 = 790°K \text{ or } 517°C}$ Ans.

$PV = mRT$

$P_3 \times 10^4 \times 11{\cdot}5 \times 10^{-3} = 0{\cdot}2048 \times 0{\cdot}0685 \times 427 \times 790$

$\therefore$ $\underline{P_3 = 41{\cdot}1 \text{ kgf/cm}^2}$ Ans.

1-5. Polytropic compression : m ; T_{end} ; n ; ΔU ; Q.

A cylinder contains 168 litres of gas at a pressure 1 kgf/cm² and temperature 47°C. If this gas is compressed to 1/12th of its volume and the pressure is then 21 kgf/cm², find (a) the mass of the air, (b) the temperature at the end of compression, (c) the index of compression, (d) the change in internal energy, and (e) the heat rejected during compression. Take $C_p = 0{\cdot}26$ and $C_v = 0{\cdot}2$.

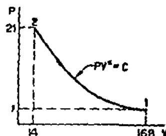


Fig 1·19.

(a) $C_p - C_v = \dfrac{R}{J}$, $0{\cdot}26 - 0{\cdot}2 = \dfrac{R}{427}$ $\therefore$ $R = 25{\cdot}6$

Mass of air, $\underline{m} = \dfrac{PV}{RT} = \dfrac{1 \times 10^4 \times 168 \times 10^{-3}}{25{\cdot}6 \times 320} = \underline{0{\cdot}2051 \text{ kg}}$ Ans.

(*b*) Volume after compression

$$V_2=\frac{168}{12}=14 \text{ litres}$$

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}, \quad \frac{1\times168}{320}=\frac{21\times14}{T_2} \quad \therefore \quad \underline{T_2=560°K} \quad \text{Ans.}$$

(*c*) $P_1V_1^n=P_2V_2^n$

$\therefore$ $1\times168^n=21\times14^n$ $\therefore$ $n=\underline{1{\cdot}226.}$ Ans.

(*d*)
$$\underline{U_2-U_1}=mC_v(T_2-T_1)$$
$$=0{\cdot}2051\times0{\cdot}2(560-320)$$
$$=\underline{9{\cdot}845 \text{ kcal}} \quad \text{Ans.}$$

(*e*)
$$W=\frac{P_1V_1-P_2V_2}{J(n-1)}$$
$$=\frac{10(1\times168-21\times14)}{427(1{\cdot}226-1)}$$
$$=-13{\cdot}07 \text{ kcal}$$

$\therefore$ Heat rejected, $Q=W+(U_2-U_1)$

$$=-13{\cdot}07+9{\cdot}845=\underline{-3{\cdot}225 \text{ kcal.}} \quad \text{Ans.}$$

(—ve sign indicates rejection of heat).

1-6. Polytropic expansion : m ; w ; heat interchange.

Prove that in a polytropic expansion the work done by the gas is equal to $\frac{P_1V_1}{n-1}\left(1-\frac{1}{r^{n-1}}\right)$ *and the heat interchange by the gas is equal to* $\frac{\gamma-n}{\gamma-1}\times$ *work done, where* P_1 *and* V_1 *are initial pressure and volume respectively, r is the expansion ratio, n is the index of the expansion and* γ *is the ratio of two specific heats. Hence, derive an expression for the rate of heat interchange per unit change of volume.*

60 litres of air at 70°C expands from 7 kgf/cm² to 1·05 kgf/cm² according to the law PV^n*=constant. The volume of air after expansion is 300 litres. Determine (a) the mass of the air, (b) the work done, and (c) the heat interchange. Given* $C_p=0{\cdot}238$ *and* $C_v=0{\cdot}169$*.*

For theory—see text.

(*a*)
$$\gamma=\frac{C_p}{C_v}=\frac{0{\cdot}238}{0{\cdot}169}=1{\cdot}41$$

$$C_p-C_v=\frac{R}{J}$$

$$0{\cdot}238-0{\cdot}169=\frac{R}{427} \quad \therefore \quad R=29{\cdot}45$$

$$PV = mRT$$

$$7\times10^4\times60\times10^{-3} = m\times29{\cdot}45\times343$$

$\therefore$ $\underline{m = 0\ 416 \text{ kg.}}$ Ans.

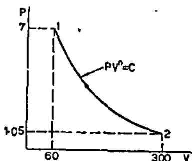


Fig. 1·20.

(b) $P_1V_1^n = P_2V_2^n$

$\therefore$ $7\times60^n = 1{\cdot}05\times300^n$, $\therefore$ $n = 1{\cdot}179$

$$W = \frac{P_1V_1 - P_2V_2}{J\ (n-1)}$$

$$= \frac{10\times(7\times60 - 1{\cdot}05\times300)}{427\times(1{\cdot}179-1)}$$

$$= \underline{13\ 75 \text{ kcal}}$$ Ans.

(c) Heat interchange

$$Q = \frac{\gamma - n}{\gamma - 1}\times\text{work done}$$

$$= \frac{1{\cdot}41 - 1{\cdot}179}{1{\cdot}41 - 1}\times13{\cdot}75$$

$$= \underline{+7{\cdot}745 \text{ kcal}}$$ Ans.

1·7. Adiabatic and constant volume process : specific heats.

Establish the relationship between the two specific heats and the characteristic gas constant for a perfect gas and hence prove

$$\frac{R}{JC_v} = \gamma - 1$$

A cylinder fitted with a movable piston contains hydrogen at a pressure and temperature of 3 5 kgf/cm² and 93°C respectively. The piston moves outward, no heat is gained or lost by gas during this process and at the end of expansion pressure is 0 7 kgf/cm². The piston is then fixed and heat is added until the gas temperature is 93°C, when the pressure is found to be 1 1 kgf/cm². Determine the specific heats of gas. The gas constant R = 424 kgf m/kg./°K.

For theory see text

$$C_p - C_v = \frac{R}{J} = \frac{424}{427} = 0{\cdot}9931 \tag{1}$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$$

Substituting, $$\frac{0{\cdot}7}{T_2} = \frac{1{\cdot}1}{366}$$

$\therefore$ $$T_2 = 233°K. \qquad [V_2 = V$$

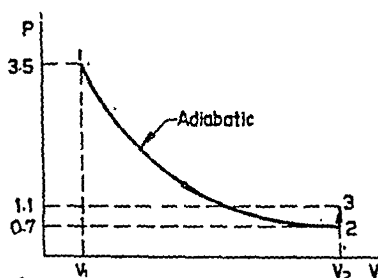


Fig. 1·21.

Substitutiug, $$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{366}{233} = \left(\frac{3{\cdot}5}{0{\cdot}7}\right)^{\frac{\gamma-1}{\gamma}}$$

$\therefore$ $$\gamma = 1{\cdot}392 = \frac{C_p}{C_v} \tag{2}$$

From equations (1) and (2)

$$\underline{C_p = 3{\cdot}526 \text{ and } C_v = 2{\cdot}533} \qquad \text{Ans.}$$

1·8. Work done in adiabatic compression.

(*a*) *Prove that the index γ in $PV^\gamma = C$ for the adiabatic expansion of a gas is the ratio of the specific heats at constant pressure and constant volume.*

(*b*) *A Diesel engine has a diameter of 17 cm and stroke of 30 cm. The compression ratio is 15. The pressure at the beginning of the compression stroke is 1 kgf/cm². Find the work done and change in internal energy during compression stroke, assuming it to be adiabatic. Given $C_p = 0{\cdot}238$ and $C_v = 0{\cdot}169$.*

$$\gamma = \frac{C_p}{C_v} = \frac{0{\cdot}238}{0{\cdot}169} = 1{\cdot}41$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$1\times 15^{1.41} = P_2\times 1^{1.41}$$
$$\therefore \quad P_2 = 45{\cdot}42 \text{ kgf/cm}^2$$

Stroke volume, $V_1 - V_2 = \frac{\pi}{4}\times(17)^2\times 30$

$$= 6{,}810 \text{ cc}$$

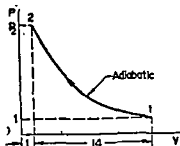


Fig. 1·22.

Clearance volume, $V_2 = \frac{V_1}{r} = \frac{6{,}810}{14} = 486$ cc

$\therefore$ Total volume of cylinder

$$V_1 = 6{,}810 + 486 = 7{,}296 \text{ cc}$$

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{10^4(1\times 7{,}296\times 10^{-6} - 45{\cdot}42\times 486\times 10^{-6})}{1{\cdot}41 - 1}$$

$$= -361{\cdot}4 \text{ kgf m or } -0{\cdot}8468 \text{ kcal} \qquad \text{Ans}$$

In adiabatic process, $Q=0$

$\therefore$ Change in internal energy

$$U_2 - U_1 = -W = 0{\cdot}8468 \text{ kcal} \qquad \text{Ans.}$$

Note. The work done on the gas is stored as internal energy.

1·9. Polytropic and constant volume process : n ; T_2 ; W and Q

An oil engine has a volume of 60 litres and a compression ratio of 14·2 to 1. At the beginning of the compression stroke the pressure and temperature are 1 kgf/cm² and 80°C respectively. At the end of compression the pressure is 30 kgf/cm². The charge is now heated at constant pressure until the volume is doubled. Find : (a) the index of compression, (b) the temperature at the end of compression, (c) the heat transfer, (d) the heat received in constant pressure operation. Assume $C_v = 0{\cdot}17$ and $R = 29{\cdot}9$ kgf m units.

(a) $P_1V_1^n = P_2V_2^n$

$\therefore$ $1 \times 14{\cdot}2^n = 30 \times 1^n$ $\therefore$ $n = 1{\cdot}281$ Ans.

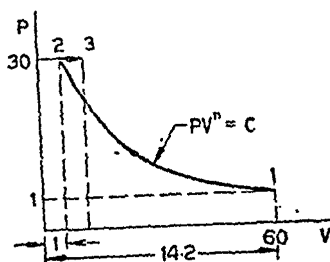


Fig. 1·23.

(b) $\frac{T_2}{T_1} = r^{n-1}$

$\frac{T_2}{353} = 14{\cdot}2^{(1.281-1)}$ $\therefore$ $T_2 = 744°K$ or $471°C$ Ans.

(c) $W = \frac{P_1V_1 - P_2V_2}{J(n-1)}$

$= \frac{P_1V_1(1 - r^{n-1})}{J(n-1)}$

$= \frac{1 \times 10^4 \times 60 \times 10^{-3}[1 - 14{\cdot}2^{(1.281-1)}]}{427(1{\cdot}281 - 1)}$

$= -5{\cdot}55$ kcal

$PV = mRT$

$\therefore$ $1 \times 10^4 \times 60 \times 10^{-3} = m \times 29{\cdot}9 \times 353$ $\therefore$ $m = 0{\cdot}0568$ kg

$U_2 - U_1 = mC_v(T_2 - T_1)$

$= 0{\cdot}0568 \times 0{\cdot}17(744 - 353) = 3{\cdot}78$ kcal

$Q = W + (U_2 - U_1) = -5{\cdot}55 + 3{\cdot}78 = -1{\cdot}77$ kcal Ans.

(—ve sign indicates rejection of heat)

(d) $\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$

$T_3 = 2 \times 744 = 1{,}488°K$ [$P_2 = P_3$ and $V_3 = 2V_2$

$C_p - C_v = \frac{R}{J}$, $C_p - 0{\cdot}17 = \frac{29{\cdot}9}{427}$ $\therefore$ $C_p = 0{\cdot}24$

Heat received $= mC_p(T_3 - T_2)$

$= 0{\cdot}0568 \times 0{\cdot}24(1{,}488 - 744) = 10{\cdot}12$ kcal Ans.

1.10. Heating gas under spring loaded piston.

An 8 cm internal diameter cylinder open at one end is fitted

with a piston which is loaded by a coil spring, the strength of which is 16 kgf/cm² of compression. The cylinder contains 420 cc of air at a temperature of 15 C and a pressure of 2 8 kgf/cm². Find the amount of heat which must be given to the air in order to move piston forward by 4 cm. Given $C_p = 0\ 238$, $C_v = 0{\cdot}169$.

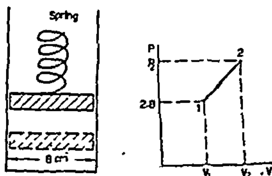


Fig. 1·24.

$$\text{Force} = P \times \text{area} \qquad \therefore \qquad F \propto P$$

$$\text{and movement } \frac{V}{A} \propto V \qquad \therefore \qquad P \propto V$$

Movement of the spring is linearly proportional to the force applied. Hence the curve is a straight line

Since movement is 4 cm against spring

$$\text{Force} = 16 \times 4 = 64 \text{ kg}$$

Increase in pressure due to spring

$$= \frac{64}{\pi/4 \times 8^2} = 1{\cdot}273 \text{ kgf/cm}^2$$

$$\text{Change in volume, } V_2 - V_1 = \frac{\pi}{4}\, d^2 l$$

$$= \frac{\pi}{4} \times 8^2 \times 4 = 201 \text{ cc}$$

$$\text{Work done} = \text{area under the line}$$

$$= \frac{(V_2 - V_1)(P_2 + P_1)}{2 \times J}$$

$$= \frac{201 \times 10^{-6}(4\ 073 + 2{\cdot}8) \times 10^4}{2 \times 427}$$

$$= +0\ 01619 \text{ kcal}$$

$$C_p - C_v = \frac{R}{J}, \quad 0\ 238 - 0\ 169 = \frac{R}{427} \quad \therefore R = 29\ 45$$

$$P_v = mRT$$

$\therefore\ 2{\cdot}8 \times 10^4 \times 420 \times 10^{-6} = m \times 29{\cdot}45 \times 288 \quad \therefore\ m = 0{\cdot}001387$ kg.

$$PV = mRT$$

$4{\cdot}073 \times 10^4 \times 621 \times 10^6 = 0{\cdot}001387 \times 29{\cdot}45 \times T_2, \quad \therefore\ T_2 = 619°K.$

$U_2 - U_1 = mC_v(T_2 - T_1)$

$= 0{\cdot}001387 \times 0{\cdot}169(619 - 288) = 0{\cdot}07767$ kcal

Heat required to be given $= W + (U_2 - U_1) = 0{\cdot}01619 + 0{\cdot}07767$

$= 0{\cdot}09386$ kcal Ans.

1·11. Adiabatic, constant volume and isothermal processes.

60 litres of hydrogen at 20°C and 1 03 kgf/cm² is compressed adiabatically to 9·8 kgf/cm². It is then cooled at constant volume to preseure P_3 and further expanded isothermally so as to reach the initial condition. Find, (a) the value of pressure P_3, (b) the arithmetical difference between the work done in adiabatic compression and isothermal expansion, and (c) the change in internal energy in constant volume process. Assume $C_p = 3{\cdot}44$ and $C_v = 2{\cdot}46$ for hydrogen.

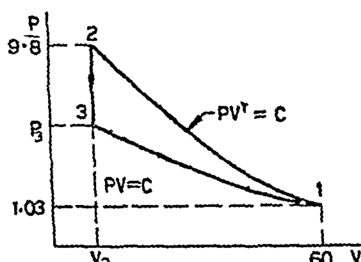


Fig. 1·25.

(a) $$\gamma = \frac{C_p}{C_v} = \frac{3{\cdot}44}{2{\cdot}46} = 1{\cdot}4$$

$$C_p - C_v = \frac{R}{J}, \quad 3{\cdot}44 - 2{\cdot}46 = \frac{R}{427}, \quad \therefore\ R = 418{\cdot}5$$

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$1{\cdot}03 \times 60^{1{\cdot}4} = 9{\cdot}8 \times V_2^{1{\cdot}4} \quad \therefore\ V_2 = 12$ litres

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \quad \frac{1{\cdot}03 \times 60}{293} = \frac{9{\cdot}8 \times 12}{T_2}, \quad \therefore\ T_2 = 557{\cdot}5°K$$

$$P_1V_1 = P_3V_3$$

$1{\cdot}03 \times 60 = P_3 \times 12 \quad \therefore\ P_3 = 5{\cdot}15$ kgf/cm² Ans.

(b) Work done in adiabatic compression $= \dfrac{P_1V_1 - P_2V_2}{n - 1}$

$$=\frac{10(1{\cdot}03\times60-9{\cdot}8\times12)}{1{\cdot}4-1}$$

$=-1,395$ kgf m

. Work done in isothermal expansion

$=P_1V_1 \log_e \gamma$

$=1\ 03\times10^4\times60\times10^{-3}\log_e \frac{5{\cdot}15}{1{\cdot}03}$

$=995$ kgf m

Arithmetic difference in work done

$=1,395-995 \quad =400$ kgf m   Ans.

(c) $PV=mRT$

$1\ 03\times10^4\times60\times10^{-3}=m\times418{\cdot}5\times293$

$\therefore \quad m=0\ 005043$ kg

$U_3-U_2=mC_v(T_3-T_2)$

$=0\ 005043\times2{\cdot}46\times(293-557{\cdot}5)$

$=-3\ 281$ kcal   Ans.

Note. In constant volume cooling temperature is reduced and hence the internal energy is reduced, which is equal to the heat lost since work done is zero.

1-12. Constant volume cooling : leakage expansion.

An air vessel of 0 115 m³ capacity was pumped with air till the pressure reached 66 kgf/cm² and a temperature of 46°C. The air was then cooled to 17°C, after which a leakage occured, and pressure of air was reduced to 30 kgf/cm² and temperature to 5°C. Find (a) how much heat was lost by all the air in the vessel between the end of pumping and the beginning of leakage, and (b) how much heat was lost or gained during leakage by the air remaining in the vessel, assuming the index of expansion of the air during leakage to be constant. $R=29\ 27$; $C_v=0{\cdot}17$.

(a) Let suffix 1 represent the initial condition of air and suffixes 2 and 3 represent the conditions before and after leakage respectively.

$PV=mRT$

$66\times10^4\times0{\cdot}115=m\times29{\cdot}27\times319$

$\therefore \quad m=8{\cdot}128$ kg.

Heat lost $\quad Q=mC_v\ (T_2-T_1)$

$=8{\cdot}128\times0{\cdot}17\times(46-17)$

$=40{\cdot}05$ kcal

(b) $$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}, \quad \frac{66}{319}=\frac{P_2}{290}$$

$\therefore$ Pressure after cooling $P_2=60$ kgf/cm²

$$\frac{T_2}{T_3}=\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}, \frac{290}{278}=\left(\frac{60}{30}\right)^{\frac{n-1}{n}} \quad \therefore \quad n=1{\cdot}065.$$

Work done $$W=\frac{P_2V_2-P_3V_3}{J(n-1)}$$

$$=\frac{mR\,(T_2-T_3)}{J(n-1)}$$

$$=\frac{8{\cdot}128\times29{\cdot}27\times(17-5)}{427\times(1{\cdot}065-1)}$$

$$=102{\cdot}9 \text{ kcal}$$

Change in internal energy,

$$U_2-U_1=8{\cdot}128\times0{\cdot}17\,(5-17)=-16{\cdot}58 \text{ kcal}$$

Heat received $$Q=W+(U_2-U_1)$$

$$=102{\cdot}9-16{\cdot}58=86{\cdot}32 \text{ kcal}$$

Volume at the end of expansion (full)

$$=0{\cdot}115\times\left(\frac{60}{30}\right)^{\frac{1}{1{\cdot}065}}$$

$$=0{\cdot}2205 \text{ m}^3$$

$\therefore$ Heat received by the air remaining in the vessel, *i.e.* 0·115 m³

$$Q=\frac{86{\cdot}32\times0{\cdot}115}{0{\cdot}2205}$$

$$=45 \text{ kcal (gain)}$$ Ans.

Note. (1) During constant volume cooling work done is zero and heat lost is equal to reduction in internal energy.

(2) Leakage is like polytropic expansion and hence heat will flow in the cylinder. Some of the heat received in the cylinder has gone out with the leakage air. The answer is the heat gained in the cylinder by the remaining air.

1-13. Molar heat : density ; adiabatic index ; partial pressures.

Define the terms mol, volumetric heat and universal gas constant.

A gas has the following composition by volume : H_2 42 4 per cent ; CH_4, 21·7 per cent ; CO, 17·1 per cent ; CO_2, 4 8 per cent ; N_2, 14 0 per cent. Calculate the mean molecular weight and specific heats of this gas and find its density at N.T.P. in kg/m^3. Assume a volumetric heat of 5 0 for diatomic gases, 8·7 for CH_4 and 7·3 for CO_2. Take the difference between the molecular heats for all gases as 1·98. What is the value of adiabatic index for this gas ? If the pressure of the mixture be 1 kgf/cm^2, calculate the partial pressures of the constituent gases.

For theory—see text.

Assume 100 mol of gas

Gas	*No. of moles* (a)	$C_{v\,mol}$ (b)	*Proportional heat* (c)=(a)×(b)	*Mol. wt.* (d)	*Proportional mass* (e)=(a)×(d)
H_2	42 4	5	212 0	2	84·8
CH_4	21·7	8 7	188 9	16	347·2
CO	17·1	5	85·5	28	478 8
CO_2	4 8	7·3	35 0	44	211·2
N_2	14 0	5	70 0	28	392 0
Total	100 0		591·4		1,514 0

$$C_{v_{mol}}(\text{average}) = \frac{591\cdot4}{100} = 5\,914. \qquad \text{Ans.}$$

$$C_{p_{mol}}(\text{average}) = R_{mol} + C_{v\,mol}(\text{average})$$

$$= 1\cdot98 + 5\cdot914 = 7\,894 \qquad \text{Ans.}$$

Molecular weight (average)

$$M = \frac{1{,}514}{100} = 15\cdot14 \qquad \text{Ans.}$$

$$\therefore \quad C_v = \frac{C_{v\,mol}}{M} = \frac{5\,914}{15\,14} = 0\,3906 \qquad \text{Ans.}$$

$$C_p = \frac{C_{p\,mol}}{M} = \frac{7\,894}{15\,14} = 0\,5214 \qquad \text{Ans.}$$

$$\gamma = \frac{C_{p\,mol}}{C_{v\,mol}} = \frac{C_p}{C_v} = \frac{0\,5214}{0\cdot3906} = 1\cdot335 \qquad \text{Ans.}$$

As a mol volume of a gas at *N.T.P.* is 22·4 m³, the density of mixture at *N.T.P.* will be

$$\frac{15 \cdot 14}{22 \cdot 4} = \underline{0 \cdot 676 \text{ kg/m}^3} \qquad \text{Ans.}$$

The partial pressure of the constituent gases are proportional to mol fraction and as the total pressure is 1 kgf/cm², the numerical value of the partial pressure of the constituent gas will be equal to the value of the mol fraction.

The partial pressure of H_2 will be 0·424 kgf/cm² ; of CH_4=0·217 kgf/cm² ; of CO=0·171 kgf/cm² : of CO_2=0·048 kgf/cm²; and of N_2=0·14 kgf./cm.² Ans.

1-14. Partial pressures.

A closed vessel of 0·5 m³ capacity contained air at 1 kgf/cm² pressure and 27°C. Hydrogen was added and the total pressure in the vessel was thereby raised to 1·04 kgf/cm² at the same temperature.

Find the masses of oxygen, nitrogen and hydrogen finally in the vessel, and their respective partial pressures.

Air contains 77 per cent by mass of nitrogen. Take R for air as 29·27 kgf m/kg°K, and for hydrogen 420 kgf m/kg°K.

Let mass of air in the vessel be m_1 kg

$$PV = mRT$$

$$1 \times 10^4 \times 0 \cdot 5 = m_1 \times 29 \cdot 27 \times 300$$

$$\therefore \qquad m_1 = 0 \cdot 57 \text{ kg}$$

Mass of oxygen $= 0 \cdot 57 \times 0 \cdot 23 = \underline{0 \cdot 1311 \text{ kg}}$ Ans.

Mass of nitrogen $= 0 \cdot 57 \times 0 \cdot 77 = \underline{0 \cdot 4389 \text{ kg}}$ Ans.

Let mass of H_2 introduced in the vessel be m_2 kg

Partial pressure of $H_2 = 1 \cdot 04 - 1 = 0 \cdot 04$ kgf/cm²

By Dalton's law of partial pressures, volume occupied by H_2 is the same as that occupied by air which is equal to the volume of the vessel itself.

$$PV = mRT$$

$$0 \cdot 04 \times 10^4 \times 0 \cdot 5 = m_2 \times 420 \times 300$$

$$\therefore \qquad \underline{m_2 = 0 \cdot 001586 \text{ kg}} \qquad \text{Ans.}$$

100 kg of air contains $\frac{23}{32}$ kg mols of O_2 and $\frac{77}{28}$ kg mols of N_2.

Percentage O_2 in air by volume

$$=\frac{(23/32)}{(23/32)+(77/28)}=20{\cdot}72\%$$

By Dalton's law, partial pressure is proportional to number of mols.

Partial pressure of $O_2=0{\cdot}2072\times1$

$=0{\cdot}2072\ \text{kgf/cm}^2$ Ans.

Partial pressure of $N_2=1-0{\cdot}2072$

$=0{\cdot}7928\ \text{kgf/cm}^2$ Ans.

1-15. Power developed by gas turbine : S.F.E.E

Discuss the principle of conservation of energy as applied to an engine system. Include in the discussion a reference to the first law of thermodynamics.

Air passes through a gas turbine at the rate of 5 kg/s. It enters with a velocity of 200 m/s and an enthalpy of 1,600 kcal/kg. At exit the velocity is 150 m/s and the enthalpy is 1,250 kcal/kg. The air has a loss of heat to the surroundings of 15 kcal/kg as it passes through the turbine. Determine the horse-power developed by the turbine.

For theory—see text.

From the steady-flow energy equation, for 1 kg of substance flowing through the system (*i.e.* gas turbine), and neglecting any change in the P.E., Eq. (1·35) gives

$$q+h_1+\frac{V_1^2}{2g}=w+h_2+\frac{V_2^2}{2g}$$

or

$$w=q-\left[(h_2-h_1)+\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)\right]$$

$$=-15-\left[(1{,}250-1{,}600)+\left(\frac{150^2-200^2}{2\times9{\cdot}81\times427}\right)\right]$$

$$=337{\cdot}1\ \text{kcal/kg}$$

$\therefore$ h.p developed $=\dfrac{337{\cdot}1\times427\times5}{75}$

$=9{,}593$ hp Ans.

1.16. Steady flow : throttling velocity downstream of restriction.

Air flows at the rate of 2·3 kg/s in a 15 cm diameter pipe. It has a pressure of 7 kgf/cm² and a temperature of 95°C before it is

throttled by a valve to 3·5 kgf/cm². Using the steady flow energy equation, find the velocity of the air downstream of the restriction.

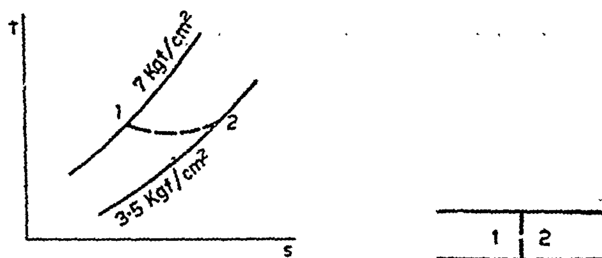


Fig. 1·26.

The initial specific volume

$$v_1 = \frac{RT_1}{P_1} = \frac{29{\cdot}27 \times 368}{7 \times 10^4} = 0{\cdot}154 \text{ m}^3/\text{kg}.$$

$$\text{Initial velocity} = \frac{mv_1}{A}$$

$$= \frac{2{\cdot}3 \times 0{\cdot}154}{\frac{\pi}{4}(0{\cdot}15)^2} = 20{\cdot}05 \text{ m/s}.$$

$$\text{Similarly, final velocity} = \frac{mRT_2}{AP_2} = \frac{2{\cdot}3}{\frac{\pi}{4} \times (0{\cdot}15)^2} \times \frac{29{\cdot}27 \times T_2}{3{\cdot}5 \times 10^4}$$

$$= 0{\cdot}1089\ T_2 \text{ m/s}$$

Since Q and W are zero, the energy equation reduces to

$$h_1 + \tfrac{1}{2}\overline{V_1}^2 = h_2 + \tfrac{1}{2}\overline{V_2}^2.$$

For a gas this becomes

$$C_p\,T_2 + \frac{(0{\cdot}1089\ T_2)^2}{2 \times 9{\cdot}81 \times 427} = C_p \times 368 + \frac{(20{\cdot}05)^2}{2 \times 9{\cdot}81 \times 427}$$

$$0{\cdot}24\ T_2 + \ldots\ldots\ldots = 0{\cdot}24 \times 368 + \ldots\ldots\ldots$$

Without calculation it is evident that K.E. terms can be ignored and $T_2 \simeq T_1$

Hence $\qquad \overline{V_2} = 0{\cdot}1089 \times 368 = \underline{40 \text{ m/s}}$ Ans.

1-17. Flow of air through change of section : nozzle ; heat added ; velocity.

Explain briefly why enthalpy is a useful concept when dealing with open systems.

Air at atmospheric pressure and 15°C enters a duct with a velocity of 300 m/s. The pipe at first diverges so that the velocity of

the air is reduced to a negligible value, and the pressure and temperature are consequently increased. The air then passes into a length of duct where heat is supplied at constant pressure raising the temperature still further to 1,000°C. At this temperature, and still with negligible velocity the air enters a nozzle wherein it expands to atmospheric pressure. Assuming that the process in the diffuser and nozzle are isentropic, find (a) the heat supplied per unit of mass flow, and (b) the velocity of the air leaving the nozzle.

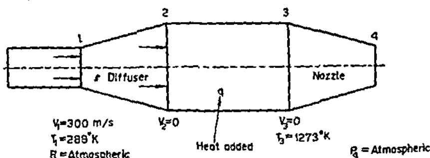


Fig. 1·27.

Steady-flow energy equation (1·36) for unit mass, neglecting P.E.

$$q+h_1+\frac{V_1^2}{2g}=w+h_2+\frac{V_2^2}{2g}$$

or
$$q-w=(h_2-h_1)+\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)$$

Applying equation to planes 1 and 2

$$0-0=0\cdot24\ (T_2-288)+\left(0-\frac{300^2}{2\times9\cdot81\times427}\right)$$

$\therefore$ $T_2=332\cdot8°K$

Applying equation to planes 2 and 3

$$q-0=0\cdot24\ (1{,}273-332\cdot8)+0$$

$\therefore$ $\underline{q=225\cdot6\ \text{kcal/kg}}$ Ans.

$$P_3=P_2$$

$$=1\times\left(\frac{332\cdot8}{288}\right)^{\frac{n}{n-1}}$$

$$\frac{P_4}{P_3}=\left(\frac{T_4}{T_3}\right)^{\frac{n}{n-1}}$$

or $$T_4=\left(\frac{P_4}{P_3}\right)^{\frac{n}{n-1}}\times T_3$$

$\therefore$ $$T_4=\frac{1,273\times 288}{332\cdot 8}=1,102°K$$

Applying *SFEE* equation at planes 3 and 4

$$0-0=0\cdot 24\,(1,102-1,273)+\left(\frac{V_4^2}{2\times 9\cdot 81\times 427}-0\right)$$

$\therefore$ Velocity at outlet, $V_4=586$ m/s Ans.

EXAMPLES 1

1-1. First law equation : heat, work and energy.

A block stands on the top of a rough inclined plane. The block slides slowly down the plane. State, giving reasons, whether the heat, work and increase in energy are positive, negative or zero for each of the following systems (a) the block, (b) the plane, and (c) the block and the plane.

Assume the block to be non-conductor of heat and plane to be good conductor of heat.

[(a) $Q=0$, $W=+$ve, $E=-$ve (b) $Q=0$, $W=-$ve, $E=+$ve (c) $Q=W=E=0$]

1-2. First law equation : heat, work and energy.

Solve the above problem (a) assuming the block to be good conductor of heat and plane to be non-conductor of heat.

[$Q=W=\Delta E=0$ in all cases]

1-3. Specific heat for polytropic expansion.

A gas expands behind a piston according to the law $PV^n=$ constant. Show that the effective specific heat 'C' for an expansion of this kind is given by

$$C=\frac{n\,C_v-C_p}{n-1}=C_v\left(1-\frac{\gamma-1}{n-1}\right)=C_v\left(\frac{n-\gamma}{n-1}\right)$$

Determine the value of 'C' and explain the result when (a) $n=0$; (b) $n=1$; (c) $n=\gamma$; (d) $n=\infty$.

[(a) C_p ; constant pressure (b) infinite ; isothermal. (c) 0 ; adiabatic. (d) C_v ; constant volume]

1-4. Isothermal, polytropic and adiabatic expansion : T_2 ; W ; Q.

Show that when a gas is compressed in a cylinder by a piston according to the law $PV^n=$constant, the heat rejected, increase in

internal energy and work done in compression are in the ratio $(\gamma-n):(n-1):(\gamma-1)$.

A perfect gas has specific heats at constant pressure and constant volume of 3·5 and 2·5 respectively. 0·1 kg of this gas at a pressure of 30 kgf/cm² and temperature of 1,000°C expands behind a piston to a volume five times the original volume. Find the final temperature, work done and heat interchange if the expansion is (a) isothermal, (b) $PV^{1.25}$ *= constant, and (c) adiabatic.*

[$R=427$; $\gamma=1{\cdot}4$; $V_2=0{\cdot}9055$ m³ (a) Isothermal : $T_2=T_1=1{,}273°K$; $Q=W=204{\cdot}7$ kcal (b) $PV^{1.25}=C$: $T_2=851°K$; $W=168{\cdot}8$ kcal ; $Q=63{\cdot}3$ kcal (c) adiabatic : $T_2=668{\cdot}6°K$ $W=151{\cdot}1$ kcal , $Q=0$]

1-5. Change in internal energy and enthalpy

Explain the term 'work of introduction' in a flow process.

30 litres of air expands from an initial pressure of 8·7 kgf/cm² and temperature 850°C to a pressure of 1·03 kgf/cm² and temperature 200°C. Using reference temperature 0°C, find the change in internal energy and enthalpy during the process. Assume $C_p=0{\cdot}24$ *and* $C_v=0{\cdot}171$.

[$R=29{\cdot}45$; $m=0{\cdot}07894$ kg ; $U_2-U_1=-8{\cdot}77$ kcal ; $H_2-H_1=-12{\cdot}28$ kcal ; check $H_2-H_1=mc_p\,(T_2-T_1)$]

1-6. Combination of non-flow ond flow processes.

A cylinder contains 0·25 m³ of air at 6 kgf/cm² and 90°C. The gas is expanded to a volume of 1 m³ the final pressure being 1·2 kgf/cm². Find the heat interchange and the change in enthalpy.

If 0·2 kg of air at 90°C is introduced in from a separate source into this space of 1 m³ find the temperature and pressure at the end of introduction. For air $R=29{\cdot}27$ *kgf m/kg°K and* $C_v=0{\cdot}17$

[$n=1{\cdot}161$; $m=1{\cdot}412$ kg ; $T_2=290{\cdot}4$ °K ; $W=43{\cdot}67$ kcal ; $\Delta U=-17{\cdot}32$ kcal ; $Q=26{\cdot}35$ kcal ; $H_2-H_1=-24{\cdot}35$ kcal ; $C_p=0{\cdot}238$; $T_3=318°K$; $P_3=1{\cdot}5$ kgf/cm²]

1-7. Isothermal, adiabatic and throttling operation : V_2 ; T_2 ; W.

Define the following processes (a) isothermal, (b) reversible adiabatic, and (c) throttling.

One kg of air at 15 kgf/cm² and 1,000°C may have the pr[essure] reduced to 1·5 kgf/cm² by ane of the above processes. Fin[d in] each case (a) the final volume and temperature and (b) work d[one]. R=29·27 in kgf m units and γ=1·4.

[(a) $V_2=2{\cdot}84$ m³ ; $t_2=t_1=1{,}000°C$; $W=85{,}800$ kgf [m]
(b) $V_2=1{\cdot}286$ m³ ; $t_2=386°C$; $W=44{,}900$ kgf m ; (c) $V_2=2{\cdot}484$ m[³]
$t_2=1{,}000°C$; $W=0$]

1-8. Polytropic compression : mass of mixture ; n ; heat interchange.

State the first law of thermodynamics.

A gas engine has a cylinder diameter of 18 cm and a stroke of 30 cm, the ratio of compression is 5. The pressure of mixture is 1·05 kgf/cm² at the beginning of compression and 6·3 kgf/cm² a[t the] end of compression. Find the index 'n' for the compression stroke [and] the mass of mixture in cylinder if the temperature at the beginnin[g of] compression is 100°C. Find also the work done and the heat in[ter]change through the cylinder walls during this stroke. R for mixt[ure] =29·27, γ=1·4.

[$V_1=9{,}545$ cc ; $m=0{\cdot}00916$ kg ; $n=1{\cdot}114$; $W=-0{\cdot}41$ kcal
$Q=\frac{\gamma-n}{\gamma-1}\times W=0{\cdot}292$ kcal ; check $Q=W+\Delta U$]

1-9. Polytropic expansion : n>γ ; W ; ΔU ; Q.

A quantity of gas at 150°C expands from 0·37 m³ to 2·16 m³ according to the law PVⁿ=constant. The initial pressure is 14 kgf/cm² and the final pressure 1 kgfcm². The value of specific heat at constant volume is 0·179 at 150°C and at this temperature the value of γ is 1·39. The specific heats at constant pressure and constant volume are of the form a+0·000045 T and b+0·000045 T respectively, where T is the absolute temperature.

Find : (a) the work done by the gas during the expansion, (b) th[e] change in internal energy, and (c) the heat received or rejected during the process.

[$n=1{\cdot}496$; $W=60{,}880$ kgf m ; $R=29{\cdot}8$; $m=4{\cdot}11$ kg ;
$T_2=176{\cdot}4°K$; $\Delta U=-175{\cdot}8$ kcal ; $Q=-33{\cdot}1$ kcal. Note $N>\gamma$,
[Q] is negative because $Q=\frac{\gamma-n}{n-1}\times W$]

1-10. Constant volume, adiabatic and constant pressure processes : W ; Q ; ΔU.

A quantity of air at 1 06 kgf/cm² and 7°C is heated at constant volume in a cylinder until its temperature has risen to 847°C. It is then expanded adiabatically until the pressure falls to 1·06 kgf/cm², following which heat is rejected at constant pressure until the temperature is again equal to 7°C. Determine per kg of air (a) the pressure, volume and temperature at the end of each operation, (b) the change in internal energy in each operation, (c) the heat input of the cycle, and (d) the work output of the cycle. Assume $R=29{\cdot}27$ and $C_v=0{\cdot}171$.

[$v_1=v_2=0{\cdot}7732$ m³ ; $P_2=4{\cdot}24$ kgf/cm² ; $T_3=754°K$; $v_3=2{\cdot}082$ m³ ; $\Delta u_{2-1}=143\ 7$ kcal ; $\Delta U_{3-2}=-62{\cdot}6$ kcal ; $\Delta u_{1-3}=-81\ 1$ kcal ; (net change in complete cycle is zero) ; $q_{added}=143{\cdot}7$ kcal ; $w=$area of P-V diagram$=30{\cdot}1$ kcal]

1-11. Adiabatic and isothermal compression : R ; v ; C_p ; C_v.

0 5 kg of gas at a pressure and temperature of 1·06 kgf/cm² and 20°C occupies a volume of 410 litres. When the gas is compressed adiabatically to one fifth of its initial volume the final temperature is 267°C. Determine the values of R, γ, C_v and C_p for the gas. State the units.

If the gas is compressed isothermally from the same initial condition and through the same compression ratio, determine the heat rejected by the gas.

[$\gamma=1{\cdot}38$; $R=29\ 66$ kgf m/kg °K ; $C_v=0{\cdot}1829$; $C_p=0\ 2524$, $Q=-16{\cdot}39$ kcal]

1-12. Polytropic and straight line compression : work done and heat interchange.

A certain gas occupies a volume of 1·6 m³ at 1 05 kgf/cm² It is compressed to a volume of 0·2 m³, the final pressure being 11 kgf/cm² Determine the work done and heat absorbed or rejected by the gas if the compression is (a) $PV^n=$constant, and (b) $P=aV+b$, where a and b are constants. $C_p=0{\cdot}238$ and $C_v=0{\cdot}169$.

[$m=1\ 902$ kg ; $\Delta U=29{\cdot}9$ kcal (a) $n=1{\cdot}13$, $W=-93\ 8$ kcal ; $Q=-63\ 9$ kcal (rejected) (b) $W=-98{\cdot}9$ kcal ; $Q=-69$ kcal]

1-13. Leak from an air vessel ; heat interchange.

A torpedo air chamber contains initially 40 kg of air at a pressure of 120 kgf/cm² and 15°C and at the end of the run the pressure is 36 kgf/cm² and the temperature 2°C. How much of the heat in the air which is left in the chamber has been abstracted from the sea ? $R=29 \cdot 27$ *and* $C_v=0 \cdot 169$.

[$n=1 \cdot 04$; $W=891 \cdot 7$ kcal ; $\Delta U=-87 \cdot 9$ kcal ; Q for whole mass$=803 \cdot 8$ kcal ; mass remaining$=12 \cdot 432$ kg ; Q for remaining mass$=249 \cdot 8$ kcal]

1-14. Expression for rise in pressure : load lifted by a balloon.

(a) *Show that if a given quantity of air receives a fixed amount of heat at constant volume, the rise in pressure produced will be directly proportional to the initial absolute pressure, and inversely proportional to the initial absolute temperature.*

(b) *A balloon is 12·2 m in diameter and may be considered spherical. The temperature of the surrounding air is 17°C and the barometer reads 755 mm of Hg. If the balloon is filled with hydrogen at a temperature of 24°C and atmospheric pressure, what load may the balloon lift ? Neglect weight of balloon and take R for hydrogen 420 and for air 29·22.*

[Equate (T_2-T_1) derived from $Q=W+\Delta U$, and $\frac{PV}{T}$ = constant. $P_2-P_1=\frac{P_1}{T_1}\times\frac{Q}{m\,C_v}$, $\frac{Q}{m\,C_v}$ is constant $\therefore P_2-P_1 \propto \frac{P_1}{T_1}$; V of $H_2=951 \cdot 3$ m³ ; $m=78 \cdot 25$ kg ; mass of equal volume of air $=1,152$ kg ; load lifted$=1,073 \cdot 75$ kg]

1-15. Polytropic compression ; n_1 ; n_2 ; heat rejected.

An oil engine is 22 cm bore by 40 cm stroke and has clearance volume of 900 cc. At the start of compression stroke the pressure is 1 kgf/cm² and temperature 90°C. At 3/4th of compression stroke the pressure is 5 kgf/cm² and at the end of compression the pressure is 40 kgf/cm². Find for each part of the curve the index of compression. Calculate also the amount of heat rejected during compression. $\gamma=1 \cdot 4$ *and* $C_v=0 \cdot 17$.

[$n_1=1 \cdot 307$; $Q_1=-0 \cdot 131$ kcal ; $n_2=1 \cdot 258$; $Q_2=-0 \cdot 402$; $Q=Q_1+Q_2=-0 \cdot 533$ kcal]

1-16. Partial pressures.

What are the partial pressures of oxygen and nitrogen in air at 1 kgf/cm² ? If one kg of air is confined in a vessel of 200 litres capacity at a temperature of 27°C and one kg of CO_2 is introduced at the same temperature, what pressure of CO_2 at entry to the vessel, and what volume at this pressure and temperature will be required ?

Air contains 21 per cent oxygen by volume. For air $R=29.27$ kgf m/kg °K and $R_{mol}=1.986$ kcal/mol °K.

[pp of $O_2=0.21$ kgf/cm² ; pp of $N_2=0.79$ kgf/cm² ; pp of air $=4.39$ kgf/cm² ; pp of $CO_2=2.89$ kgf/cm² ; pressure of CO_2 at entry$=7.28$ kgf/cm² ; $P_1V_1=$pp of $CO_2\times200$ $\therefore$ $V_1=79.5$ litres]

1-17. Steady flow : work done.

In a steady-flow system a substance flows at the rate of 5 kg/s. It enters the system at a pressure of 6 kgf/cm², velocity 300 m/s, internal energy 500 kcal/kg and specific volume 0.38 m³/kg It leaves the system at a pressure of 1.5 kgf/cm², velocity 150 m/s, internal energy 420 kcal/kg and specific volume 1.26 m³/kg. During its passage through the system the substance loses 20 kcal/kg to the surroundings. Determine the horse-power of the system stating whether it is from or to the system. Neglect any change in the protential energy.

[$w=32{,}960$ kgf m/kg ; h p $=2{,}197$]

1-18. Nozzle : final velocity.

10 kg of air per minute enter a nozzle with negligible velocity and expand from a pressure of 4 to 2 kgf/cm². The temperature falls from 950°C to 770°C in the process. Find the velocity of air as it leaves the nozzle, the velocity which would have been reached had the expansion being frictionless, and the nozzle efficiency.

[$V=536.2$ m/s ; $T_2'=1{,}005$°K , $V'=662$ m/s ; $\eta=82.5\%$]

1-19. Heat transfer from compressor.

Air is drawn into a system at the rate of 15 m³/min through a 5 cm diameter pipe and it leaves the system through a 2.5 cm diameter pipe. The inlet temperature and pressure are 20°C and 1 kgf/cm² respectively and the outlet values are 200°C and 6 kgf/cm². If the compressor producing the pressure rise absorbs 40 hp find the heat transfer to or from the surroundings. Take $R=29.27$.

[$m=17.49$ kg ; $V_1=127.3$ m/s ; $V_2=136.9$ m/s. ; $Q=339.29$ kcal/min]

2

The Second Law of Thermodynamics : Entropy

2·1. The Second Law of Thermodynamics. The first law of thermodynamics states that net work cannot be produced during

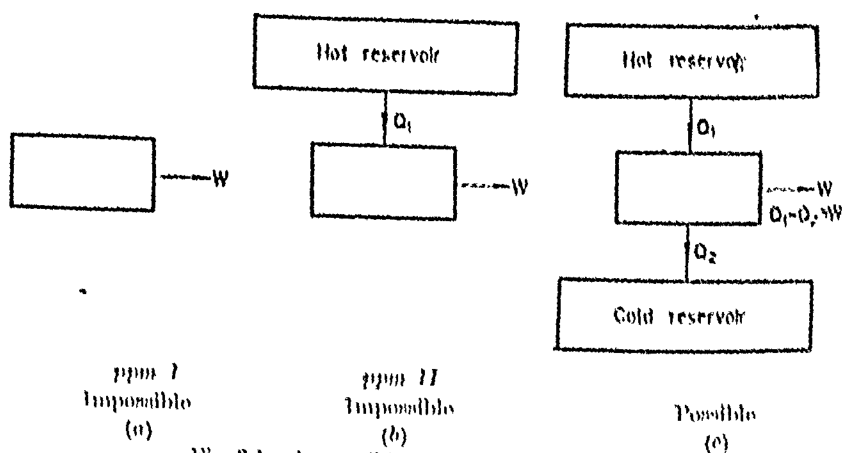


Fig. 2·1. Impossible and possible machines.

cycle without some supply of heat. *A perpetual motion machine of the first kind*, defined as a machine which produces power without consuming any energy [shown in Fig. 2·1 (*a*)], is impossible according to the first law of thermodynamics. The first law of thermodynamics only fixes the exchange rate between the heat and work done but does not place any restriction on their direction. The second law of thermodynamics puts the restriction that processes proceed in a certain direction and not in the opposite direction. According to the second law of thermodynamics the whole of the heat energy cannot be converted into work and part of the energy, therefore, must be rejected to the surroundings. Thus according to the Second Law a *perpetual motion machine of the second kind* [defined as 100 per

cent efficient engine shown in Fig. 2·1 (*b*)] is impossible. Fig. 2·1 (*c*) shows the possible machine in which heat is supplied from the hot reservoir and part heat is rejected to the cold reservoir, and the work equal to the difference of heat supplied and rejected is done on the surroundings.

The second law of thermodynamics can be stated in different ways but the two classical statements are as follows :—

1. *Kelvin-Planck Statement.* It is impossible to construct an engine which operating in a cycle will produce no effect other than the exchange of heat from a single reservoir and produce work. This implies that perpetual motion machine of the second kind is impossible.

2. *Clausius Statement.* Heat cannot flow from lower temperature to higher temperature without the aid of an external agency. This implies that a heat pump cannot operate without the input of work.

Essentially, both the above statements and the other statements of the Second Law are one and the same. Any one statement can be derived from the other. Like the First Law, Second Law also cannot be proved. The only proof of the thermodynamic laws is that no macroscopic phenomenon has been found to violate them.

2·2. Reversible Processes. A process is thermodynamically reversible if it can be carried out in reverse direction in exactly the same manner as in the forward direction and if the surroundings before and after the process are exactly in the same condition The conditions of thermodynamic reversibility are (*a*) the working substance at any instant must have same temperature as that of the heat source while heat is being absorbed *i e.* the cylinder head must be a perfect conductor. Also the working substance must be perfect conductor of heat so that the temperature throughout the working fluid is same, (*b*) the working substance at any instant must have the same temperature as that of the cold body while heat is being rejected, (*c*) there should be no frictional and mechanical losses, and (*d*) there should be no free or imperfectly resisted expansion. It can be shown by the help of Second Law that a process cannot be reversible if it involves (*i*) friction, (*ii*) unresisted expansion, and (*iii*) heat transfer with a finite temperature difference. For example consider leakage occurs through a hole from one vessel conta a substance under pressure into another vessel at low

2

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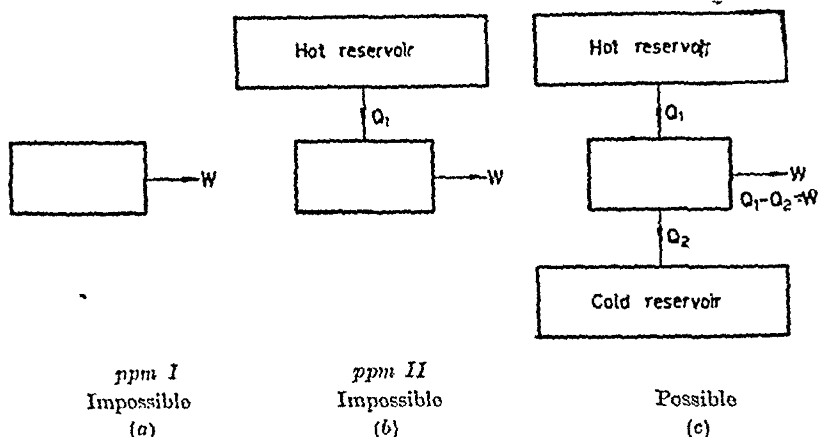


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cent efficient engine shown in Fig. 2·1 (*b*)] is impossible. Fig. 2·1 (*c*) shows the possible machine in which heat is supplied from the hot reservoir and part heat is rejected to the cold reservoir, and the work equal to the difference of heat supplied and rejected is done on the surroundings.

The second law of thermodynamics can be stated in different ways but the two classical statements are as follows :—

1. *Kelvin-Planck Statement.* It is impossible to construct an engine which operating in a cycle will produce no effect other than the exchange of heat from a single reservoir and produce work. This implies that perpetual motion machine of the second kind is impossible.

2. *Clausius Statement.* Heat cannot flow from lower temperature to higher temperature without the aid of an external agency. *This implies that a heat pump cannot operate without the input of* work.

Essentially, both the above statements and the other statements of the Second Law are one and the same. Any one statement can be derived from the other. Like the First Law, Second Law also cannot be proved. The only proof of the thermodynamic laws is that no macroscopic phenomenon has been found to violate them.

2-2. Reversible Processes. A process is thermodynamically reversible if it can be carried out in reverse direction in exactly the same manner as in the forward direction and if the surroundings before and after the process are exactly in the same condition. The conditions of thermodynamic reversibility are (*a*) the working substance at any instant must have same temperature as that of the heat source while heat is being absorbed *i.e.* the cylinder head must be a perfect conductor. Also the working substance must be perfect conductor of heat so that the temperature throughout the working fluid is same, (*b*) the working substance at any instant must have the same temperature as that of the cold body while heat is being rejected, (*c*) there should be no frictional and mechanical losses, and (*d*) there should be no free or imperfectly resisted expansion It can be shown by the help of Second Law that a process cannot be reversible if it involves (*i*) friction, (*ii*) unresisted expansion, and (*iii*) heat transfer with a finite temperature difference. For example, consider leakage occurs through a hole from one vessel containting a substance under pressure into another vessel at low pressure. It

is obvious that the process cannot be reversed to reach the original state again without any external aid. Similarly, where friction is involved the process is irreversible because friction produces heat and heat can never be reconverted into mechanical work by reversing a frictional effect.

Isothermal and adiabatic processes are the only processes which are reversible. But even adiabatic process becomes irreversible when internal friction (*i.e.* friction between the molecules and cylinder surface or between molecules and molecules) is generated. An example of this is in turbines and rotary compressors where there is friction due to high speed.

The concept of reversibility, though hypothetical, is very important in thermodynamics because a reversible process is the most efficient process. Reversible processes alone can be truely represented on P-V diagram. In practice thermodynamic reversibility can be approached but cannot be achieved.

The criterion of irreversibility is as follows : *A process is irreversible if a perpetual motion machine of the second kind would result from its being reversible.*

2-3. Carnot cycle. A thermodynamic cycle consists of a series of thermodynamic operations such that the working agent is restored finally to the original state. It is represented by a closed figure on P-V diagram, the area of which is equal to the work done.

The Carnot cycle consists of two isothermal and two adiabatic processes. Both isothermal and adiabatic processes are reversi-

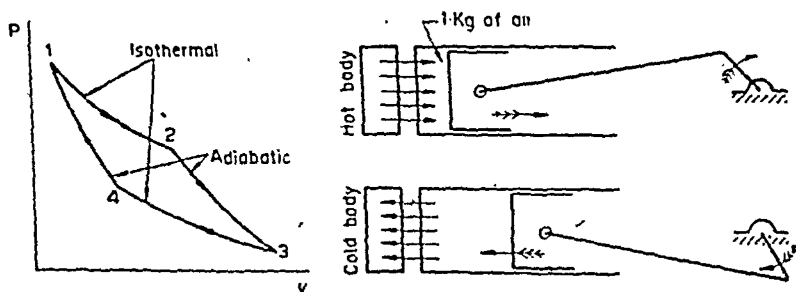


Fig. 2·2. The Carnot cycle.

ble and hence, the cycle is a reversible cycle. Fig. 2·2 shows the Carnot cycle on P-V diagram.

Consider the cylinder to contain one kg of air at temperature T_1. Let the initial conditions be represented by point 1 in the diagram. Heat is added isothermally from an external hot body at the same temperature as air *i.e.* T_1, *the air expanding to* 2. During the expansion work is done on the piston. At 2 the source of heat is removed and the air now expands adiabatically to point 3, cooling it to a temperature T_2. During this process further work is done on the piston. At 3 the piston reverses and heat is rejected isothermally to an external cold body at constant temperature T_2. At 4 the cold body is withdrawn and the remainder of the compression is done adiabatically to restore the air to the original state 1 at temperature T_1.

Let, P_1, v_1 and T_1 represent the condition of air at 1
and P_2, v_2 and T_2 ,, ,, ,, ,, ,, 3

Further, let r be the ratio of expansion during 1-2 which is equal to the ratio of compression during 3-4. If the ratio of expansion and compression are not equal it would not be a closed cycle.

Heat supplied $= P_1 v_1 \log_e r = RT_1 \log_e r$

Heat rejected $= P_2 v_2 \log_e r = RT_2 \log_e r$

Work done $=$ heat supplied $-$ heat rejected

$$= RT_1 \log_e r - RT_2 \log_e r$$

$$\therefore \text{ Efficiency of the cycle} = \frac{\text{work done}}{\text{heat supplied}}$$

$$= \frac{RT_1 \log_e r - RT_2 \log_e r}{RT_1 \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \qquad (2\cdot1)$$

It is seen that for the highest efficiency heat should be taken at the highest possible temperature and rejected at the lowest possible temperature.

The Carnot cycle cannot be realised in actual practice because of the following reasons :—

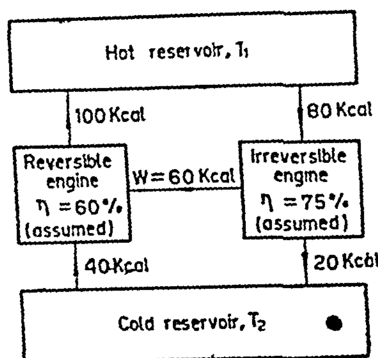
1. Isothermal process can only be achieved if the piston moves *very slowly to allow heat interchange* so that temperature remains constant. Again, adiabatic process can only be achieved if the piston moves very fast so that heat interchange is negligible due to very short time available. In the Carnot cycle isothermal and adiabatic processes take place in the same stroke which means that for part of

the stroke the piston should move very slowly and for the remaining part very fast. This variation of speed in the same stroke is not possible.

2. Compared to the length of the stroke the area of the indicator diagram is very small and hence the work done per cycle is very small and may not be even able to overcome the friction of the reciprocating parts.

2·4. Reversibility and Efficiency. Reversible cycles are the most efficient cycles and all reversible cycles have the same efficiency. It can be proved as follows :—

Let there be a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2. Imagine a reversible engine R taking



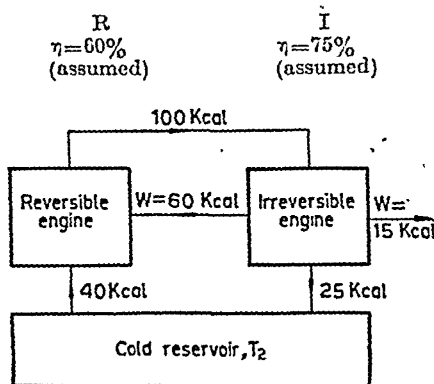


Fig. 2·3. Fig. 2·4.

The assumption of higher efficiency for irreversible engine contradicts the Second Law.

100 kcal from the hot reservoir and converting 60 kcal into work and rejecting 40 kcal to the cold reservoir, the engine efficiency being 60 per cent. If this engine is reversed it will take 60 kcal to drive it, another 40 kcal will be taken from the cold reservoir and 100 kcal will be delivered to the hot reservoir as shown in Fig. 2·3.

For the sake of argument assume an irreversible engine I having efficiency greater than that of the reversible engine, say 75 per cent, and driving the reversible engine without any losses. Now for the same 60 kcal work done, heat taken by irreversible engine $= \dfrac{60}{0{\cdot}75} = 80$ kcal from the hot reservoir and heat rejected to the cold reservoir $=(80-60)=20$ kcal. In this system reversible engine delivers $(100-80)=20$ kcal heat more to hot reservoir than the irreversible

engine takes from it. Also reversible engine takes (40−20) =20 kcal heat more from the cold reservoir than the irreversible engine delivers to it and this heat is being pumped from the cold reservoir to the hot reservoir without any external power, which is contrary to the second law of thermodynamics. Hence the assumption that irreversible engine is more efficient is wrong.

It can also be shown by directing the energy from reversible engine to irreversible engine (*see* Fig. 2·4) that the system delivers power while exchanging heat from one source only, which is again contrary to the second law of thermodynamics.

By replacing the irreverisible engine with another reversible engine and applying the same reasoning it will be seen that one reversible engine cannot be more efficient than another when working between the same temperature limits. Hence, the conclusion is derived that reversible engines are most efficient and all reversible engines have the same efficiency working in the same temperature range.

2·5. Entropy. Entropy of a substances is a property* which increases with the addition of heat. Entropy itself cannot be defined but change of entropy can be defined. Mathematically, in a reversible process the increase of entropy when multiplied by the absolute temperature gives the heat received by the fluid from an external source. The change of entropy with temperature is shown on a diagram known as temperature-entropy (T-S) diagram, the base

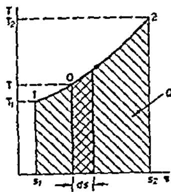


Fig. 2·5. Temperature-entropy diagram.

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*Entropy is an extensive property. For, if the mass of the system is doubled twice the amount of heat is necessary to bring about the same change in the state of mass.

representing units of entropy and the ordinate representing absolute temperature. Consider certain mass of gas, represented on T-S diagram (Fig. 2·5) by the point 1, receiving heat in some manner. As the heating proceeds the absolute temperature and entropy increase. The curve is plotted and the final condition is represented by point 2.

Considering any point O on the curve, a small addition of heat dQ under reversible conditions increases entropy by dS. If T is the absolute temperature at this instant, by the mathematical definition of entropy, we get

$$dQ_{rev}=TdS \text{ or } dS=\frac{dQ_{rev}}{T} \qquad (2\cdot2)$$

dQ_{rev} means that the heat transfer is by reversible process. In subsequent treatment the subscript rev is omitted for simplitcity.

From Fig, 2·5 TdS is the area under the curve during the change of entropy dS.

$$\therefore \qquad Q=\int_{T_1}^{T_2} TdS=\text{area under the curve 1-2} \qquad (2\cdot3)$$

Thus for any reversible heating and expansion of a gas the area under the curve in T-S diagram gives the total heat absorbed.

The change in entropy is also equal to maximum amount of work obtainable for a unit temperature drop in a heat engine. The maxinum work dW obtainable from an amount of heat dQ is given by the Carnot efficiency

$$dW=dQ\times\frac{T_1-T_2}{T_1}$$

If temperature drop (T_1-T_2) is unity

$$dW=\frac{dQ}{T}=\text{change of entropy } dS \qquad (2\cdot4)$$

Thus the change of entropy may be regarded as a measure of the rate of availability of heat for transformation into work.

It may be noted that (*i*) the entropy is a property and hence the change of entropy is independent of the path of the process, and (*ii*) the definition of entropy is only in terms of the change of entropy.

Change of Entropy in Irreversible Processes. The change of entropy, by definition, is in terms of a reversible process. However, as entropy is a property it is independent of the path of the process. Therefore, change of entropy in any irreversible process can be found out by assuming any reversible path between the end states and

calculating $\int_1^2 \frac{dQ_{rev}}{T}$.

For simplicity entropy is taken zero at a particular temperature; then with that temperature as datum change of entropy becomes entropy itself.

2.6. Isentropic and Polytropic Processes. In the reversible adiabatic process i.e. without friction, as no heat is supplied or rejected the entropy remains constant and reversible adiabatic process is, therefore, known as *isentropic process* ($dQ=0 \therefore dS=0$). The other processes where entropy is not constant are known as *polytropic processes*. Entropy can therefore be defined as a thermodynamic function which does not change during a reversible adiabatic processes.

Fig. 2·6 shows the reversible process on T-S diagrams. On T-S diagram the reversible adiabatic or isentropic expansion is represented by vertical line. There is no area under the line as no heat is supplied during an adiabatic process. An isothermal expansion of a gas is represented by a horizontal line on T-S diagram. The area under the line represents the heat absorbed during the isothermal expansion which is also equal to the work done in this case. The constant volume and constant pressure expansions are represented by firm and

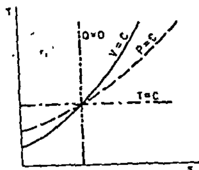


Fig. 2·6. Process curves passing through one state point of a gas on T-S diagrams.

dotted lines respectively on T-S diagram. These curves are of logarithmic nature. The derivations for change of entropy for the above processes are given in article 2·8.

2-7. Physical Concept of Entropy. The entropy of a substance is a real physical quantity defining the state of the body and

can be easily evaluated for a solid or a perfect gas except for an additive constant representing the entropy at the absolute zero. However, it cannot be felt like other physical quantities *i.e.* temperature, pressure, volume, etc., and hence there is difficulty in forming a tangible concept of entropy.

The addition of heat to a system increases its entropy. From the kinetic point of view addition of heat produces a more violent agitation of the molecules. The growth of entropy implies a transition from an ordered to a less ordered state of affairs, from a more to less available energy and from a less probable to a more probable state.

The idea of entropy is necessitated due to the existence of irreversible processes. Entropy remains constant in a reversible process and increases in an irreversible process.

2-8. Change of Entropy in any Thermodynamic Process. The following assumptions are made in deriving expressions for change of entropy of a gas in a thermodynamic process.

(*i*) Throughout the process the substance remains a perfect gas and follows the usual gas laws.

(*ii*) Small addition of heat dQ takes place under reversible conditions. In irreversible process $dQ \neq TdS$. Therefore, cases like fluid friction are not admissible.

Let unit mass of gas at a pressure P_1, absolute temperature T_1 and volume v_1 be heated in any general manner to pressure P_2, absolute temperature T_2 and volume v_2.

From the first law of thermodynamics

$$q = w + (u_2 - u_1)$$

$$dq = \frac{Pdv}{JT} + C_v \, dT$$

By definition, $$ds = \frac{dq}{T} = \frac{Pdv}{JT} + \frac{C_v \, dT}{T}$$

$$\int_{s_1}^{s_2} ds = \frac{R}{J} \int_{v_1}^{v_2} \frac{dv}{v} + C_v \int_{T_1}^{T_2} \frac{dT}{T} \qquad [\text{as } Pv = RT$$

(i) $\therefore\ s_2-s_1=\dfrac{R}{J}\log_e\dfrac{v_2}{v_1}+C_v\log_e\dfrac{T_2}{T_1}$ (2·5)

Also, $s_2-s_1=\dfrac{R}{J}\log_e\dfrac{v_2}{v_1}+C_v\log_e\dfrac{P_2v_2}{P_1v_1}$

$$=C_v\log_e\frac{P_2}{P_1}+\left(C_v+\frac{R}{J}\right)\log_e\frac{v_2}{v_1}$$

$$=C_v\log_e\frac{P_2}{P_1}+C_p\log_e\frac{v_2}{v_2} \quad (2·6)$$

(ii) *When pressure remains constant*

$$P_1=P_2 \quad \text{or} \quad C_v\log_e\frac{P_2}{P_1}=0$$

$$\therefore \quad s_2-s_1=C_p\log_e\frac{v_2}{v_1}=C_p\log_e\frac{T_2}{T_1} \quad (2·7)$$

(iii) *When volume remains constant*

$$v_1=v_2 \quad \text{or} \quad C_p\log_e\frac{v_2}{v_1}=0$$

$$\therefore \quad s_2-s_1=C_v\log_e\frac{P_2}{P_1}=C_v\log_e\frac{T_2}{T_1} \quad (2·8)$$

(iv) *When temperature remains constant*

$$T_1=T_2 \quad \text{or} \quad C_v\log_e\frac{T_2}{T_1}=0$$

$$\therefore \quad s_2-s_1=\frac{R}{J}\log_e\frac{v_2}{v_1}=\frac{R}{J}\log_e\frac{P_1}{P_2} \quad (2·9)$$

Note. $\dfrac{v_2}{v_1}=\dfrac{V_2}{V_1}$ and hence instead of specific volumes ratio actual volume ratio can be used

2·9. Other Expressions for Change of Entropy in a Polytropic Process.

(i) $s_2-s_1=C_v\ \dfrac{\gamma-n}{n-1}\log_e\dfrac{T_2}{T_1}$

From Eq. (2·5) for 1 kg of gas

$$s_2-s_1=\frac{R}{J}\log_e\frac{v_2}{v_1}+C_v\log_e\frac{T_2}{T_1}$$

$$=\frac{R}{J}\log_e\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}+C_v\log_e\frac{T_2}{T_1}$$

$$=\log_e\frac{T_1}{T_2}\left[\frac{R}{J\,(n-1)}-C_v\right]$$

$$=C_v \log_e \frac{T_1}{T_2}\left[\frac{C_p-C_v}{C_v\,(n-1)}-1\right]$$

$$=C_v\,\frac{\gamma-n}{n-1}\log_e\frac{T_1}{T_2} \qquad (2\cdot10)$$

Q.E.D.

(*ii*) $s_2-s_1=(C_p-n\,C_v)\log_e r$,
where r is the ratio of expansion and the law of expansion is $Pv^n=C$.

From Eq. (2·5) for 1 kg of gas

$$s_2-s_1=\frac{R}{J}\log_e\frac{v_2}{v_1}+C_v\log_e\frac{T_2}{T_1}$$

$$=\frac{R}{J}\log_e\frac{v_2}{v_1}-C_v\,(n-1)\log_e\frac{v_2}{v_1}$$

$$=\log_e\frac{v_2}{v_1}\left(\frac{R}{J}-n\,C_v+C_v\right)$$

$$=(C_p-n\,C_v)\log_e r \qquad \text{Q.E.D.} \qquad (2\cdot11)$$

(*iii*) $s_2-s_1=\left(b-\dfrac{a-b}{n-1}\right)\log_e\dfrac{T_2}{T_1}+k\,(T_2-T_1)$,

where the values of specific heat are $C_p=a+kT$ and $C_v=b+kT$, a, b and k being constants.

Considering 1 kg of gas

$$q=w+u_2-u_1$$

$$dq=\frac{Pdv}{J}+C_v\,dT$$

From definition, $ds=\dfrac{dq}{T}=\dfrac{Pdv}{JT}+\dfrac{C_v\,dT}{T}$

Integrating

$$s_2-s_1=\frac{R}{J}\int_{v_1}^{v_2}\frac{dv}{v}+\int_{T_2}^{T_1}C_v\frac{dT}{T} \qquad (i)$$

Since, $C_p-C_v=\dfrac{R}{J}$

$$\therefore \qquad \frac{R}{J}=(a+kT)-(b+kT)=a-b \qquad (ii)$$

From Eq. (*i*) and (*ii*)

$$s_2-s_1=(a-b)\int_{v_1}^{v_2}\frac{dv}{v}+\int_{T_1}^{T_2}\left[(b+kT)\frac{dT}{T}\right]$$

$$=(a-b)\log_e\frac{v_2}{v_1}+b\log_e\frac{T_2}{T_1}+k\,(T_2-T_1) \qquad (iii)$$

Since $\quad \dfrac{v_2}{v_1}=\left(\dfrac{T_1}{T_2}\right)^{\frac{1}{n-1}}$

$$\therefore \quad s_2-s_1=(a-b)\log_e\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}+b\log_e\frac{T_2}{T_1}+k(T_2-T_1)$$

$$=-\left(\frac{a-b}{n-1}\right)\log_e\frac{T_2}{T_1}+b\log_e\frac{T_2}{T_1}+k(T_2-T_1)$$

$$=\left(b-\frac{a-b}{n-1}\right)\log_e\frac{T_2}{T_1}+k(T_2-T_1) \qquad (2\cdot12)$$

Q.E.D.

2.10. Approximation for Heat Absorbed.

Let the curve 1·2 represent the heating of a gas on temperature-entropy diagram from temperature T_1 to T_2 (*see* Fig. 2·7). The area under the curve 1·2 represents the heat absorbed.

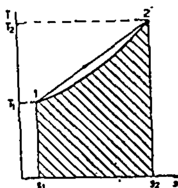


Fig. 2·7. Approximation for heat absorbed.

The curvature of the curve is usually small and the curve may therefore be assumed a straight line.

Approximately

Heat absorbed, $\quad Q=(S_2-S_1)\times$ Mean temperature

$$=(S_2-S_1)\times\frac{(T_1+T_2)}{2} \qquad (2\cdot13)$$

Important Points

1. Entropy increases with the addition of heat and decreases with the rejection of heat. Therefore, in all polytropic expansions where n is $<\gamma$ and in isothermal expansion, entropy will increase

the heat flows in the system. Similarly in all polytropic compression where n is $>\gamma$ and in isothermal compression, entropy will decrease as the heat is rejected or flows out of the system.

In adiabatic expansion or compression entropy remains constant.

2. The general mistake in solving problems in entropy is to neglect the mass of the substance. Therefore it should be made a habit to put down the formula for m kg. 'm' must always be calculated unless it is mentioned as 1 kg.

3. The equation for change of entropy, by definition, is in terms of a reversible process. However, it is possible to find the change of entropy in any irreversible process provided its initial and final conditions are known. As entropy is a property it is independent of the path of the process. Therefore change of entropy in any irreversible process can be found out by assuming any reversible path between the end states and calculating $\int_1^2 \frac{dQ_{rev}}{T}$.

4. The change of entropy in polytropic expansion is given in terms of V and T, T and P and P and V, but it is best to remember only one form and, if necessary, derive other forms.

ILLUSTRATIVE EXAMPLES

2-1. Constant pressure and constant volume processes : ΔS.

Show that the slope of the constant pressure line through a point on the temperature-entropy diagram for a perfect gas is proportional to the absolute temperature at that point. What is the slope of a constant-volume line through the same point ?

0·34 m³ of a perfect gas at constant pressure of 2·8 kgf/cm² is heated from 100°C to 300°C and is then cooled at constant volume to its initial temperature. Calculate the overall change of entropy. Given $C_p=0{\cdot}25$ and $C_v=0{\cdot}18$.

For constant pressure process

$$ds=\frac{dq}{T}=\frac{C_p dT}{T}$$

∴ Slope of the constant pressure line on T-s diagram, Fig. 2·8

$$\frac{dT}{ds}=\frac{T}{C_p} \text{ i.e. } \frac{dT}{ds} \propto T \qquad [C_p \text{ being constant}$$

Q.E.D.

For constant volume process

$$ds=\frac{dq}{T}=\frac{C_v dT}{T}$$

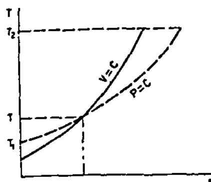


Fig. 2 8.

∴ Slope of the constant volume line on T-s diagram, Fig. 2·8.

$$\frac{dT}{ds}=\frac{T}{C_v} \quad i\,e \quad \frac{dT}{dS} \propto T \qquad [C_v \text{ being constant}$$

Q.E.D.

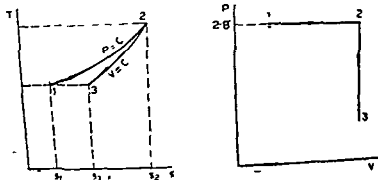


Fig. 2 9.

$$C_p-C_v=\frac{R}{J}$$

$$0\,25-0{\cdot}18=\frac{R}{427} \quad \therefore\ R=29\,89 \text{ kgf m}$$

$$PV=mRT$$

$$2\,8\times10^4\times0{\cdot}340=m\times29\,89\times373$$

$$\therefore \qquad m=0{\cdot}8541 \text{ kg}$$

$$S_2-S_1=mC_p\log_e\frac{T_2}{T_1}$$

$$=0{\cdot}8541\times0\,25\times\log_e\frac{573}{373}$$

$$=0{\cdot}0917$$

$$S_3-S_2=m\ C_v \log_e \frac{T_3}{T_2}$$

$$=0{\cdot}8541\times0{\cdot}18\times\log_e \frac{373}{573}$$

$$=-0{\cdot}066$$

$\therefore$ Overall change of entropy

$$\underline{S_3-S_1}=0{\cdot}0917-0{\cdot}066$$

$$=\underline{0{\cdot}0257}$$ Ans.

Note. On T-s diagram the constant volume curve is steeper than the constant pressure curve.

2·2. Polytropic expansion : n ; ΔS.

What is available and unavailable energy ? State the second law of thermodynamics.

The initial pressure, volume and temperature of a gas are $P_1=28$ kgf/cm², $V_1=0{\cdot}01$ m³ and $T_1=2{,}200°K$. The gas expands in an engine cylinder to a volume of 0·17 m³ and a pressure of 4·2 kgf/cm². Find the mean index of expansion, the heat flow to or from the gas and the change of entropy. Take gas constant $R=30{\cdot}5$ and $C_v=0{\cdot}19$.

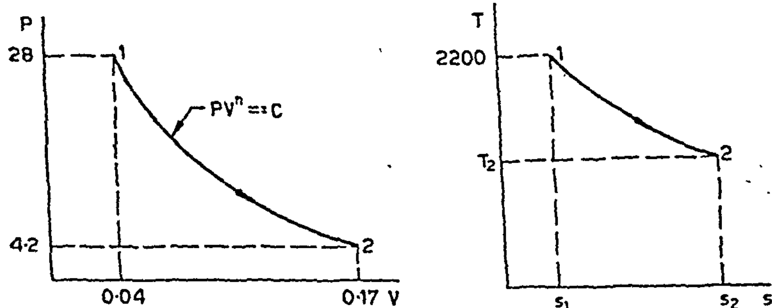


Fig. 2·10.

$$PV=mRT$$

$$28\times10^4\times0{\cdot}04=m\times30{\cdot}5\times2{,}240 \quad \therefore \quad m=0{\cdot}1669 \text{ kg}$$

$$P_1V_1^n=P_2V_2^n$$

$$28\times40^n=4{\cdot}2\times170^n \quad \therefore \quad \underline{n=1{\cdot}311}$$ Ans.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$\frac{28\times0{\cdot}04}{2{,}200}=\frac{4{\cdot}2\times0{\cdot}170}{T_2}$$

$$\therefore \quad T_2=1403°K.$$

Heat flow, $\underline{Q}=\frac{P_1V_1-P_2V_2}{J(n-1)}+mC_v(T_2-T_1)$

$$=\frac{10\times(28\times40-4\cdot2\times270)}{427\times(1\cdot311-1)}+0\cdot1669\times0\cdot19\times(1{,}403-2{,}200)$$

$=\underline{5\cdot32\text{ kcal}}$ Ans.

$$\underline{S_2-S_1}=m\left[C_v\log_e\frac{T_2}{T_1}+\frac{R}{J}\times\log_e\frac{V_2}{V_1}\right]$$

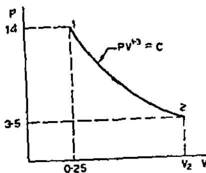
$$=0\cdot1669\left[0\cdot19\times\log_e\frac{1{,}403}{2{,}200}+\frac{30\cdot5}{427}\times\log_e\frac{170}{40}\right]$$

$=\underline{0\cdot00301}$ units Ans.

2-3. Polytropic expansion : Q ; ΔS.

Define entropy. Is it a property of a working fluid ? Can entropy be measured directly by an instrument ?

0·25 m³ of gas under a pressure of 14 kgf/cm² and temperature 60°C is expanded to a pressure of 3·5 kgf/cm² along a curve, the equation of which is $PV^{1\cdot3}$=constant. If specific heats at constant pressure and at constant volume are 0·52 and 0·37 respectively, find (a) the heat added or rejected during the expansion, and (b) the change of entropy.



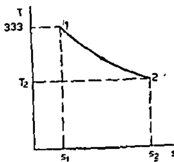


Fig 2 11.

(a) $$C_p-C_v=\frac{R}{J}$$

$$0\cdot52-0\cdot37=\frac{R}{427}\quad\therefore\ R=64\cdot1$$

$$PV=mRT$$

$$14\times10^4\times0\cdot25=m\times64\cdot1\times333$$

$$\therefore\quad m=1\cdot64\text{ kg}$$

$$P_1V_1^n=P_2V_2^n$$

$$14\times0\cdot25^{1\cdot3}=13\cdot5\times V_2^{1\cdot3}$$

$$V_2=0\cdot7261\text{ m}^3$$

$$\frac{P_1V_1}{T_2}=\frac{P_2V_2}{T_2}$$

$$\frac{14\times0{\cdot}25}{333}=\frac{3{\cdot}5\times0{\cdot}7261}{T_2}$$

$$\therefore \quad T_2=241{\cdot}9°K$$

$$W=\frac{P_1V_1-P_2V_2}{J(n-1)}$$

$$=\frac{10^4\times(14\times0{\cdot}25-3{\cdot}5\times0{\cdot}7261)}{427\times(1{\cdot}3-1)}$$

$$=74{\cdot}85 \text{ kcal}$$

$$U_2-U_1=mC_v\,(T_2-T_1)$$

$$=1{\cdot}64\times0{\cdot}37\times(241{\cdot}9-333)$$

$$=-55{\cdot}27 \text{ kcal.}$$

$$\therefore \quad Q=W+(U_2-U_1)$$

$$=74{\cdot}85-55{\cdot}27$$

$$=19{\cdot}58 \text{ kcal (added)} \qquad \text{Ans.}$$

(b) $$S_2-S_1=m\left[\frac{R}{J}\log_e\frac{V_2}{V_1}+C_v\log_e\frac{T_2}{T_1}\right]$$

$$=1{\cdot}64\left[(0{\cdot}52-0{\cdot}37)\times\log_e\frac{0{\cdot}7261}{0{\cdot}25}+0{\cdot}37\times\log_e\frac{241{\cdot}9}{333}\right]$$

$$=0{\cdot}0684 \qquad \text{Ans.}$$

2-4. Polytropic compression : ΔS ; ΔU.

Prove that the change of entropy per kg of a gas in polytropic expansion is given by $C_v\frac{\gamma-n}{n-1}\log_e\frac{T_1}{T_2}$ *where all the notations have the usual meaning. State the assumptions made.*

A petrol engine has a bore and stroke of 8 cm and 10 cm respectively and compression ratio 8 : 1. The temperature and pressure at the beginning of the compression stroke are 70°C and 1·033 kgf/cm² respectively. The compression index is 1·3. Find (a) the change in entropy and (b) the change in internal energy during the compression stroke. Assume $R=29{\cdot}27$ *and* $C_v=0{\cdot}169$.

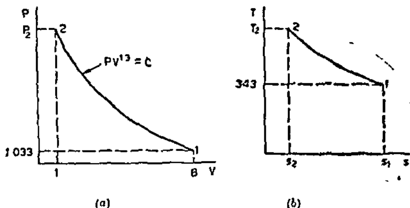


Fig. 2·12.

Stroke volume $= \frac{\pi}{4} \times 8^2 \times 10 = 503$ cc

Compression ratio, $8 = \frac{503+V_2}{V_2}$ $\therefore$ $V_2 = 72$ cc

$\therefore$ *Total cylinder volume*

$$V_1 = 503 + 72 = 575 \text{ cc}$$

$$PV = mRT$$

$$1{\cdot}033 \times 10^4 \times 575 \times 10^{-6} = m \times 29\ 27 \times 343 \quad \therefore \quad m = 0\ 000592 \text{ kg}$$

$$\frac{T_2}{T_1} = r^{n-1}, \quad \frac{T_2}{343} = 8^{1\ 3-1} \quad \therefore \quad T_2 = 640^\circ K$$

$$C_p - C_v = \frac{R}{J}$$

Dividing by C_v and rearranging

$$\gamma = \frac{R}{JC_v} + 1 = \frac{29\ 27}{427 \times 0\ 169} + 1 = 1{\cdot}406$$

(a) $$\underline{S_2 - S_1} = mC_v \frac{\gamma - n}{n-1} \log_e \frac{T_1}{T_2}$$

$$= 0\ 000592 \times 0\ 169 \times \frac{1{\cdot}406 - 1\ 3}{1\ 3 - 1} \times \log_e \frac{343}{640}$$

$$= \underline{-0\ 000022}$$ Ans.

(—ve sign indicates decrease in entropy)

$$\frac{P_1V_1}{T_2}=\frac{P_2V_2}{T_2}$$

$$\frac{14\times0{\cdot}25}{333}=\frac{3{\cdot}5\times0{\cdot}7261}{T_2}$$

$$\therefore \quad T_2=241{\cdot}9°K$$

$$W=\frac{P_1V_1-P_2V_2}{J(n-1)}$$

$$=\frac{10^4\times(14\times0{\cdot}25-3{\cdot}5\times0{\cdot}7261)}{427\times(1{\cdot}3-1)}$$

$$=74{\cdot}85 \text{ kcal}$$

$$U_2-U_1=mC_v(T_2-T_1)$$

$$=1{\cdot}64\times0{\cdot}37\times(241{\cdot}9-333)$$

$$=-55{\cdot}27 \text{ kcal.}$$

$$\therefore \quad Q=W+(U_2-U_1)$$

$$=74{\cdot}85-55{\cdot}27$$

$$=19{\cdot}58 \text{ kcal (added)}$$ Ans.

(b) $$S_2-S_1=m\left[\frac{R}{J}\log_e\frac{V_2}{V_1}+C_v\log_e\frac{T_2}{T_1}\right]$$

$$=1{\cdot}64\left[(0{\cdot}52-0{\cdot}37)\times\log_e\frac{0{\cdot}7261}{0{\cdot}25}+0{\cdot}37\times\log_e\frac{241{\cdot}9}{333}\right]$$

$$=0{\cdot}0684$$ Ans.

2-4. Polytropic compression : ΔS ; ΔU.

Prove that the change of entropy per kg of a gas in polytropic expansion is given by $C_v\frac{\gamma-n}{n-1}\log_e\frac{T_1}{T_2}$ *where all the notations have the usual meaning. State the assumptions made.*

A petrol engine has a bore and stroke of 8 cm and 10 cm respectively and compression ratio 8 : 1. The temperature and pressure at the beginning of the compression stroke are 70°C and 1·033 kgf/cm² respectively. The compression index is 1·3. Find (a) the change in entropy and (b) the change in internal energy during the compression stroke. Assume R=29·27 and C_v=0·169.

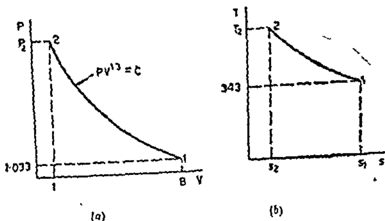


Fig. 2·12.

Stroke volume $= \frac{\pi}{4} \times 8^2 \times 10 = 503$ cc

Compression ratio, $8 = \frac{503+V_2}{V_2}$ $\therefore$ $V_2 = 72$ cc

$\therefore$ Total cylinder volume

$$V_1 = 503 + 72 = 575 \text{ cc}$$

$$PV = mRT$$

$1\cdot033 \times 10^4 \times 575 \times 10^{-6} = m \times 29\cdot27 \times 343$ $\therefore$ $m = 0\cdot000592$ kg

$$\frac{T_2}{T_1} = r^{n-1}, \quad \frac{T_2}{343} = 8^{1\cdot3-1} \quad \therefore \quad T_2 = 640°K$$

$$C_p - C_v = \frac{R}{J}$$

Dividing by C_v and rearranging

$$\gamma = \frac{R}{JC_v} + 1 = \frac{29\cdot27}{427 \times 0\cdot169} + 1 = 1\cdot406$$

(a) $$\underline{S_2 - S_1} = mC_v \frac{\gamma - n}{n-1} \log_e \frac{T_1}{T_2}$$

$$= 0\cdot000592 \times 0\cdot169 \times \frac{1\cdot406 - 1\cdot3}{1\cdot3 - 1} \times \log_e \frac{343}{640}$$

$$= \underline{-0\cdot000022}$$ Ans.

(–ve sign indicates decrease in entropy)

(b) $$U_2-U_1=mC_v\ (T_2-T_1)$$
$$=0{\cdot}000592\times0{\cdot}169\times(640-343)$$
$$=0{\cdot}02972 \text{ kcal}$$ Ans.

Note.—During compression heat is given out and therefore the entropy decreases. Temperature, however, rises in compression and therefore the internal energy increases.

2-5. Polytropic compression : universal gas constant.

Show that when one kg of a perfect gas is expanded through a volume ratio of r, the law of expansion being PV^n=constant, the change in entropy is given by $(C_p-nC_v)\log_e r$.

A gas having a molecular weight of 32 is compressed from an initial pressure and temperature of 1·06 kgf/cm² and 30°C respectively to a find temperature of 120°C. If the decrease of entropy during the process is 0·01 units per kg find the final pressure. Assume $C_v=0{\cdot}17$.

For theory—see text.

$$MC_p-MC_v=R_{mol}$$

or
$$C_p=\frac{R_{mol}}{M}+C_v$$

$$=\frac{1{\cdot}986}{32}+0{\cdot}17=0{\cdot}232$$

$$s_2-s_1=(C_p-nC_v)\log_e r$$

$$=(C_p-nC_v)\ \log_e\left(\frac{T_2}{T_1}\right)^{\frac{1}{n-1}}$$

Substituting,
$$0{\cdot}01=(0{\cdot}232-n\times0{\cdot}17)\times\log_e\left(\frac{393}{303}\right)^{\frac{1}{n-1}}$$

$\therefore$
$$n=1{\cdot}297$$

$$\frac{P_2}{P_1}=\left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$\frac{P_2}{1{\cdot}06}=\left(\frac{393}{303}\right)^{\frac{1{\cdot}297}{1{\cdot}297-1}}$$

$\therefore$
$$P_2=3{\cdot}306 \text{ kgf/cm}^2$$ Ans.

2·6. Adiabatic expansion : pressure ratio ; γ ; ΔU.

Prove that the change of entropy in a polytropic expansion is given by

$$\left(b-\frac{a-b}{n-1}\right)\log_e \frac{T_2}{T_1}+k(T_2-T_1)$$

where T_1 and T_2 are temperatures before and after compression and the values of specific heats are $C_p=a+kT$ and $C_v=b+kT$, where a, b and k are constants.

0·145 kg of a gas is expanded adiabatically from 527°C to 227°C. Using the expression derived, determine (a) the ratio of pressure before and after expansion, (b) the average value of γ, and (c) the change in the internal energy. Assume that $a=0{\cdot}23$, $b=0{\cdot}165$ and $k=0{\cdot}000054$, T being the absolute temperature in °C.

Compare the above value of γ with that obtained from the average values of the specific heats over the particular temperature range.

For theory—see text.

(*a*) In numerical portion the pressure range is required so the expression derived may be put in terms of pressures.

Reproducing Equation

$$s_2-s_1=(a-b)\log_e \frac{v_2}{v_1}+b\log_e \frac{T_2}{T_1}+k(T_2-T_1)$$

$$\frac{P_1v_1}{T_1}=\frac{P_2v_2}{T_2}$$

$$\therefore \quad \frac{v_2}{v_1}=\frac{P_1}{P_2}\times\frac{T_2}{T_1}$$

or $$s_2-s_1=(a-b)\log_e \frac{P_1T_2}{P_2T_1}+b\log_e \frac{T_2}{T_1}+K(T_2-T_1)$$

$$=(a-b)\log_e \frac{P_1}{P_2}+a\log_e \frac{T_2}{T_1}+k(T_2-T_1)$$

In adiabatic expansion, $s_2-s_1=0$. By substituting the values

$$O=(0{\cdot}23-0{\cdot}165)\log_e \frac{P_1}{P_2}+0{\cdot}23\log_e \frac{500}{800}+0{\cdot}000054\,(500-800)$$

$$\therefore \quad \frac{P_1}{P_2}=6{\cdot}769 \qquad \textbf{Ans.}$$

(*b*) $$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{500}{800}=\left(\frac{1}{6{\cdot}769}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \underline{\gamma=1{\cdot}326}$$

Average value of $C_p = a + kT$

$$= 0{\cdot}23 + 0{\cdot}000054 \left(\frac{800+500}{2}\right) = 0{\cdot}2651$$

Average value of $C_v = b + kT$

$$= 0{\cdot}165 + 0{\cdot}000054 \times \left(\frac{800+500}{2}\right) = 0{\cdot}2001$$

$$\therefore \qquad \gamma = \frac{C_p}{C_v} = \frac{0{\cdot}2651}{0{\cdot}2001} = 1{\cdot}325$$

The two values of γ are not fundamentally identical but the numerical difference is negligible.

(c) Change in internal energy

$$\Delta U = mC_v\,(T_2 - T_1)$$
$$= 0{\cdot}145 \times 0{\cdot}2001 \times (500 - 800)$$
$$= -8{\cdot}7 \text{ kcal} \qquad \text{Ans.}$$

2-7. Polytropic expansion : ΔS by approximate method.

Prove that the approximate change of entropy in a polytropic expansion is equal to the quantity of heat exchanged divided by the mean absolute temperature.

One kg of air is compressed in a cylinder according to the law $PV^{1{\cdot}3}$=constant. If compression ratio is 15 and the initial temperature is 100°C, find the change in entropy of air compressed stating whether it is an increase or a decrease. What is the percentage error if change in entropy is calculated by the approximate method ? $C_p = 0{\cdot}238$ and $C_v = 0{\cdot}169$.

For theory—see text.

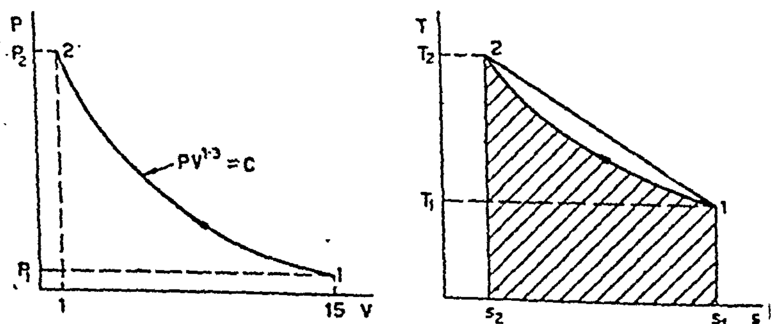


Fig. 2·13.

$$\gamma=\frac{C_p}{C_v}=\frac{0\ 238}{0\ 169}=1{\cdot}41$$

$$\frac{T_2}{T_1}=\gamma^{n-1}$$

$$\frac{T_2}{373}=15^{1\ 3-1} \quad \therefore\ T_2=840°K$$

$$\underline{S_2-S_1}=m\ C_v\ \frac{\gamma-n}{n-1}\log_e\frac{T_1}{T_2}$$

$$=1\times0\ 169\times\left(\frac{1\ 41-1{\cdot}3}{1{\cdot}3-1}\right)\times\log_e\frac{373}{840}$$

$$=\underline{-0\ 0503}$$ Ans.

(—ve sign indicates decrease in entropy)

$$\underline{Q}=m\ C_v\ \frac{\gamma-n}{n-1}(T_1-T_2)$$

$$=1\times0\ 169\times\left(\frac{1\ 41-1\ 3}{1{\cdot}3-1}\right)\times(373-840)$$

$$=\underline{-29\ \text{kcal}}$$ Ans

Approximate change of entropy

$$=\frac{Q}{(T_1+T_2)/2}$$

$$=\frac{-29}{(373+840)/2}=-0\ 0478$$

$\therefore$ Percentage error$=\dfrac{0\ 0503-0\ 0478}{0\ 0503}=\underline{4{\cdot}97\%}$ Ans

Note.—The approximate value is lower, because in the formula $Q=\Delta S\times T_{mean}$, actual value of Q is substituted instead of approximate value (*i.e.*, area under straight line AB) which is higher.

2·8. Polytropic and constant volume process : Q ; ΔS.

The following data refer to the condition of air in the cylinder of an internal combustion engine. At the beginning of compression : pressure, 1 033 kgf/cm²; volume, 50 litres ; temperature, 100°C. At the end of compression : pressure, 28 kgf/cm² ; volume, 4 litres. After constant volume heat addition : pressure=56 kgf/cm².

Assuming that the specific heats of air, $C_p=0{\cdot}238$ and $C_v=0\ 169$, remain constant over whole range of working, calculate the change of entropy of contents of cylinder during each operation, indicating

whether this is an increase or a decrease. Calculate also the interchange of heat between air and cylinder during compression and state the direction of flow.

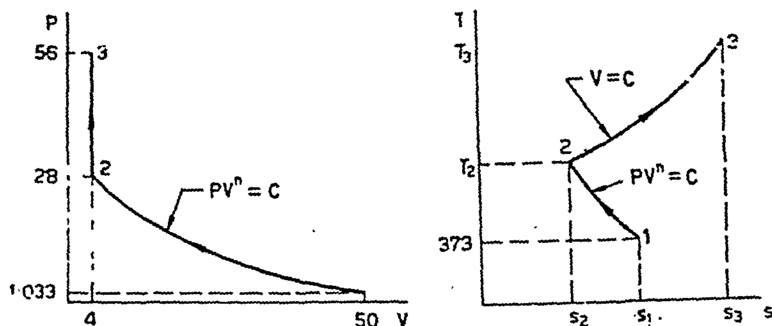


Fig. 2·14.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$\frac{1\cdot033\times50}{373}=\frac{28\times4}{T_2} \qquad \therefore \quad T_2=809°K$$

$$P_1V_1^n=P_2V_2^n$$

$$1\cdot033\times50^n=28\times4^n \qquad \therefore \quad n=1\cdot306$$

$$\gamma=\frac{0\cdot238}{0\cdot169}=1\cdot408$$

$$R=(0\cdot238-0\cdot169)\times427=29\cdot46$$

$$PV=mRT$$

$$1\cdot033\times10^4\times0\cdot05=m\times29\cdot46\times373$$

$$\therefore \qquad m=0\cdot047 \text{ kg}$$

$$S_2-S_1=m\ (C_p-nC_v)\log_e\frac{V_2}{V_1}$$

$$=0\cdot047\times(0\cdot239-1\cdot306\times0\cdot169)\times\log_e\frac{4}{50}$$

$$=\underline{-0\cdot002173}$$

Ans.

(∵ -ve sign indicates decrease in entropy)

$$Q=m\ \frac{\gamma-n}{n-1}C_v\ (T_1-T_2)$$

$$=0\cdot047\ \times\left(\frac{1\cdot408-1\cdot306}{1\cdot306-1}\right)\times0\cdot169\times(373-809)$$

$$=\underline{-1\cdot154 \text{ kcal}}$$

Ans.

(—ve sign indicates loss of heat)

Process 2·3 is a constant volume process

$$\therefore \quad \frac{P_2}{T_2}=\frac{P_3}{T_3}$$

$$\frac{28}{809}=\frac{56}{T_3} \quad \therefore \quad T_3=1{,}618°K$$

$$S_3-S_2=m\,C_v \log_e \frac{T_3}{T_2}$$

$$=0{\cdot}047\times 0{\cdot}169\times \log_e\left(\frac{1{,}618}{908}\right)$$

$$=\underline{0{\cdot}005505} \qquad \text{Ans.}$$

Note.—During compression 1-2 heat is rejected and hence entropy decreases. During constant volume process heat is added and hence entropy increases.

2·9 Adiabatic and throttling processes : change in entropy.

One m³ of air at 30°C and 1 kgf/cm² is compressed adiabatically to a volume of 0·2 m³. It is throttled such that its pressure reduces to 4·837 kgf/cm². Find the change in entropy in each operation. Assume $C_p=0{\cdot}238$ and $C_v=0{\cdot}169$. Sketch the operation on P-V and T-S diagrams

Operation 1-2 represents adiabatic compression. Throttling *process is similar to adiabatic process in the sense that no heat is* supplied or rejected. However, in the former no work is done and the final temperature becomes equal to initial temperature due to

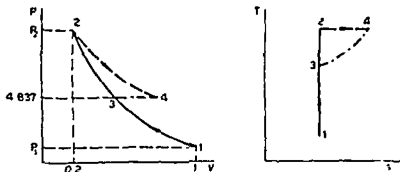


Fig. 2·15.

reheating. Throttling process may be assumed consisting of two processes : adiabatic expansion (operation 2-3) upto pressure

4·837 kgf/cm² and constant pressure heating to initial temperature (operation 3-4). Throttling is an irreversible process.

The change in entropy is therefore same as in an isothermal process. Heat interchange is not given by the area under T-S diagram as in an irreversible process $dQ \neq T \times dS$.

$$\gamma = \frac{C_p}{C_v} = \frac{0{\cdot}238}{0{\cdot}169} = 1{\cdot}41$$

$$C_p - C_v = \frac{R}{J}$$

$$0{\cdot}238 - 0{\cdot}169 = \frac{R}{427} \quad \therefore \; R = 29{\cdot}26$$

$$PV = mRT$$

$$1 \times 10^4 \times 1 = m \times 29{\cdot}26 \times 303 \quad \therefore \; m = 1{\cdot}141 \text{ kg}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$1 \times 1^{1{\cdot}41} = P_2 \times 0{\cdot}2^{1{\cdot}41} \quad \therefore \; P_2 = 9{\cdot}674 \text{ kgf/cm}^2$$

Change in entropy in adiabatic operation is zero Ans.

In throttling process, $T_2 = T_4$

$$\therefore \quad S_4 - S_2 = m \frac{R}{J} \log_e \frac{P_2}{P_4}$$

$$= 1{\cdot}141 \times \frac{29{\cdot}26}{427} \times \log_e \left(\frac{9{\cdot}674}{4{\cdot}837} \right)$$

$$= 0{\cdot}06742$$ Ans.

Note.—Though no external heat is supplied entropy increases because of internal reheating due to friction.

2-10. ΔU ; ΔS, for different thermodynamic processes.

What do areas mean on T-S graphs ?

One kilogram of air ($C_p = 0{\cdot}24$, $C_v = 0{\cdot}17$) initially at 227°C and 21 kgf/cm² receives 100 kcal of heat. Determine the changes in internal energy and entropy if the heat is received (a) at constant temperature, (b) at constant volume, (c) at constant pressure, (d) when the air expands according to the law $PV^{1{\cdot}15}$ = constant. Find also for each case the product of the increase in entropy and the mean absolute temperature.

(a) *Constant temperature*

$$U_2 - U_1 = m\,C_v\,(T_2 - T_1) = 0 \qquad [T_2 = T_1$$ Ans.

$$S_2 - S_1 = \frac{Q}{T} = \frac{100}{(227 + 273)}$$

$$= 0{\cdot}2/\text{kg}$$ Ans.

$$T\text{-}S \text{ product} = 500 \times 0{\cdot}2$$

$$= 100 \text{ kcal/kg}$$ Ans.

(b) *Constant volume*

$$Q=W+\Delta U=0+m\ C_v\ (T_2-T_1)$$

$$100=1\times0{\cdot}17\times(T_2-500)$$

$$\therefore \quad T_2=1{,}088°K$$

$$U_2-U_1=Q=m\ C_v\ (T_2-T_1)$$

$$=100 \text{ kcal/kg} \qquad \text{Ans.}$$

$$S_2-S_1=m\ C_v \log_e \frac{T_2}{T_1}$$

$$=1\times0{\cdot}17\times\log_e \frac{1{,}088}{500}$$

$$=0{\cdot}1322/\text{kg} \qquad \text{Ans.}$$

$$T\text{-}S \text{ product}=\left(\frac{1{,}088+500}{2}\right)\times0{\cdot}1322$$

$$=105 \text{ kcal/kg} \qquad \text{Ans.}$$

(c) *Constant pressure*

$$Q=m\ C_p\ (T_2-T_1)$$

$$100=1\times0{\cdot}24\times(T_2-500)$$

$$\therefore \quad T_2=916{\cdot}7°K$$

$$U_2-U_1=m\ C_v\ (T_2-T_1)$$

$$=1\times0{\cdot}17\times(916{\cdot}7-500)$$

$$=70{\cdot}74 \text{ kcal/kg} \qquad \text{Ans.}$$

$$S_2-S_1=m\ C_p \log_e \frac{T_2}{T_1}$$

$$=1\times0{\cdot}24\times\log_e\left(\frac{916{\cdot}7}{500}\right)$$

$$=0{\cdot}14551 \text{ kg} \qquad \text{Ans.}$$

$$T\text{-}S \text{ product}=\left(\frac{916{\cdot}7+500}{2}\right)\times0{\cdot}1455$$

$$=103 \text{ kcal/kg} \qquad \text{Ans.}$$

(d) $PV^{1{\cdot}15}=$*Constant*

$$Q=\frac{P_1V_1-P_2V_2}{J\ (n-1)}+mC_v(T_2-T_1)$$

Putting $PV=mRT$ and rearranging, we have

$$Q=m\ C_v\ \frac{\gamma-n}{n-1}\ (T_1-T_2)$$

$$\gamma=\frac{C_p}{C_v}=\frac{0{\cdot}24}{0{\cdot}17}=1{\cdot}41$$

$$\therefore \quad 100 = 1 \times 0{\cdot}17 \times \left(\frac{1{\cdot}41 - 1{\cdot}15}{1{\cdot}15 - 1}\right) \times (500 - T_2)$$

$$\therefore \quad T_2 = 161°K$$

$$\underline{U_2 - U_1} = m\, C_v\, (T_2 - T_1)$$

$$= 1 \times 0{\cdot}17 \times (161 - 500)$$

$$= \underline{-57{\cdot}6 \text{ kcal/kg}} \qquad \text{Ans.}$$

$$\underline{S_2 - S_1} = m\, C_v\, \frac{\gamma - n}{n - 1} \log_e \frac{T_1}{T_2}$$

$$= 1 \times 0{\cdot}17 \times \left(\frac{1{\cdot}41 - 1{\cdot}15}{1{\cdot}15 - 1}\right) \times \log_e \frac{500}{161}$$

$$= \underline{0{\cdot}3339/\text{kg}} \qquad \text{Ans.}$$

$$\underline{T\text{-}S \text{ product}} = \left(\frac{161 + 500}{2}\right) \times 0{\cdot}3339$$

$$= \underline{110 \text{ kcal/kg}} \qquad \text{Ans.}$$

2·11. Straight line expansion : change of entropy ; maximum internal energy and heat interchange.

The expansion of a perfect gas is so controlled that the pressure changes according to the law $P = aV + b$, *where a and b are constants and V is the volume. The mass of the gas is 0·68 kg and the initial and final pressures are 7 kgf/cm² and 2·1 kgf/cm² and the corresponding volumes are 0·084 m³ and 0·28 m³. The characteristic gas constant is 26·5 meter kg °C units, and* $\gamma = 1{\cdot}39$. *Find (a) the change in entropy per kg during the expansion, (b) the maximum value of the internal energy per kg reckoned from 0°C, (c) the heat added upto the point of maximum internal energy, (d) heat rejected during the rest of the operation and hence (e) the net heat added or removed during the process.*

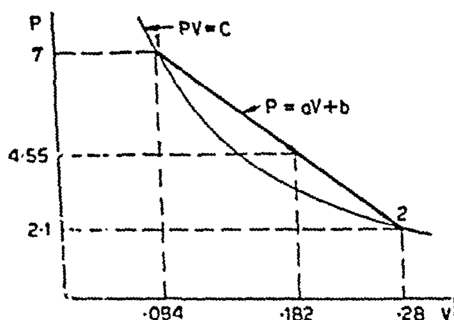


Fig. 2·16.

$P=aV+b$ is an equation of straight line 1-2 in the P-V diagram

$$7=84\,a+b$$

and $$2{\cdot}1=280\,a+b$$

$\therefore$ $$a=-0{\cdot}025 \quad \text{and} \quad b=9{\cdot}1$$

Note. Though $P_1V_1=P_2V_2$ and hence $T_1=T_2$, the expansion is not hyperbolic (or isothermal). It is only incidental that initial and final points also lie on hyperbolic curve.

(a) Change of entropy per kg

$$s_2-s_1=C_v \log_e \frac{T_2}{T_1}+\frac{R}{J}\log_e \frac{V_2}{V_1}$$

$$=0+\frac{26\ 5}{427}\log_e \frac{280}{84}$$

$$=\underline{0\ 07471}$$ Ans.

(b) $$C_v=\frac{R}{J\ (\gamma-1)}$$

$$=\frac{26\ 5}{427\times(1{\cdot}39-1)}=0\ 159$$

Maximum internal energy is at the point of maximum temperature. To find this point differentiate P with respect to T

$$P=aV+b=a\,\frac{mRT}{P}+b$$

or $$P^2-Pb=a\,mRT$$

Differentiating

$$2\,PdP-bdP=a\,mRT$$

or $$\frac{dT}{dP}=\frac{2P-b}{amR}=0$$

If $P=\frac{b}{2}=4{\cdot}55$ kgf/cm² and $V=182$ litres

($4{\cdot}55\times182$ is greater than any other P-V product)

$$T_{max}=\frac{PV}{mR}=\frac{4\ 55\times10^4\times182\times10^{-3}}{0\ 68\times26\ 5}=460\ K$$

Internal energy per kg reckoned from 0°C

$$=C_v\ (T_{max}-T)$$

$$=0\ 159\ (460-273)$$

$$=\underline{29\ 7\ \text{kcal}}$$ Ans.

(c) $$PV=mRT$$

$$7\times10^4\times84\times10^{-3}=0\ 68\times26\ 5\times T_1$$

$\therefore$ $$T_1=326°K$$

From 1 to T_{max}

$Q_1 = W_1 + \Delta U$

$$= \frac{(182-84)\ 10^{-3} \times (4{\cdot}55+7) \times 10^4}{427 \times 2} + 0{\cdot}68 \times 0{\cdot}159\ (460-326)$$

$= 13{\cdot}25 + 14{\cdot}49$

$= \underline{27{\cdot}74 \text{ kcal.}}$ Ans.

(*d*) From T_{max} to 2

$Q_2 = W_2 + \Delta U$

$$= \frac{(280-182)\ 10^{-3} \times (4{\cdot}55+2{\cdot}1) \times 10^4}{427 \times 2} + 0{\cdot}68 \times 0{\cdot}159\ (326-460)$$

$= 7{\cdot}63 - 14{\cdot}49$

$= \underline{-6{\cdot}86 \text{ kcal}}$ Ans.

(—ve sign indicates heat flows out)

(*e*) Net heat during the whole process

$$Q = Q_1 + Q_2$$

$$= 27{\cdot}74 - 6{\cdot}86 = \underline{20{\cdot}88 \text{ kcal}}$$ Ans.

Note.—The increase in internal energy from 1 to T_{max} is equal to the decrease in internal energy from T_{max} to 2.

[Check. From 1 to 2

$Q = W + \Delta U$

$$= \frac{(280-84) \times 10^{-3} \times (7+2{\cdot}1) \times 10^4}{427 \times 2} + 0 = 20{\cdot}88 \text{ kcal}]$$

2·12. Joule free expansion. ΔS of universe in revesible and irreversible process.

Explain why reversibility is used as a criterion of an ideal process.

1·5 kg of air undergoes a Joule free expansion from a volume of 2 m³ to a volume of 10 m³. Calculate the change of entropy of (a) the air, and (b) the surroundings. Also find the change of entropy for a reversible isothermal expansion between the same two states. Hence compare the net change of entropy of the universe in the two cases and comment on the results. Take R=29·27.

In Joule free expansion the final temperature is same as initial temperature. To find the change of entropy in Jolue expansion,

which is an irreversible process, assume a reversible path between the same end states. In this case reversible isothermal path can be assumed.

(a) ∴ Change of entropy

$$S_2-S_1=m\ \frac{R}{J}\log_e\frac{V_2}{V_1}$$

$$=1{\cdot}5\times\frac{29{\cdot}27}{427}\times\log_e\frac{10}{2}$$

$$=0{\cdot}1655$$

Ans.

(b) In case of Joule free expansion no heat is gained or lost to the surroundings.

∴ Change of entropy of surroundings

$$S_2-S_1=0$$

Hence, net change of entropy of the universe $=+0{\cdot}1655$

Reversible isothermal expansion

(a) Change of entropy

$$S_2-S_1=m\ \frac{R}{J}\log_e\frac{V_2}{V_1}$$

$$=1{\cdot}5\times\frac{29{\cdot}27}{427}\times\log_e\frac{10}{2}$$

$$=0{\cdot}1655$$

Ans.

(b) For a reversible isothermal expansion of air the heat is gained from the surroundings or the surroundings lose equal amout of heat.

∴ Change of entropy of surroundings

$$S_2-S_1=-0{\cdot}1655$$

Hence, net change of the entropy of the universe $=0$

From the problem it is seen that in case of reversible processes net change of entropy of the universe i.e. of the system and the surroundings, is zero. In case of irreversible processes net change of entropy of the universe is positive.

EXERCISES 2

2-1. Constant volume process : Change of entropy ; heat interchange.

Develop from first principles an expression in terms of P and V for gain of entropy of a perfect gas during a change from condition P_1, V_1 and T_1 to P_2, V_2 and T_2. Hence, find the change in entropy when (a) the pressure remains constant, (b) the volume remains constant. State the assumptions made.

A tank holds 1·1 m³ of air at 15 kgf/cm². This air is cooled in the tank to 27°C and its pressure is now 12 kgf/cm². Determine the heat extracted and decrease in entropy. Given $R=29{\cdot}26$, $\gamma=1{\cdot}4$.

[$m=15{\cdot}04$ kg ; $C_v=0{\cdot}1714$; $Q=193{\cdot}3$ kcal ; $S_2-S_1=0{\cdot}575$]

2-2. Constant volume and constant pressure heating : ΔS ; P_2 ; V_2.

Derive an expression for the change of entropy of a perfect gas receiving heat at constant volume.

One kg of air is heated from a temperature of 30°C to 120°C. Find the change of entropy if the gas is heated at (i) constant volume, (ii) constant pressure. For air $C_p=0{\cdot}24$ and $C_v=0{\cdot}171$.

If the initial pressure is 1·05 kgf/cm², find the final pressure in constant volume heating and the final volume in constant pressure heating.

[Constant pressure : $S_2-S_1=0{\cdot}06245$; $V_2=1{\cdot}102$ m³. Constant volume : $S_2-S_1=0{\cdot}04434$; $P_2=1{\cdot}362$ kgf/cm².]

2-3. Polytropic compression and constant pressure expansion : ΔS ; ΔU.

0·05 m³ of a gas at 1·05 kgf/cm² and 50°C is compressed to a volume of 0·01 m³ and 8·4 kgf/cm². Heat is then added at constant pressure until its volume is 0·015 m³. Determine the change of entropy and internal energy during each of these operations. State in each case whether the change is an increase or a decrease. $C_p=0{\cdot}24$, $C_v=0{\cdot}18$.

[$m=0{\cdot}06346$ kg ; $S_2-S_1=-0{\cdot}000759$; $T_2=516{\cdot}8°K$; $U_2-U_1=2{\cdot}214$ kcal ; $S_3-S_2=0{\cdot}006176$; $U_3-U_2=2{\cdot}952$ kcal.]

2-4. Polytropic expansion : change of entropy and heat interchange.

Define entropy and explain its physical significance. What is its importance in thermodynamics ?

0·4 m³ of gas at pressure 10 kgf/cm² and temperature 200°C expands according to the law $PV^{1{\cdot}25}=$ constant. During the process there is a loss in internal energy equal to 90 kcal. Calculate the change in entropy and heat interchange through cylinder walls. State the direction of flow of heat. Assume $C_p=0{\cdot}25$ and $C_v=0{\cdot}18$.

[$m=2{\cdot}829$ kg ; $T_2=296{\cdot}3°K$; $S_2-S_1=0{\cdot}1323$; $Q=50$ kcal]

2-5. Constant pressure heating and polytropic compression ; ΔS ; n.

2 m³ of air at 1·05 kgf/cm² and 27°C is heated at constant pressure until its volume is 2·5 m³. It is then compressed according

to the law PV^n=constant until its pressure is 5·25 kgf/cm² and its volume 0·6 m³. Calculate (i) the change of entropy for constant pressure, (ii) the index 'n' for the compression stroke, and (iii) the change of entropy for the compression stroke. R=29·27 and C_v=0·17.

[C_p=0·24 ; m=2·392 kg ; S_2-S_1=0·128 ; n=1·128 ; S_3-S_2 =−0·1647]

2-6. ΔS in different reversible processes between the same initial and final state.

A perfect gas undergoes two consecutive reversible processes changing the state of the gas from T_1, V_1 to T_3, V_3. Assuming different reversible processes between the end states prove that entropy change is same in all cases.

2-7. Isothermal process : change in entropy ; heat interchange.

The increase in entropy of 0·5 kg of oxygen undergoing an isothermal expansion at 300°C was 0·1. Determine the final volume and pressure if the initial pressure was 10 kgf/cm². Also calculate the heat exchanged during the process. Assume universal gas constant as 848 kgf m/kilo mol/°K.

[V_1=0·07593 m³ ; V_2=1·906 m³ , P_2=0·3984 kgf/cm² ; $Q=W$ =57·31 kcal.]

2-8. Isothermal, constant volume and constant pressure processes; Δs.

0·1 kg of air is contained in a cylinder behind a piston at 1·05 kgf/cm² and 57°C. It is heated at constant volume until its pressure is 5·25 kgf/cm², then expanded isothermally to the original pressure and finally cooled at constant pressure to its original volume. Illustrate the cycle on P-V and T-S diagrams and determine the change of entropy in each stage. C_p=0·238 and C_v=0·169.

[T_2=1,650 °K ; $V_1=V_2$=0·0925 m³ ; S_2-S_1=0·0272 ; S_3-S_2= 0·0111 and S_3-S_1=−0·0383]

2-9. ΔS of universe in constant pressure adiabatic mixing.

A mass m of water at T_1 is isobarically and adiabatically mixed with an equal mass of water at T_2. Show that the entropy change of the

A tank holds 1·1 m³ of air at 15 kgf/cm². This air is cooled in the tank to 27°C and its pressure is now 12 kgf/cm². Determine the heat extracted and decrease in entropy. Given R=29·26, γ=1·4.

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Derive an expression for the change of entropy of a perfect gas receiving heat at constant volume.

One kg of air is heated from a temperature of 30°C to 120°C. Find the change of entropy if the gas is heated at (i) constant volume, (ii) constant pressure. For air $C_p=0\cdot24$ and $C_v=0\cdot171$.

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[Constant pressure : $S_2-S_1=0\cdot06245$; $V_2=1\cdot102$ m³. Constant volume : $S_2-S_1=0\cdot04434$; $P_2=1\cdot362$ kgf/cm².]

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[$m=0\cdot06346$ kg ; $S_2-S_1=-0\cdot000759$; $T_2=516\cdot8$°K ; $U_2-U_1=2\cdot214$ kcal ; $S_3-S_2=0\cdot006176$; $U_3-U_2=2\cdot952$ kcal.]

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[$m=2\cdot829$ kg ; $T_2=296\cdot3$°K ; $S_2-S_1=0\cdot1323$; $Q=50$ kcal]

2-5. Constant pressure heating and polytropic compression ; ΔS ; n.

2 m³ of air at 1·05 kgf/cm² and 27°C is heated at constant pressure until its volume is 2·5 m³. It is then compressed according

to the law PV^n=constant until its pressure is 5·25 kgf/cm² and its volume 0·6 m³. Calculate (i) the change of entropy for constant pressure, (ii) the index 'n' for the compression stroke, and (iii) the change of entropy for the compression stroke. R=29·27 and C_v=0·17.

[C_p=0 24 ; m=2·392 kg ; S_2-S_1=0·128 ; n=1·128 ; S_3-S_2 =−0 1647]

2-6. ΔS in different reversible processes between the same initial and final state.

A perfect gas undergoes two consecutive reversible processes changing the state of the gas from T_1, V_1 to T_3, V_3. Assuming different reversible processes between the end states prove that entropy change is same in all cases.

2-7. Isothermal process : change in entropy ; heat interchange.

The increase in entropy of 0·5 kg of oxygen undergoing an isothermal expansion at 300°C was 0·1. Determine the final volume and pressure if the initial pressure was 10 kgf/cm². Also calculate the heat exchanged during the process. Assume universal gas constant as 848 kgf m/kilo mol/°K.

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0·1 kg of air is contained in a cylinder behind a piston at 1·05 kgf/cm² and 57°C. It is heated at constant volume until its pressure is 5·25 kgf/cm², then expanded isothermally to the original pressure and finally cooled at constant pressure to its original volume. Illustrate the cycle on P-V and T-S diagrams and determine the change of entropy in each stage. C_p=0 238 and C_v=0 169.

[T_2=1,650 °K ; $V_1=V_2$=0 0925 m³ ; S_2-S_1=0 0272 ; S_3-S_2= 0 0111 and S_3-S_1=−0 0383]

2·9. △S of universe in constant pressure adiabatic mixing

A mass m of water at T_1 is isobarically and adiabatically mixed with an equal mass of water at T_2. Show that the entropy change of the

universe is given by

$$\Delta s_{\text{universe}} = 2m\, C_v \log_e \frac{T_1 + T_2}{\sqrt{T_1 T_2}}$$

and that this quantity is positive.

2·10. Maximum efficiency of an engine.

An inventor claims to have developed an engine which receives heat at a temperature of 800°C and rejects heat to atmosphere at a temperature of 30°C and which has an efficiency of 75 per cent. How do you evaluate his claim ? How would you evaluate his claim of an efficiency of 70 per cent ?

[Carnot efficiency=71·4% ; first claim, impossible ; second claim, possible theoretically.]

3

Air Cycles

3-1. Air Standard Efficiency. In order to compare the efficiencies of various cycles a hypothetical efficiency called *air standard efficiency* is calculated on the following assumptions :—

(*i*) The working substance is pure dry air, which in addition is assumed to be a perfect gas.

(*ii*) The specific heat remains constant at all temperatures.

(*iii*) To eliminate the effect of calorific value of fuel the heat is supplied by a hot reservoir and is rejected to a cold reservoir. The hot reservoir or the cold reservoir is brought in contact with the end of the cylinder, which is assumed perfectly conducting for these operations. The hot reservoir and the cold reservoir are of large capacities and their temperatures do not change during the heat interchange. Further, heat interchange causes no increase or decrease in the number of molecules.

(*iv*) No account is taken how the heat is supplied or rejected. The supply and rejection of heat is assumed to take place exactly as required *i.e.* instantaneously for the constant volume process and at such a rate that the pressure remains absolutely constant for the constant pressure process.

(*v*) There is no dissociation at higher temperatures.

(*vi*) Apart from intentional changes in heat, no heat is either gained or lost during the cycle.

(*vii*) No mechanical losses like friction, etc., occur in the working.

3-2. Engine Efficiencies. The formulae for various engine efficiencies are as follows :

(i) *Air standard or ideal efficiency*

$$=\frac{\text{ideal work done (from hypothetical } P\text{-}V \text{ diagram)}}{\text{heat supplied}} \qquad (3\cdot1)$$

$$=\frac{\text{heat supplied}-\text{heat rejected}}{\text{heat supplied}}$$

(ii) *Thermal efficiency.* The thermal efficiency of an engine is defined as

$$\frac{\text{indicated work done}}{\text{heat supplied}} \qquad (3\cdot2)$$

(iii) *Relative efficiency.* The relative efficiency is defined as

$$\frac{\text{thermal efficiency}}{\text{air standard efficiency}} \qquad (3\cdot3)$$

(iv) *Overall efficiency or brake thermal efficiency.* The overall or brake thermal efficiency is defined as

$$\frac{\text{brake or actual work done}}{\text{heat supplied}}$$

Calculations of work done and heat supplied are generally done per cycle.

Mean Effective Pressure. It is the mean height of the pressure-volume (P-V) diagram as shown in Fig. 3·1.

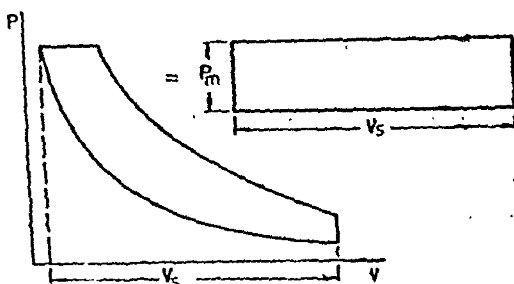


Fig. 3·1. Mean effective pressure.

Mean effective pressure

$$\text{m.e.p.}=\frac{\text{work done per cycle}}{\text{stroke volume}}$$

$$=\frac{\text{area of the diagram}}{\text{length of the diagram}} \qquad (3\cdot4)$$

3·3. Reversible Cycles. A cycle is said to be thermodynamically reversible if the engine is capable of being thermodynamically reversed. (It does not mean mechanical reversibility simply by change of rotation). The engine then abstracts heat from the cold body and rejects to the hot body and requires external power for its drive. Thus a reversed cycle converts a power producing engine into a heat pump or a refrigerator.

A reversible cycle consists of thermodynamically reversible processes. Adiabatic and isothermal processes are reversible processes. The conditions of reversibility have already been dealt in the article 2-2.

A reversible cycle is the most efficient cycle because (*i*) all the heat supplied *is at the highest temperature and all the heat rejected* is at the lowest temperature and, therefore, there is maximum scope of work being done, (*ii*) there are no mechanical losses like friction, etc. The fact that the reversible cycle is the most efficient cycle explains the importance of the concept of reversibility. All reversible cycles have the same efficiency (see article 2·4).

The examples of reversible cycles are Carnot, Stirling and Ericsson cycles

3·4. Carnot Cycle. See article 2·3.

Fig. 3·2 shows the Carnot cycle on P-V and T-S diagrams. For the details of calculation see article 2-3.

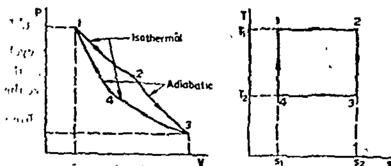


Fig. 3·2. Carnot cycle.

3·5. Stirling Cycle. The Stirling cycle consists of two isothermal and two constant volume processes. The constant volume

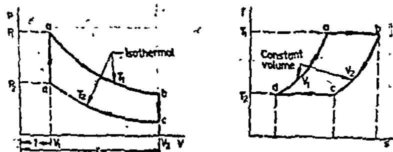


Fig. 3·3. Stirling cycle

processes are performed with the aid of regenerator to make the cycle reversible. Fig. 3·3 shows the Stirling cycle on P-V and T-S diagrams.

Assuming the regenerator efficiency to be 100 per cent the heat supplied in constant volume process da and heat rejected in constant volume process bc will be equal. Thus the work done will be the difference between the heat supplied in the isothermal process ab and the heat rejected in the isothermal process cd. Assuming unit mass of the working fluid :

Heat supplied from the external source

$$=RT_1 \log_e r$$

Heat rejected to the external source

$$=RT_2 \log_e r$$

$$\therefore \text{ Work done } =RT_1 \log_e r-RT_2 \log_e r$$

$$\text{Air standard efficiency}=\frac{\text{work done}}{\text{heat supplied}}$$

$$=\frac{RT_1 \log_e r-RT_2 \log_e r}{RT_1 \log_e r}$$

$$=\frac{T_1-T_2}{T_1} \tag{3·5}$$

It is seen that the air standard efficiency of the Stirling cycle is the same as that of the Carnot cycle. This must be so because the Stirling cycle is a reversible cycle and all reversible cycles have the same efficiency (see article 2-4).

If regenerator efficiency is e, the heat received by the air from the regenerator during the process da is $e\,C_v\,(T_1-T_2)$, and

Air standard efficiency

$$=\frac{R\,(T_1-T_2)\log_e r}{RT_1 \log_e r+(1-e)\,C_v\,(T_1-T_2)} \tag{3·6}$$

This cycle has an application in hot air engines.

3·6. Ericsson Cycle. The Ericsson cycle is also a reversible cycle and consists of two isothermal and two constant pressure processes

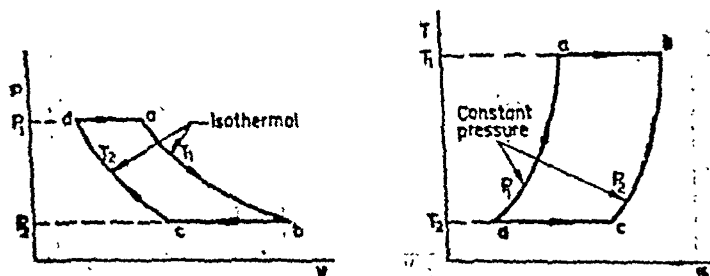


Fig. 3·4. Ericsson cycle.

Fig. 3·4 shows the Ericsson cycle on *P-V* and *T-S* diagrams. The air standard efficiency of this cycle is also equal to that of Carnot cycle.

The Ericsson cycle is the ideal cycle for gas turbines using multi-stage compression and expansion.

3·7. Joule or Brayton Cycle. The Joule or Brayton cycle consists of two constant volume and two adiabatic processes. Fig. 3·5 shows the Joule cycle on *P-V* and *T-S* diagrams.

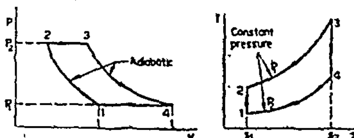


Fig. 3·5. Joule or Brayton cycle.

Consider unit mass of air

Heat supplied $= C_p\,(T_3 - T_2)$

Heat rejected $= C_p\,(T_4 - T_1)$

$$\text{Air standard efficiency} = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

$$= \frac{C_p\,(T_3 - T_2) - C_p\,(T_4 - T_1)}{C_p\,(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now $\quad T_4 = \dfrac{T_3}{r^{\gamma-1}} \quad$ and $\quad T_1 = \dfrac{T_2}{r^{\gamma-1}}$.

Substituting the values of T_4 and T_1, we get

Air standard efficiency

$$= 1 - \frac{\left(\dfrac{T_3}{r^{\gamma-1}} - \dfrac{T_2}{r^{\gamma-1}}\right)}{(T_3 - T_2)}$$

$$= 1 - \frac{1}{r^{\gamma-1}} \qquad (3\cdot8)$$

This cycle is used in open-cycle constant pressure gas turbines.

Reversed Joule cycle, also known as Bell-Coleman cycle, has application in air refrigerators.

3·8. Otto Cycle. The Otto cycle consists of two constant volume and two adiabatic processes. Fig. 3·6 shows the Otto cycle on *P-V* and *T-S* diagrams.

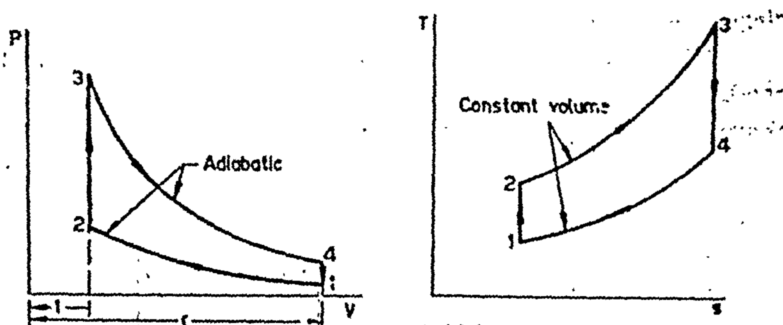


Fig. 3·6. Otto cycle.

Consider the cylinder to contain one kg of air. At the point 1 the piston is at the beginning of instroke. It is compressed adiabatically to the clearance volume to point 2. Heat is now added at constant volume from an external hot reservoir upto point 3. At point 3 the hot reservoir is withdrawn and the air expands adiabatically to point 4 doing external work. Heat is now rejected at constant volume to an external cold reservoir restoring the air to the initial condition 1.

Heat is supplied during the process 2-3 and is rejected during the process 4-1. No heat interchange takes place during the adiabatic processes 1-2 and 3-4.

Let the clearance be unity and the ratio of compression and expansion, which are equal, be *r*.

$$\therefore \qquad r=\frac{v_1}{v_2}=\frac{v_4}{v_3}$$

Heat supplied during the process 2-3 $= C_v\,(T_3-T_2)$

Heat rejected during the process 4-1 $= C_v\,(T_4-T_1)$

$$\therefore \quad \text{Air standard efficiency} = \frac{C_v\,(T_3-T_2)-C_v\,(T_4-T_1)}{C_v\,(T_3-T_2)}$$

$$=1-\frac{(T_4-T_1)}{(T_3-T_2)}$$

Now $\qquad T_1=\dfrac{T_2}{r^{\gamma-1}}$ and $\quad T_4=\dfrac{T_3}{r^{\gamma-1}}$

Substituting the values of T_1 and T_4, we get

$$\text{Air standard efficiency} = 1 - \frac{\left(\frac{T_3}{r^{\gamma-1}} - \frac{T_2}{r^{\gamma-1}}\right)}{(T_3 - T_2)}$$

$$= 1 - \frac{1}{r^{\gamma-1}} \tag{3·9}$$

The Otto cycle is the theoretical cycle on which petrol and gas engines run. The high speed Diesel engines also approach Otto cycle.

Mean effective pressure of Otto cycle.

Since clearance volume is unity, $v_2 = v_3 = 1$ and $v_1 = v_4 = r$

then $\frac{v_4}{v_3} = \frac{v_1}{v_2} = r$

Let $\frac{P_3}{P_2} = \alpha$

Now $\frac{P_2}{P_1} = r^{\gamma} = \frac{P_3}{P_4}$ $\qquad \therefore \frac{P_3}{P_2} = \frac{P_4}{P_1} = \alpha$

Work done = area of the diagram

$$= \frac{P_3v_3 - P_4v_4}{\gamma-1} - \frac{P_2v_2 - P_1v_1}{\gamma-1}$$

$$= \frac{1}{\gamma-1}\left[P_4 r\left(\frac{P_3}{P_4 r} - 1\right) - P_1 r\left(\frac{P_2}{P_1 r} - 1\right)\right]$$

$$= \frac{r}{\gamma-1}\left[P_4\left(r^{\gamma-1} - 1\right) - P_1\left(r^{\gamma-1} - 1\right)\right]$$

$$= \frac{r}{\gamma-1}\,(r^{\gamma-1} - 1)\,(P_4 - P_1)$$

$$= \frac{P_1 r(\alpha-1)\,(r^{\gamma-1} - 1)}{\gamma-1}$$

Length of the diagram $= r - 1$

$\therefore$ Mean effective pressure $= \frac{\text{work done}}{\text{length of diagram}}$

$$= \frac{P_1 r(\alpha-1)\,(r^{\gamma-1} - 1)}{(\gamma-1)\,(r-1)} \tag{3·10}$$

Efficiencies of Carnot and Otto Cycles in Terms of Compression Ratio.

The ideal efficiency of the Carnot cycle is given by

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{r^{\gamma-1}} \tag{i}$$

The ideal efficiency of the Otto cycle is given by

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{r^{\gamma - 1}} \quad \text{(ii)}$$

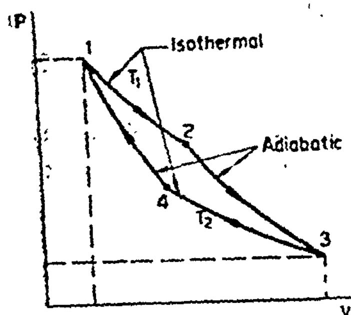


Fig. 3·7. Carnot cycle.

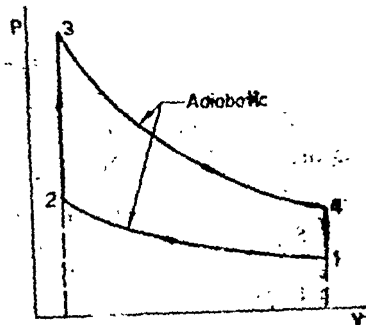


Fig. 3·8. Otto cycle.

From the Eqs. (*i*) and (*ii*) it seems as though the efficiency of the Carnot cycle is equal to that of the Otto cycle. But this is not true because in the Carnot cycle all the heat is given at the highest temperature ; however, T_2 is not the highest temperature in the Otto cycle. The Carnot cycle efficiency for the same temperature range as that of Otto cycle would be $\left(1 - \frac{T_1}{T_3}\right)$.

3·9. Diesel Cycle. The Diesel cycle consists of a constant pressure, a constant volume and two adiabatic processes. Fig. 3·9 shows the Diesel cycle on *P*-*v* and *T*-*s* diagrams.

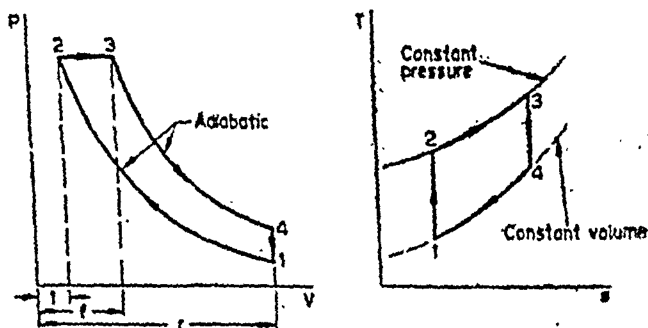


Fig. 3·9. Diesel cycle.

In this cycle heat is supplied during constant pressure process 2-3 and is rejected during the constant volume process 4-1. Compression 1-2 and expansion 3-4 are adiabatics.

Let the clearance volume be unity, cylinder volume $= r$ and cut-off volume $= \rho$

$\therefore$ Ratio of compression $= \dfrac{v_1}{v_2} = r$

Ratio of expansion $= \dfrac{v_4}{v_3} = \dfrac{r}{\rho}$

Ratio of point of cut-off $= \dfrac{v_3}{v_2} = \rho$

Consider unit mass of air

Heat supplied $= C_p (T_3 - T_2)$

Heat rejected $= C_v (T_4 - T_1)$

$$\text{Air standard efficiency} = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

$$= \frac{C_p (T_3 - T_2) - C_v (T_4 - T_1)}{C_p (T_3 - T_2)}$$

$$= 1 - \frac{C_v}{C_p} \left(\frac{T_4 - T_1}{T_3 - T_2} \right)$$

Now $\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$ and $P_1 v_1^{\gamma} = P_2 v_2^{\gamma}$

Hence $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2} \right)^{\gamma - 1}$ $\therefore$ $T_2 = T_1 \times r^{\gamma - 1}$

Also $\dfrac{v_3}{v_2} = \dfrac{T_3}{T_2} = \rho$

$\therefore$ $T_3 = T_2 \rho = T_1 \rho r^{\gamma - 1}$

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3} \right)^{\gamma - 1} = \left(\frac{r}{\rho} \right)^{\gamma - 1}$$

$\therefore$ $T_4 = \dfrac{T_3}{\left(\dfrac{r}{\rho} \right)^{\gamma - 1}}$

Substituting these values of temperatures in the expression for the air standard efficiency, we get

$$\text{Air standard efficiency} = 1 - \frac{1}{\gamma} \; \frac{(\rho^{\gamma} - 1)}{(\rho r^{\gamma - 1} - r^{\gamma - 1})}$$

$$=1-\frac{1}{\gamma r_2^{\gamma-1}}\frac{(\rho^{\gamma}-1)}{(\rho-1)} \qquad [3\cdot11\ (a)]$$

If r_1 is the expansion ratio $\rho=\frac{r}{r_1}$

and

$$\text{Air standard efficiency}=1-\frac{1}{\gamma r^{\gamma-1}}\left[\frac{\left(\frac{r}{r_1}\right)^{\gamma}-1}{\left(\frac{r}{r_1}\right)-1}\right] \qquad [3\cdot11(b)]$$

In all Diesel engines except some slow speed engines, mixed cycle is used with combustion partly at constant volume and partly at constant pressure. It allows more time for the combustion of fuel without adversely effecting the efficiency.

The proportion of the charge burnt at constant volume depends on the injection setting, engine speed and the rate of combustion. In most of the modern high speed Diesel engines constant pressure process is so small that the cycle is nearly the Otto cycle. Therefore, modern high-speed compression-ignition engines may be compared with the Otto cycle instead of the Diesel cycle.

Mean effective pressure of Diesel cycle. Refer to Fig. 3·9.

Work done=area of the diagram

$$=P_2(v_3-v_2)+\frac{P_3v_3-P_4v_4}{\gamma-1}-\frac{P_2v_2-P_1v_1}{\gamma-1}$$

$$=P_2(\rho-1)+\frac{P_2\rho-P_4r-(P_2-P_1r)}{\gamma-1}$$

$$=\frac{P_2(\rho-1)(\gamma-1)+P_2(\rho-\rho^{\gamma}r^{1-\gamma})-P_2(1-r^{1-\gamma})}{\gamma-1}$$

$$=\frac{P_2(\rho\gamma-\rho-\gamma+1+\rho-\rho^{\gamma}r^{1-\gamma}-1+r^{1-\gamma})}{(\gamma-1)}$$

$$=\frac{P_2}{\gamma-1}\left[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\gamma}-1)\right]$$

$$\text{Mean effective pressure}=\frac{\text{work done}}{\text{length of diagram}}$$

$$=\frac{P_2[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\gamma}-1)]}{(\gamma-1)(r-1)} \qquad [3\cdot12(a)]$$

$$=\frac{P_1r^{\gamma}[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\gamma}-1)]}{(\gamma-1)(r-1)} \qquad [3\cdot12(b)]$$

3.10. Dual Cycle or Mixed Cycle or Limited Pressure Cycle. The limited pressure cycle is a combination of Otto and

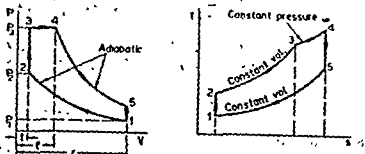


Fig. 3·10. Dual cycle or limited pressure cycle

Diesel cycles and as such is known as dual or mixed cycle. Fig. 3·10 shows the cycle on P-v and T-s diagrams The process 1-2 is adiabatic compression, 2-3 heat addition at constant volume, 3-4 heat addition at constant pressure, 4-5 adiabatic expansion and 5-1 heat rejection at constant volume.

Let the clearance volume be unity, cylinder volume$=r$, cut-off volume$=\rho$

$$\frac{P_3}{P_2}=\alpha$$

$\therefore$ Ratio of compression$=\frac{v_1}{v_2}=r$

and Ratio of expansion$=\frac{v_5}{v_4}=\frac{r}{\rho}$

and Ratio of point of cut-off$=\frac{v_4}{v_3}=\rho$

Consider unit mass of air.

Total heat supplied $=C_v(T_3-T_2)+C_p(T_4-T_3)$

Heat rejected $=C_v(T_5-T_1)$

Air standard efficiency $=\frac{\text{heat supplied}-\text{heat rejected}}{\text{heat supplied}}$

$$=\frac{C_v(T_3-T_2)+C_p(T_4-T_3)-C_v(T_5-T_1)}{C_v(T_3-T_2)+C_p(T_4-T_3)}$$

$$=1-\frac{(T_5-T_1)}{(T_3-T_2)+\gamma(T_4-T_3)}$$

Now $\quad \frac{P_3}{T_3}=\frac{P_2}{T_2} \qquad \therefore T_2=T_3\times\frac{P_2}{P_3}=\frac{T_3}{\alpha} \qquad$ (i)

$$\frac{v_4}{T_4}=\frac{v_3}{T_3} \qquad \therefore T_4=T_3\times\frac{v_4}{v_3}=\rho T_3 \qquad (ii)$$

$$\frac{T_4}{T_5}=\left(\frac{v_5}{v_4}\right)^{\gamma-1} \therefore T_5=\frac{T_4}{(r/\rho)^{\gamma-1}}=\frac{T_3\rho^{\gamma}}{r^{\gamma-1}} \qquad (iii)$$

$$\frac{T_2}{T_1}=r^{\gamma-1} \quad \therefore T_1=\frac{T_2}{r^{\gamma-1}}=\frac{T_3}{\alpha r^{\gamma-1}} \qquad (iv)$$

Substituting the above values of temperatures in the expression for air standard efficiency, we get

$$\text{Air standard efficiency}=1-\frac{\left(\frac{T_3\rho^{\gamma}}{r^{\gamma-1}}\right)-\left(\frac{T_3}{\alpha r^{\gamma-1}}\right)}{(T_3-T_3/\alpha)+\gamma(\rho T_3-T_3)}$$

$$=1-\frac{1}{r^{\gamma-1}}\left[\frac{\alpha\rho^{\gamma}-1}{(\alpha-1)+\alpha\gamma(\rho-1)}\right] \qquad (3\cdot13)$$

It should be noted that if $\rho=1$, the cycle becomes the Otto cycle and substituting this value, we get

Air standard efficiency of Otto cycle

$$=1-\frac{1}{r^{\gamma-1}}.$$

If $\alpha=1$, the cycle becomes the Diesel cycle and substituting this value, we get

Air standard efficiency of Diesel cycle

$$=1-\frac{1}{r^{\gamma-1}}\left(\frac{\rho^{\gamma}-1}{\rho-1}\right).$$

Mean effective pressure of dual or mixed pressure cycle. Refer to Fig. 3·10.

Work done=area of the diagram

$$=P_3(v_4-v_3)+\frac{P_4v_4-P_5v_5}{\gamma-1}-\frac{P_2v_2-P_1v_1}{\gamma-1}$$

$$=P_3(\rho-1)+\frac{P_3\rho-P_5r-P_2+P_1r}{\gamma-1}$$

$$=P_3(\rho-1)+\frac{P_3(\rho-\rho^{\gamma}r^{1-\gamma})-P_2(1-r^{1-\gamma})}{\gamma-1}$$

$$= \frac{P_2[(\rho-1)(\gamma-1)+\rho-\rho^{\gamma} r^{1-\gamma}-1/\alpha(1-r^{1-\gamma})]}{\gamma-1}$$

$$= \frac{P_2[\rho\gamma-\rho-\gamma+1+\rho-\rho^{\gamma} r^{1-\gamma}-1/\alpha\,(1-r^{1-\gamma})]}{\gamma-1}$$

$$= \frac{P_2[\alpha\gamma(\rho-1)+\alpha-\alpha\rho^{\gamma} r^{1-\gamma}-(1-r^{1-\gamma})]}{\alpha(\gamma-1)}$$

$$= \frac{P_2[\alpha\gamma(\rho-1)+(\alpha-1)-r^{1-\gamma}\,(\alpha\rho^{\gamma}-1)]}{\alpha(\gamma-1)}$$

Mean effective pressure $= \dfrac{\text{work done}}{\text{length of diagram}}$

$$= \frac{P_2[\alpha\gamma(\rho-1)+(\alpha-1)-r^{1-\gamma}\,(\alpha\rho^{\gamma}-1)]}{\alpha(\gamma-1)(r-1)} \quad [3\cdot14(a)]$$

$$= \frac{P_1 r^{\gamma}[\alpha\gamma(\rho-1)+(\alpha-1)-r^{1-\gamma}(\alpha\rho^{\gamma}-1)]}{(\gamma-1)(r-1)} \quad [3\cdot14(b)]$$

3·11. Comparison of Otto, Diesel and Dual Cycles.

(a) *For same compression ratio and same heat input.* For same compression ratio and heat input the thermal efficiencies are in the following order :

1. Otto cycle (*abcd*) 2. Dual cycle (*abb'c'd'*) 3. Diesel cycle (*abc''d''*)

The cycles are shown in Fig. 3·11 on P-v and T-s diagrams.

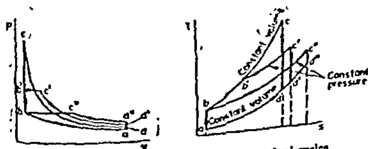


Fig. 3·11. Comparison of Otto, Diesel and dual cycles.

As the heat added in each cycle is the same

Area *fbce* = area *fbb'c'e'* = area *fbc''e''*

Heat rejected is proportional to areas as given below

Otto cycle	Dual cycle	and	Diesel cycle
fade	*fad'e'*		*fad''e''*

As the heat added in the three cases is same it is clear ... greater the efficiency lesser is the heat rejected. Thus, Otto ...

the most efficient and Diesel cycle is the least efficient of the three cycles. The figures bring out the importance of the mode of adding heat so as to permit maximum possible expansion of the working fluid for maximum thermal efficiency.

That the Otto cycle is more efficient than the Dieceleycle can be proved mathematically as follows :—

Let r_1=compression ratio for the both cycles.
r_2=expansion ratio for the Diesel cycle.

Air standard efficiency of Otto cycle $=1-\dfrac{1}{r_1^{\gamma-1}}$

Air standard efficiency of Diesel cycle

$$=1-\frac{1}{\gamma r_1^{\gamma-1}}\left[\frac{(r_1/r_2)^\gamma-1}{(r_1/r_2)-1}\right]$$

Let $r_2=r_1-\Delta r$, where Δr is a small quantity.

$$\frac{r_1}{r_2}=\frac{r_1}{r_1\left(1-\dfrac{\Delta r}{r_1}\right)}=\left(1-\frac{\Delta r}{r_1}\right)^{-1}$$

$$=1-1\times\left(\frac{-\Delta r}{r_1}\right)-\frac{1(-1-1)}{\underline{|2}}\left(\frac{-\Delta r}{r_1}\right)^2\ldots$$

$$=1+\frac{\Delta r}{r_1}+\frac{\Delta r^2}{r_1^2}+\frac{\Delta r^3}{r_1^3}+\ldots\ldots$$

and

$$\left(\frac{r_1}{r_2}\right)^\gamma=\frac{r_1^\gamma}{r_1^\gamma\left(1-\dfrac{\Delta r}{r_1}\right)^\gamma}=\left(1-\frac{\Delta r}{r_1}\right)^{-\gamma}$$

$$=1-\gamma\left(\frac{-\Delta r}{r_1}\right)-\frac{\gamma(-\gamma-1)}{\underline{|2}}\left(\frac{-\Delta r}{r_1}\right)^2-\ldots$$

$$=1+\frac{\gamma\Delta r}{r_1}+\frac{\gamma(\gamma+1)}{\underline{|2}}\left(\frac{\Delta r^2}{r_1^2}\right)+\frac{\gamma(\gamma+1)(\gamma+2)}{\underline{|3}}\left(\frac{\Delta r^3}{r_1^3}\right)+\ldots$$

∴ Air standard efficiency of Diesel cycle

$$=1-\frac{1}{\gamma r^{\gamma-1}}\left[\frac{1+\dfrac{\gamma\Delta r}{r_1}+\dfrac{\gamma(\gamma+1)}{\underline{|2}}\left(\dfrac{\Delta r^2}{r_1^2}\right)+\dfrac{\gamma(\gamma+1)(\gamma+2)}{\underline{|3}}\left(\dfrac{\Delta r^3}{r_1^3}\right)+\ldots-1}{1+\dfrac{\Delta r}{r_1}+\dfrac{\Delta r^2}{r_1^2}+\dfrac{\Delta r^3}{r_1^3}+\ldots-1}\right]$$

$$=1-\frac{1}{r_1^{\gamma-1}}\left[\frac{\dfrac{\Delta r}{r_1}+\dfrac{(\gamma+1)}{\underline{|2}}\left(\dfrac{\Delta r^2}{r_1^2}\right)+\dfrac{(\gamma+1)(\gamma+2)}{\underline{|3}}\left(\dfrac{\Delta r^3}{r_1^3}\right)+\ldots}{\dfrac{\Delta r}{r_1}+\dfrac{\Delta r^2}{r_1^2}+\dfrac{\Delta r^3}{r_1^3}+\ldots}\right]$$

Since the coefficients of the terms $\frac{\Delta r}{r_1}$, $\frac{\Delta r^2}{r_1^2}$, etc., in the numerator are >1 the ratio in brackets is >1 and hence the above expression for the efficiency of the Diesel cycle yields a lower result than the efficiency of the Otto cycle for the same compression ratio. In Diesel engines, therefore, fuel should be cut-off early in the outward stroke.

Note.—In practice the efficiency of Diesel engines is higher than petrol engines as higher compression ratios are used in the former. (Diesel engine compression ratios are generally from 15-20 whereas petrol engine compression ratios are from 5-8). It is because compression of air alone in Diesel engine puts no limit on compression ratio whereas compression of mixture of fuel and air in petrol engine should not produce temperature greater than the ignition temperature of the mixture.

(*b*) *For constant maximum pressure and heat input.* From practical point of view this condition is important. For this condition the efficiencies are in the following order :

1. Diesel cycle 2. Dual cycle 3. Otto cycle

Fig. 3·12 shows the Diesel and Otto cycles on *P-v* and *T-s* diagrams for constant pressure and heat input respectively.

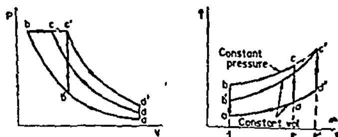


Fig. 3·12. Diesel and Otto cycle for constant maximum pressure.

For the maximum pressure the point c and c' must be on a constant pressure line and on the T-s plane areas $fbce$ and $fb'c'e'$ must be equal. It is clear from T-s diagram that the heat rejected by Diesel cycle (area $fade$) is less than that rejected by Otto cycle (area $fad'e'$) ; hence, the Diesel cycle is more efficient than the Otto cycle for the condition of constant maximum pressure and heat input.

(c) *Constant maximum pressure and output.*

$$\text{Efficiency} = \frac{\text{work done}}{\text{work done} + \text{heat rejected}}$$

$$= \frac{\text{constant}}{\text{constant} + \text{heat rejected}}$$

The work done (*abcd* and *ab'c'd'*) can be equal only if the point c' has a greater entropy than c (see Fig. 3·12) and in that case the heat rejected by the Diesel cycle is less than that of Otto cycle ; hence, Diesel cycle will be more efficient than Otto cycle.

(d) *Constant maximum pressure and temperature.* Fig. 3·13 shows the Diesel cycle and the Otto cycle with constant maximum pressure and temperature on *P-v* and *T-s* diagrams respectively. For

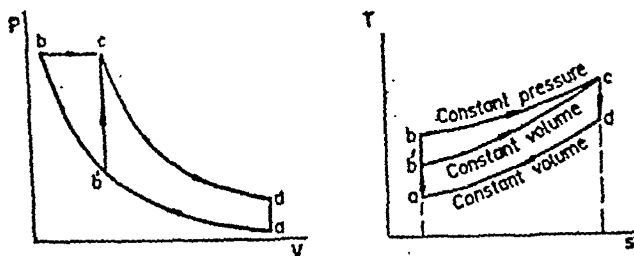


Fig. 3·13. Diesel and Otto cycle for constant maximum pressure and temperature.

constant maximum pressure and temperature heat rejected will be same in both the cases.

$$\text{Efficiency} = 1 - \left(\frac{\text{heat rejected}}{\text{heat supplied}}\right).$$

As greater heat is supplied in Diesel cycle, it is more efficient than Otto cycle.

IMPORTANT POINTS

1. For solving problems it is not necessary to remember the formulae of air standard efficiencies of various cycles.

All problems, except on the Otto cycle, should be solved from the fundamental equation. For the Otto cycle the formula is simple and may be memorised. For other cycles temperature at all cardinal points should be calculated and then the heat supplied and rejected should be determined.

$$\text{Efficiency} = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

Table 3·1. Various cycles and their efficiencies

S.N.	Name of cycle	Heat received at	Heat rejected at	P-V and T-S diagrams	Air standard efficiency in terms of temperature	Efficiency in terms of volumes and pressures
1.	Carnot	Constant temperature (T_1), 1-2	Constant temperature (T_2), 3-4	Fig. 3·2	$\frac{T_1-T_2}{T_1}$	$1-\frac{1}{r^{\gamma-1}}$
2.	Stirling	Constant temperature (T_1) ab	Constant temperature (T_2) cd	Fig. 3·3	$\frac{T_1-T_2}{T_1}$	$1-\frac{P_d}{P_a}$
3.	Ericsson	Constant temperature (T_1) ab	Constant temperature (T_2), cd	Fig. 3·4	$\frac{T_1-T_2}{T_1}$	$1-\frac{V_4}{V_1}$
4.	Joule or Brayton	Constant pressure 2-3	Constant pressure 4-1	Fig. 3·5	$1-\frac{(T_4-T_1)}{(T_3-T_2)}$	$1-\frac{1}{r^{\gamma-1}}$
5.	Otto or constant volume	Constant volume 2-3	Constant volume 4-1	Fig. 3·6	$1-\frac{(T_4-T_1)}{(T_3-T_2)}$	$1-\frac{1}{r^{\gamma-1}}$
6.	Diesel or constant pressure	Constant pressure 2-3	Constant volume 4-1	Fig. 3·9	$1-\frac{1}{\gamma}\left[\frac{T_4-T_1}{T_3-T_2}\right]$	$1-\frac{1}{\gamma r^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right]$ where $r=\frac{V_1}{V_2}$ and $\rho=\frac{V_3}{V_2}$
7.	Dual or mixed	Partly constant volume and partly constant pressure 2-3 and 3-4	Constant volume 5-1	Fig. 3·10	$1-\frac{T_5-T_1}{(T_3-T_2)+\gamma(T_4-T_3)}$	$1-\frac{1}{r^{\gamma-1}}\frac{x\rho^{\gamma}-1}{(x-1)+x\gamma(\rho-1)}$ where $r=\frac{V_1}{V_2}$, $\rho=\frac{V_4}{V_3}$ [illegible] P_3

The clearnce volume may be assumed unity since it is taken as a ratio of total volume.

2. Total volume=swept volume+clearance volume

and, Compression ratio$=\dfrac{\text{total volume}}{\text{clearance volume}}$

3. Sometimes standard cycles like the Otto or the Diesel cycles may be modified by having polytropic compression and expansion instead of adiabatic compression and expansion and in such cases the standard formulae must not be applied.

4. For finding m.e.p. the work done should be divided by swept volume and not by total volume.

5. In problems, for the sake of simplicity, it is assumed that the mass of charge in the cycle remains constant. In actual practice, however, mass of the charge changes due to injection of fuel in the case of Diesel engines.

6. Various cycles in tabular form are shown in Table 3·1.

ILLUSTRATIVE EXAMPLES

3.1. The Carnot cycle : mep ; hp.

Describe the Carnot cycle and explain why it cannot be realised in an actual engine. Sketch the Carnot cycle on P-v and T-s diagrams.

In a theoretical heat engine working on the Carnot cycle, pressure, volume and temperature at the beginning of isothermal expansion are 21 kgf/cm², 0·06 m³ and 277°C respectively. The ratio of isentropic compression is 5 and isothermal expansion 2. Determine (a) the pressure and temperature at each salient point, (b) the efficiency, (c) the mep, and (d) the horse-power developed when engine completes 100 cycles in one minute. γ=1·4.

For theory—see text.

(a) Isentropic compression

$$\frac{V_4}{V_1}=5=\frac{V_3}{V_2}$$

Isothermal expansion$=2=\dfrac{V_2}{V_1}=\dfrac{V_3}{V_4}$

Assuming $V_1=\text{unity},\ V_3=5\times 2=10$ units

$$P_1V_1=P_2V_2$$

$$21\times 0{\cdot}06=P_2\times 2(0{\cdot}06) \qquad \therefore \quad P_2=10{\cdot}5 \text{ kgf/cm}^2$$

$$\frac{T_4}{T_1}=\left(\frac{V_1}{V_4}\right)^{\gamma-1}, \quad \frac{T_4}{550}=\frac{1}{5^{0.4}}$$

$\therefore$ $T_4=289°K=T_3$

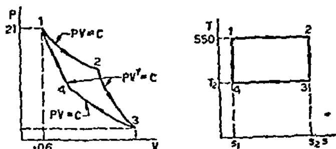


Fig. 3·14.

$$P_4V_4^{\gamma}=P_1V_1^{\gamma}, \quad [P_4\times 5^{1.4}=21\times 1^{1.4}$$

$\therefore$ $P_4=2\,206$ kgf/cm²

$$P_2V_2^{\gamma}=P_3V_3^{\gamma}, \quad 10\,5\times 1^{1.4}=P_3\times 5^{1.4}$$

$\therefore$ $P_3=1{\cdot}104$ kgf/cm²

$P_1=21$ kgf/cm², $T_1=550°K$; $P_2=10\,5$ kgf/cm², $T_2=550°K$ }
$P_3=1{\cdot}104$ kgf/cm², $T_3=289°K$; $P_4=2\,206$ kgf/cm², $T_4=289°K$ } Ans.

(b) Heat supplied during operation $1\text{-}2=P_1V_1 \log_e r$

$$=21\times 10^4\times 0\,06 \log_e 2=8{,}730 \text{ kgf m}$$

Heat rejected during operation $3\text{-}4=P_3V_3 \log_e r$

$$=1{\cdot}104\times 10^4\times 0\,6 \log_e 2=4{,}590 \text{ kgf m}$$

$$\eta=\frac{8{,}730-4{,}590}{8{,}730}=47{\cdot}43\%$$ Ans.

$\left[\text{Check : Carnot} \quad \eta=\frac{T_1-T_2}{T_1}=\frac{550-289}{550}=47\,43\%\right]$

(c) $$\text{mep}=\frac{\text{work done}}{\text{stroke volume}}=\frac{8{,}730-4{,}590}{(0{\cdot}6-0\,06)\times 10^4}$$

$$=0{\cdot}7667 \text{ kgf/cm}^2$$ Ans.

(d) $$\text{hp developed}=\frac{\text{work done}\times N}{75}$$

$$=\frac{(8{,}730-4{,}590)\times 100}{60\times 75}=92$$ Ans

Note.—The example shows that the Carnot cycle is not a practical one. Though ideal efficiency for this cycle is highest, mep is very small *viz.* work done is very small.

For theory—see text.

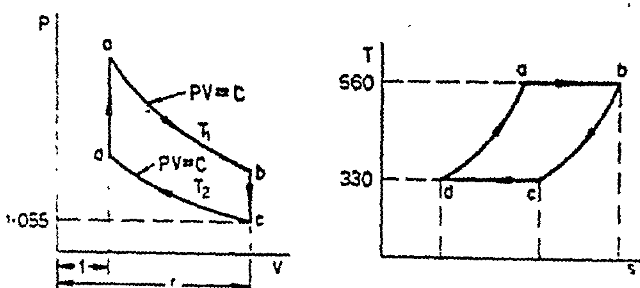


Fig. 3·16.

Theoretical work done per kg of air $= R(T_1 - T_2)\log_e r$

$$= \frac{29{\cdot}27 \times (560-330)\log_e 2{\cdot}2}{427} = 12{\cdot}43 \text{ kcal}$$

Heat supplied/kg of air

$$= RT_1 \log_e r + (1-e)\,C_v\,(T_1 - T_2)$$

$$= \frac{29{\cdot}27 \times 560 \log_e 2{\cdot}2}{427} + (1-0{\cdot}9) \times 0{\cdot}17 \times (560-330$$

$$= 34{\cdot}18 \text{ kcal}$$

Mass of air/min

$$= \frac{450}{34{\cdot}18} = 13{\cdot}16 \text{ kg}$$

Mass of air/working cycle

$$= \frac{13{\cdot}16}{95} = 0{\cdot}1386 \text{ kg}$$

Stroke volume, $\quad V = \dfrac{0\;1386 \times 29{\cdot}27 \times 330}{1{\cdot}055 \times 10^4}$

$$= 0{\cdot}1269 \text{ m}^3$$ Ans.

Actual indicated work done/min

$$= 0{\cdot}58 \times 12{\cdot}43 \times 13{\cdot}16 \times 427$$

$$= 40{,}520 \text{ kgf m}$$

$\therefore$ $\quad \text{ihp} = \dfrac{40.520}{60 \times 75} = 9$ Ans.

1-4. The Joule cycle : efficiency ; rate of air circulation.

Show that the thermal efficiency of a hot air engine, operating on the Joule cycle is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

where γ is the isentropic index of compression and expansion and r is the volume compression and expansion ratio.

An engine of this type operates between pressure limits of 1·05 kgf/cm² and 4·2 kgf/cm² and temperature limits of 537°C and 37°C. If the indicated horse power is 2000, find (a) the thermal efficiency, (b) the rate of air circulation in kg/s and (c) mean effective pressure. Assume that the mean specific heats of air at constant pressure and constant volume for the given temperature range are 0·26 and 0·19 respectively.

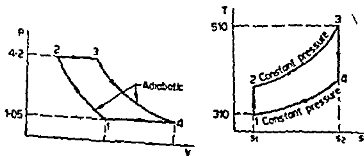


Fig. 3 17.

For theory—see text.

$$\gamma = \frac{C_p}{C_v} = \frac{0.26}{0.19} = 1.37$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}, \quad \frac{T_2}{310} = \left(\frac{4.2}{1.05}\right)^{\frac{1.37-1}{1.37}}$$

$\therefore$ $T_2 = 451°K$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}, \quad \frac{810}{T_4} = \left(\frac{4.2}{1.05}\right)^{\frac{1.37-1}{1.37}}$$

$\therefore$ $T_4 = 556°K$

Work done/kg $= C_p (T_3 - T_2) - C_p (T_4 - T_1)$

$= 0.26 (810 - 451) - 0.26 (556 - 310)$

$= 29.38$ kcal

Heat supplied/kg $= C_p (T_3 - T_2) = 0.26 (810 - 451) = 93.34$ kcal

$\therefore$ Thermal efficiency, $\eta = \frac{29.38}{93.34} = 31.5\%$. *Ans.*

Air circulation/s $= \frac{\text{heat equivalent to hp}}{\text{work done/kg}}$

$= \frac{2,000 \times 75}{427 \times 29.38} = 11.94$ kg *Ans.*

$$C_p - C_v = \frac{R}{J}, \quad 0{\cdot}26 - 0{\cdot}19 = \frac{R}{427} \quad \therefore R = 29{\cdot}89$$

$$P_2v_2 = RT_2, \quad 4{\cdot}2 \times 10^4 \times v_2 = 29{\cdot}89 \times 451$$

$$\therefore \quad v_2 = 0{\cdot}321 \text{ m}^3$$

$$P_4v_4 = RT_4, \quad 1{\cdot}05 \times 10^4 \times v_4 = 29{\cdot}89 \times 556$$

$$\therefore \quad v_4 = 1{\cdot}583 \text{ m}^3$$

Stroke volume/kg of fluid $= v_4 - v_2 = 1{\cdot}583 - 0{\cdot}321$

$$= 1{\cdot}262 \text{ m}^3$$

Mean effective pressure $= \dfrac{29{\cdot}38 \times 427}{1{\cdot}262 \times 10^4}$

$= \underline{0{\cdot}994 \text{ kgf/cm}^2}$ **Ans.**

3-5. Otto cycle : work ; efficiency ; mep.

Sketch the P-V and T-S diagrams for constant volume air cycle. Develop the formula for the ideal thermal efficiency.

A four-stroke engine having a swept volume of 0·13 m³ operates on the above cycle. The compression ratio is 6 and conditions at the beginning of compression are 1 kgf/cm² and 60°C. The heat supplied is 36 kcal/cycle. Calculate the values of pressure, volume and temperature at salient points. Also calculate the change of entropy and give dimensioned sketch of P-V and T-S diagrams of the cycle.

Calculate also the work done, efficiency and mean effective pressure of the cycle. $C_p = 0{\cdot}237$ and $C_v = 0{\cdot}169$.

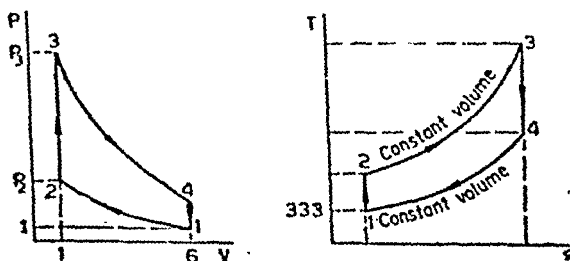


Fig. 3·18.

$$C_p - C_v = \frac{R}{J}, \quad 0{\cdot}237 - 0{\cdot}169 = \frac{R}{427} \quad \therefore R = 29$$

$$\gamma = \frac{0{\cdot}237}{0{\cdot}169} = 1{\cdot}4$$

$$r = \frac{v_s + v_c}{v_c}, \quad 6 = \frac{0.130 + V_2}{V_2}$$

$\therefore \quad V_2 = 0.026 \text{ m}^3$

$\therefore$ Total cylinder volume, $V_1 = 0.130 + 0.026 = 0.156 \text{ m}^3$

$PV = mRT, \quad 1 \times 10^4 \times 156 \times 10^{-3} = m \times 29 \times 333$

$\therefore \quad m = 0.1615 \text{ kg}$

$P_1V_1^\gamma = P_2V_2^\gamma, \quad 1 \times 6^{1.4} = P_2 \times 1^{1.4} \quad \therefore \quad P_2 = 12.28 \text{ kgf/cm}^2$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \quad \frac{1 \times 0.156}{333} = \frac{12.28 \times 0.026}{T_2} \quad \therefore T_2 = 681.5 \text{ °K}$$

$Q_{2\text{-}3} = mC_v (T_3 - T_2), \; 36 = 0.1615 \times 0.169 (T_3 - 681.5)$

$\therefore \quad T_3 = 2{,}000 \text{ °K}$

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2}, \quad \frac{P_3}{2{,}000} = \frac{12.28}{681.5} \quad \therefore P_3 = 36 \text{ kgf/cm}^2$$

$P_3V_3^\gamma = P_4V_4^\gamma, \quad 36 \times 1^{1.4} = P_4 \times 6^{1.4} \; \therefore \; P_4 = 2.93 \text{ kgf/cm}^2$

$$\frac{P_4V_4}{T_4} = \frac{P_3V_3}{T_3}, \quad \frac{2.93 \times 6}{T_4} = \frac{36 \times 1}{2{,}000} \therefore T_4 = 977 \text{ °K}$$

$$S_3 - S_2 = S_4 - S_1 = m\, C_v \log_e \frac{T_4}{T_1} = 0.1615 \times 0.169 \log_e \frac{977}{333}$$
$$= 0.0294.$$

The calculated values of P, V, T and change of entropy are shown in Fig. 3·18.

Heat rejected $= 0.1615 \times 0.169 \; (977 - 333) = 17.58$ kcal

Work done $= 36 - 17.58 = 18.42$ kcal or 7,865 kgf m **Ans.**

$$\text{Efficiency} = \frac{18.42}{36} = 51.2\% \qquad \textbf{Ans.}$$

$$\text{mep} = \frac{7{,}865}{0.13 \times 10^4} = 6.05 \text{ kgf/cm}^2 \qquad \textbf{Ans.}$$

$$\left[\text{Check: Otto cycle efficiency} = 1 - \frac{1}{6^{0.4}} = 51.2\%\right]$$

3.6. Otto cycle: efficiency; mep; specific fuel consumption.

Prove that the ideal efficiency of the Otto cycle is given by $\eta = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$ *and the mean effective pressure by*

$$p_m = \frac{Pr(\alpha-1)\,.\,(r^{\gamma-1}-1)}{(\gamma-1)(r-1)}$$

where P is the suction pressure and α the explosion ratio. What is the effect of increasing the compression ratio of the cycle ?

An engine working on the Otto cycle has a clearance of 17 per cent of stroke volume and initial pressure of 0·95 kgf/cm² and temperature 30°C. If the pressure at the end of the constant volume heating is 28 kgf/cm², find (i) the air standard efficiency, (ii) the maximum temperature in the cycle, and (iii) the ideal mean effective pressure. Assume γ=1·4, C_v=0·171.

(iv) If the relative efficiency of the engines 50 per cent, calculate the fuel consumption per bhp-hr, the calorific value of fuel used being 10,000 kcal.

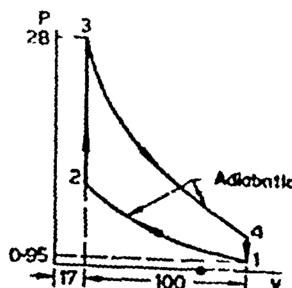


Fig. 3·19.

For theory—see text.

(i) Compression ratio, $r = \frac{V_c+V_s}{V_c} = \frac{0\cdot17+1}{0\cdot17} = 6\cdot882$

Air standard $\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{6\cdot882^{1\cdot4-1}} = 53\cdot77\%$ Ans.

(ii) $P_1V_1^{\gamma} = P_2V_2^{\gamma}$, $0\cdot95 \times 6\cdot882^{1\cdot4} = P_2 \times 1$

∴ $P_2 = 14\cdot15$ kgf/cm²

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \quad \frac{0\cdot95\times6\cdot882}{303} = \frac{14\cdot15\times1}{T_2}$$

∴ $T_2 = 655\cdot7\ °K$

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2}, \quad \frac{28}{T_3} = \frac{14\cdot15}{655\cdot7} \quad \therefore\ T_3 = 1{,}298\ °K$$ Ans.

(iii) $\gamma - 1 = \frac{R}{J\,C_v}$, $1\cdot4 - 1 = \frac{R}{427\times0\cdot171}$ ∴ $R = 29\cdot21$

For 1 kg $V = \frac{1\times29\cdot21\times303}{0\cdot95\times10^4} = 0\cdot9315\ m^3$

Stroke volume, $V_s = 0.9315 \times \frac{5.882}{6.882} = 0.796 \text{ m}^3$

Work done $= \eta \times m \times C_v (T_3 - T_2)$
$= 0.5377 \times 1 \times 0.171(1,298 - 655.7)$
$= 59.06$ k cal or 25,220 kgf m

$\therefore$ $\underline{\text{mep}} = \frac{25,220}{0.796 \times 10^4} = \underline{3.168 \text{ kgf/cm}^2}$. Ans.

(iv) $\underline{\text{Fuel consumption}} = \frac{75 \times 60 \times 60}{427 \times 0.5377 \times 0.5 \times 10,000}$
$= \underline{0.235 \text{ kg/hphr}}$ Ans.

3-7. Diesel cycle : hp : thermal efficiency

A four-stroke, single-cylinder oil engine, operating on the Diesel cycle and running at 240 rev/min has a piston diameter of 25 cm, a stroke of 40 cm and a clerance volume of 1,560 cc. Fuel oil is injected during the first 1/12th of the expansion stroke. If the pressure and temperature at the beginning of compression are 1 kgf/cm² and 47°C find the ideal indicated horse-power and the corresponding thermal efficiency. Neglect the increase in mass of the charge due to oil injection. The gas constant for the working substance may be taken as 29.2 kgfm/kg°K and the index of isentropic compression and expansion as 1.4.

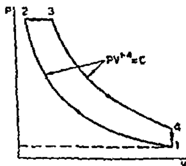


Fig. 3-20

$$\text{Stroke volume} = \frac{\pi}{4} \times 25^2 \times 40 = 19,640 \text{ cc}$$

Total cylinder volume, $V_1 = 19,640 + 1,560 = 21,200$ cc

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$, $1 \times 21,200^{1.4} = P_2 \times 1,560^{1.4}$ $\therefore$ $P_2 = 38.6 \text{ kgf/cm}^2$

Volume at cut-off, $V_3 = 1,560 + \frac{1}{12} \times 19,640 = 3,197$ cc

$P_3V_3^{\gamma}=P_4V_4^{\gamma}$, $38{\cdot}6\times3{,}197^{1{\cdot}4}=P_4\times21{,}200^{1{\cdot}4}$

$\therefore\ P_4=2{\cdot}732$ kgf/cm²

Work done/cycle=Area of the the P-V diagram

$$=P_2\,(V_3-V_2)+\frac{P_3V_3-P_4V_4}{\gamma-1}-\frac{P_2V_2-P_1V_1}{\gamma-1}$$

$$=10^4\times10^{-6}\left[38{\cdot}6\,(3{,}197-1{,}560)+\frac{38{\cdot}6\times3{,}197-2{\cdot}732\times21{,}200}{1{\cdot}4-1}-\frac{38{\cdot}6\times1{,}560-1\times21{,}200}{1{\cdot}4-1}\right]$$

=1,294 kgfm or 3·031 kcal

$$\text{hp}=\frac{1{,}294\times120}{60\times75}=\underline{39{\cdot}17}$$ Ans.

$$\text{Mass of air/cycle}=\frac{1\times10^4\times21{,}200\times10^{-6}}{29{\cdot}2\times320}=0{\cdot}02269\text{ kg}$$

$$C_v=\frac{\gamma R}{J(\gamma-1)}=\frac{1{\cdot}4\times29{\cdot}2}{427\,(1{\cdot}4-1)}=0{\cdot}2394$$

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2},\quad\frac{1\times21{,}200}{320}=\frac{38{\cdot}6\times1{,}560}{T_2}\qquad\therefore\ T_2=909°K$$

$$\frac{P_2V_2}{T_2}=\frac{P_3V_3}{T_3},\quad\frac{1{,}560}{909}=\frac{3{,}197}{T_3}\qquad\therefore\ T_3=1{,}863°K$$

Heat added/cycle $=0{\cdot}02269\times0{\cdot}2394\,(1{,}863-909)$

$=5{\cdot}214$ kcal

$$\text{Thermal efficiency}=\frac{3{\cdot}031}{5{\cdot}214}=\underline{58{\cdot}12}\%$$ Ans.

3-8. Diesel engine : compression ratio ; efficiency.

(a) *Prove that for the same compression ratio the Otto cycle is more efficient than the Diesel cycle. Why in actual practice a Diesel engine is preferred for some applications ?*

(b) *The pressures on the compression curve of a Diesel engine are, at $\frac{1}{8}$th stroke 1·4 kgf/cm² and at $\frac{7}{8}$th stroke 14 kgf/cm². Estimate the compression ratio. Calculate the air standard efficiency of the engine, if the cut-off occurs at $\frac{1}{15}$th of the stroke. Also find the fuel consumption per bhp-hr if the indicated thermal efficiency is 0·5 of ideal efficiency, machanical efficiency 0·8 and the calorific value of oil 10,000 kcal/kg. $\gamma=1{\cdot}41$.*

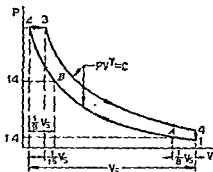


Fig. 3·21.

Let stroke volume be V_s and clearance volume be V_c

$$\therefore \quad V_A = V_c + \frac{7}{8}V_s \quad (1)$$

and

$$V_B = V_c + \frac{1}{8}V_s \quad (2)$$

$$P_A V_A^{\gamma} = P_B V_B^{\gamma}, \quad \frac{V_A}{V_B} = \left[\frac{P_B}{P_A}\right]^{\frac{1}{\gamma}} = \left[\frac{14}{1\cdot4}\right]^{\frac{1}{1\cdot41}} = 5\cdot119 \quad (3)$$

From Eq. (1), (2) and (3), $\dfrac{V_c + \frac{7}{8}V_s}{V_c + \frac{1}{8}V_s} = 5\cdot119 \quad \therefore \frac{V_s}{V_c} = 17\cdot54$

Compression ratio, $r = \dfrac{V_s + V_c}{V_c} = \dfrac{V_s}{V_c} + 1 = 17\cdot54 + 1 = 18\cdot54$ Ans.

Cut-off ratio, $\rho = \dfrac{V_c + \frac{V_s}{15}}{V_c} = 1 + \dfrac{17\cdot54}{15} = 2\cdot17$

Air standard efficiency, $\eta = 1 - \dfrac{1}{r^{\gamma-1} \times \gamma} \times \left[\dfrac{\rho^{\gamma} - 1}{\rho - 1}\right]$

$$= 1 - \frac{1}{(18\cdot54)^{1\cdot41-1} \times 1\cdot41}\left[\frac{2\cdot17^{1\cdot41} - 1}{2\cdot17 - 1}\right] = 63\cdot7\% \text{ Ans}$$

Overall efficiency = ideal η × indicated efficiency ratio × mechanical η

$$= 0\cdot637 \times 0\cdot5 \times 0\cdot8 = 0\cdot2548$$

Fuel consumption/bhp/hr $= \dfrac{75 \times 60 \times 60}{427 \times 0\cdot2548 \times 10{,}000} = 0\cdot248$ kg Ans.

3.9. Diesel cycle : cylinder diameter given different index for compression and expansion.

Obtain an expression in terms of volume ratios for the ideal efficiency of Diesel engine cycle, assuming constant specific heats.

Find the diameter of a single-acting Diesel engine working on the four-stroke cycle with combustion at constant pressure which is required to give 50 ihp at 200 rev/min from the following data :

Compression ratio 14 : 1, fuel cut-off at 5 per cent of stroke, index of compression curve 1·4, index of expansion curve 1·3, pressure at beginning of compression 0·95 kgf/cm² and ratio of stroke to bore 1·5.

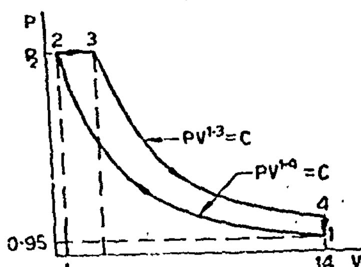


Fig. 3·22.

For theory—see text.

Compression ratio, $r=\dfrac{V_s+V_c}{V_c}=14 \quad \therefore \; V_c=0{\cdot}077\, V_s$

Cylinder volume, $V_1=V_s+V_c=V_s+0{\cdot}077\; V_s=1{\cdot}077\, V_s$

Cut-off volume, $V_3=V_c+0{\cdot}05\, V_s=0{\cdot}127\, V_s$

$P_1V_1^n=P_2V_2^n, \quad 0{\cdot}95\times 14^{1{\cdot}4}=P_2\times 1 \qquad \therefore \; P_2=38{\cdot}2$ kgf/cm²

$P_3V_3^n=P_4V_4^n, \; 38{\cdot}2\times 0{\cdot}127^{1{\cdot}3}=P_4\times 1{\cdot}077^{1{\cdot}3}$

$\therefore \; P_4=2{\cdot}374$ kgf/cm²

Work done = area $1234=P_2\,(V_3-V_2)+\dfrac{P_3V_3-P_4V_4}{n-1}-\dfrac{P_2V_2-P_1V_1}{n-1}$

$$=10^4\times 10^{-6}\times V_s\Bigg[38{\cdot}2\,(0{\cdot}127-0{\cdot}077)+\frac{38{\cdot}2\times 0{\cdot}127-2{\cdot}374\times 1{\cdot}077}{1{\cdot}3-1}-\frac{38{\cdot}2\times 0{\cdot}077-0{\cdot}95\times 1{\cdot}077}{1{\cdot}4-1}\Bigg]$$

$=0{\cdot}04763\, V_s$ kgf m $\qquad$ [V_s in cc

Now $\quad V_s=\dfrac{\pi}{4}d^2\, l=\dfrac{\pi}{4}d^2\times 1{\cdot}5\, d=1{\cdot}178\, d^3$

Work done per cycle $= 0{\cdot}04763 \times 1{\cdot}178\, d^3 = 0{\cdot}05611\, d^3$ kgf m

$$\text{ihp} = \frac{\text{work done per cycle} \times \text{no. of working cycles per sec.}}{75}$$

$$50 = \frac{0{\cdot}05611\, d^3 \times \dfrac{200}{2 \times 60}}{75}$$

$\therefore$ Diameter $\underline{d = 34{\cdot}2 \text{ cm}}$ and stroke $\underline{l = 51{\cdot}3 \text{ cm}}$ Ans

3-10. Dual cycle : mep.

The following data refer to an oil engine operating on the dual cycle. Compression ratio 11·6 ; pressure and temperature at the beginning of compression, 1 kgf/cm² and 320°K, percentage increase in pressure during constant volume burning, 53 and percentage volume increase during constant pressure burning, 38. If it is assumed that $C_p = 0{\cdot}26$ and $C_v = 0{\cdot}19$ and that the compression and expansion curves are isentropic, find the temperature of gases at the end of expansion and mean effective pressure of the cycle.

$$\gamma = \frac{C_p}{C_v} = \frac{0{\cdot}26}{0{\cdot}19} = 1{\cdot}368$$

Assuming clearance volume as unity, $V_1 = 11{\cdot}6$ and $V_4 = 1{\cdot}38$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$, $\quad 1 \times 11{\cdot}6^{1{\cdot}368} = P_2 \times 1^{1{\cdot}368}$ $\quad \therefore P_2 = 28{\cdot}6$ kgf/cm²

Fig. 3·23.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}, \quad \frac{1 \times 11{\cdot}6}{320} = \frac{28{\cdot}6 \times 1}{T_2} \qquad \therefore T_2 = 788^\circ K$$

$$P_3 = 1{\cdot}53 P_2 = 1{\cdot}53 \times 28{\cdot}6 = 43{\cdot}76 \text{ kgf/cm}^2$$

$$T_3 = 1{\cdot}53 T_2 = 1{\cdot}53 \times 788 = 1{,}206^\circ K$$

$$T_4 = 1{\cdot}38 T_3 = 1{\cdot}38 \times 1{,}206 = 1{,}663^\circ K$$

$$P_4 V_4^{\gamma} = P_5 V_5^{\gamma}, \quad 43{\cdot}76 \times 1{\cdot}38^{1{\cdot}368} = P_5 \times 11{\cdot}6^{1{\cdot}368} \quad \therefore P_5 = 2{\cdot}378 \text{ kgf/cm}^2$$

$$\frac{P_4 V_4}{T_4} = \frac{P_5 V_5}{T_5}, \quad \frac{43{\cdot}76 \times 1{\cdot}38}{1{,}663} = \frac{2{\cdot}378 \times 11{\cdot}6}{T_5}$$

$$\therefore \underline{T_5 = 760^\circ K}$$

Work done per cycle = area of the P-V diagram

$$=P_3(V_4-V_3)+\frac{P_4V_4-P_5V_5}{n-1}-\frac{P_2V_2-P_1V_1}{n-1}$$

$$=43{\cdot}76(1{\cdot}38-1)+\frac{43{\cdot}76\times1{\cdot}38-2{\cdot}378\times11{\cdot}6}{1{\cdot}368-1}-\frac{28{\cdot}6\times1-1\times11{\cdot}6}{1{\cdot}368-1}$$

$=59{\cdot}6$ units

$$\text{m.e.p.}=\frac{59{\cdot}6}{11{\cdot}6-1}=5{\cdot}622\ \text{kgf/cm}^2 \quad \textbf{Ans.}$$

3-11. Dual combustion cycle : efficiency ; mep ; specific fuel consumption.

A Diesel engine working on the dual combustion cycle has a stroke volume of 0·0084 m³ and a compression ratio of 15 to 1. The fuel has a calorific value of 10,000 kcal/kg. At the end of suction the air is at 1 kgf/cm² and 90°C. The maximum pressure in the cycle is 65 kgf/cm² and air fuel ratio is 21 : 1. Find for the ideal cycle (a) the thermal efficiency, (b) the mean effective pressure, and (c) the fuel consumption per horse power hour. Neglect the fuel mass in the constant volume part of combustion.

Assume $C_v=0{\cdot}17$, and $R=29{\cdot}2$ kgf m/kg°K.

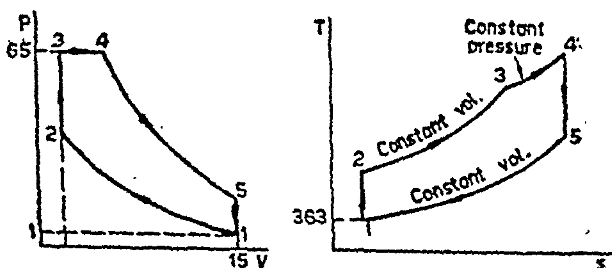


Fig. 3·24.

$$\gamma=\frac{R}{JC_v}+1=\frac{29{\cdot}2}{427\times0{\cdot}17}+1=1{\cdot}4$$

$$\frac{C_p}{C_v}=\gamma, \qquad \frac{C_p}{0{\cdot}17}=1{\cdot}4 \qquad \therefore\ C_p=0{\cdot}238$$

$$\frac{V_s+V_c}{V_c}=r, \qquad \frac{0{\cdot}0084+V_c}{V_c}=15 \qquad \therefore\ V_c=0{\cdot}0006\ \text{m}^3$$

$$V_1=V_s+V_c=0{\cdot}009\ \text{m}^3$$

$$\frac{P_2}{P_1}=r^\gamma, \qquad \frac{P_2}{1}=15^{1.4} \qquad \therefore\ P_2=44{\cdot}31\ \text{kgf/cm}^2$$

$$\frac{T_2}{T_1}=r^{\gamma-1}, \quad \frac{T_2}{363}=15^{1.4-1} \qquad \therefore\ T_2=1{,}073°K.$$

$$PV=mRT, \qquad 1\times10^4\times9\times10^{-3}=m\times29{\cdot}2\times363$$

$$\therefore \qquad m=0{\cdot}008487\ \text{kg}$$

$$\frac{P_3V_3}{T_3}=\frac{P_2V_2}{T_2}, \qquad \frac{65}{T_3}=\frac{44{\cdot}31}{1{,}073} \quad \therefore\ T_3=1{,}573°K$$

To find T_4 and T_5:

Heat added at constant volume

$=mC_v(T_3-T_2)$

$=0{\cdot}008487\times0{\cdot}17(1{,}573-1{,}073)=0{\cdot}7214$ kcal

Mass of fuel added in constant volume process

$$=\frac{0{\cdot}7214}{10{,}000}=0{\cdot}00007214\ \text{kg}$$

Total mass of fuel added $=\dfrac{0{\cdot}008487}{21}=0{\cdot}00040414$ kg

$\therefore$ Fuel added in constant pressure process

$=0{\cdot}00040414-0{\cdot}00007214=0{\cdot}000332$ kg

Mass of mixture in process 3-4

$=0{\cdot}008487+0{\cdot}00040414=0{\cdot}008891$ kg

Heat added in constant pressure process = rise in heat content

Mass of fuel in process 3-4 × C.V. = mass of mixture × $C_p(T_4-T_3)$

$$0{\cdot}000332\times10{,}000=0{\cdot}008891\times0{\cdot}238(T_4-1{,}573)$$

$$\therefore \qquad T_4=3{,}142°K$$

$$\frac{P_4V_4}{T_4}=\frac{P_3V_3}{T_3}, \qquad \frac{V_4}{3{,}142}=\frac{0{\cdot}6}{1{,}573} \qquad \therefore\ V_4=1{\cdot}2$$

$$\frac{T_5}{T_4}=\left(\frac{V_4}{V_5}\right)^{\gamma-1}, \ \frac{T_5}{3{,}142}=\left(\frac{1{\cdot}2}{9}\right)^{1.4-1} \therefore\ T_5=1{,}404°K$$

Total heat supplied $=0{\cdot}0004041\times10{,}000=4{\cdot}041$ kcal

Heat rejected = Total mixture $\times C_v\times(T_5-T_1)$

$=0{\cdot}008891\times0{\cdot}17\times(1{,}404-363)=1{\cdot}573$ kcal

Work done $=4{\cdot}041-1{\cdot}573=2{\cdot}468$ kcal

(a) $\quad$ Ideal $\eta=\dfrac{2{\cdot}468}{4{\cdot}041}=61{\cdot}06\ \%$ $\qquad$ Ans.

(b) $\quad$ mep $=\dfrac{2{\cdot}468\times427}{9\times10^{-3}\times10^4}=11{\cdot}68\ \text{kgf/cm}^2$ $\qquad$ Ans.

$$P_1'V_1'^{\gamma}=P_2'V_2'^{\gamma}$$

$$1\cdot05\times1\cdot047^{1.41}=\frac{45(15{,}280\,V_2'-1{,}183)}{15{,}280V_2'}\times V_2'^{1.41}$$

or $\quad V_2'^{1.41}-0\cdot07743\,V_2'^{0.41}-0\cdot0249=0 \qquad \therefore\ V_2'=0\cdot134\ \text{m}^3$

From Eq. (5) $\quad P_2'=\frac{45(15{,}280\times0\cdot134-1{,}183)}{15{,}280\times0\cdot134}=19\ \text{kgf/cm}^2$

From Eq. (1) $\quad T_3'=1\cdot528\times10^4\times0\cdot134=2{,}048°K$

From Eq. (4) $\quad T_2'=T_3'-\frac{200}{0\cdot169}=2{,}048-\frac{200}{0\cdot169}=865°K$

$$C_p(T_4'-T_3')=200,\quad 0\cdot238(T_4'-2048)=200$$

$$\therefore \qquad T_4'=2{,}888°K$$

$$\frac{P_4'V_4'}{T_4'}=\frac{P_3'V_3'}{T_3'},\quad \frac{V_4'}{2{,}888}=\frac{0\cdot134}{2{,}048} \qquad \therefore\ V_4'=0\cdot189\ \text{m}^3$$

$$\frac{T_4'}{T_5}=\left(\frac{V_5}{V_4'}\right)^{\gamma-1},\quad \frac{2{,}888}{T_5}=\left(\frac{1\cdot047}{0\cdot189}\right)^{1.41-1}$$

$$\therefore \qquad T_5=1{,}432°K$$

Heat rejected in constant volume operation, 5-1

$$=C_v(T_5-T_1)=0\cdot169\,(1{,}432-373)=179\ \text{kcal}$$

Ideal mixed cycle $\eta=\frac{400-179}{400}=55\cdot25\%$ **Ans.**

Note.—(1) The maximum temperature for the same heat supplied and same maximum pressure is greater in constant volume cycle than in dual combustion cycle.

(2) Dual combustion cycle is more efficient than constant volume cycle for the same maximum pressure.

3·13. Special cycle.

In an ideal cycle charge is c[illegible] explosion occurs at the end of the stroke and [illegible] to such a condition that subsequent is[illegible] charge to its initial original pressure [illegible] standard efficienc[illegible] 'c is [illegible]

$$\frac{V_1}{V_4}=x, \qquad \frac{V_2}{V_3}=r$$

$$\therefore \qquad \frac{V_1}{V_2}=\frac{x}{r}, \quad \text{since } V_3=V_4$$

Heat is supplied during the constant volume burning and is rejected during isothermal compression.

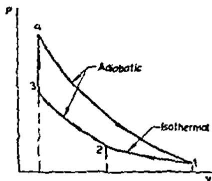


Fig. 3 26.

Consider 1 kg of air

Heat supplied $=C_v(T_4-T_3)$

Heat rejected $=\frac{P_1V_1}{J}\log_e r$

$$=\frac{RT_1\log_e\frac{V_1}{V_2}}{J}=\frac{RT_1\log_e\frac{x}{r}}{J}$$

$$\therefore \quad \text{Efficiency}=\frac{C_v(T_4-T_3)-\frac{RT_1}{J}\log_e\frac{x}{r}}{C_v(T_4-T_3)}$$

$$=1-\frac{R\log_e\frac{x}{r}}{J\times C_v\left(\frac{T_4}{T_1}-\frac{T_3}{T_1}\right)}$$

Now $\quad C_v=\frac{R}{J(\gamma-1)}$

$$\frac{T_4}{T_1}=\left(\frac{V_1}{V_4}\right)^{\gamma-1}=x^{\gamma-1} \qquad \text{and} \qquad \frac{T_3}{T_1}=\frac{T_3}{T_2}=\left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

Substituting the values

$$\eta=1-\frac{(\gamma-1)(\log_e x-\log_e r)}{x^{\gamma-1}-r^{\gamma-1}}$$

$$=1-\frac{\log_e x^{\gamma-1}-\log_e r^{\gamma-1}}{x^{\gamma-1}-r^{\gamma-1}} \qquad \text{Q.E.D}$$

3·14. Suction temperature.

Assuming that the densities and specific heats of the gases remain constant during the suction stroke of an I.C. engine, show that T_s the suction temperature is given by the expression $\dfrac{rtT}{(r-1)T+t}$ *where r is the volume compression ratio at ends of stroke, T the temperature of the exhaust and 't' the atmospheric temperature. Neglect the heat transfer to and from the walls during the suction stroke.*

A petrol engine has compression ratio of 6·75. Find the temperature at the end of suction stroke if the temperature of exhaust is 300°C. Assume atmospheric temperature to be 25°C.

As temperature of exhaust gas is T, density of exhaust gas $\propto \dfrac{273}{T}$

Similarly, density of suction air $\propto \dfrac{273}{t}$

Let the cylinder volume be unity.

Volume of exhaust gas remaining in cylinder $= \dfrac{1}{r}$

$\therefore$ Volume of air sucked in $=1-\dfrac{1}{r}=\dfrac{r-1}{r}$

$\therefore$ Proportion of exhaust gas and suction by mass is

$$=\frac{273}{T}\times\frac{1}{r} : \frac{273}{t}\times\frac{r-1}{r}$$

Equating heat content of constituents to heat content of mixture, we have

$$T_s\left(\frac{273}{T}\times\frac{1}{r}+\frac{273}{t}\times\frac{r-1}{r}\right)=\frac{273}{T}\times\frac{1}{r}\times T+\frac{273}{t}\times\frac{r-1}{r}\times t$$

$$T_s\left(\frac{1}{T}+\frac{r-1}{t}\right)=1+r-1 \qquad \left[\frac{273}{r} \text{ is common}\right.$$

$$\therefore \qquad T_s=\frac{rtT}{(r-1)\,T+t} \qquad \text{Q.E.D.}$$

Substituting the values

$$T_s=\frac{6\cdot75\times298\times573}{(6\cdot75-1)\times573+298}=321°K \qquad \text{Ans.}$$

EXERCISES 3

3.1. Efficiency of Carnot cycle with losses.

(a) *State the Kelvin-Planck statement of the second law of thermodynamics.*

(b) *A working fluid goes through a Carnot cycle of operations, the upper absolute temperature of the fluid being θ_1, and the lower absolute temperature being θ_2. The amount of heat taken in and rejected by the working fluid are Q_1 and Q_2 respectively. On account of losses of heat due to conduction, etc., the heat source temperature T_1 is higher than θ_1 and the heat sink temperature T_2 is lower then θ_2. If $T_1=(\theta_1+kQ_1)$ and $T_2=(\theta_2-kQ_2)$ where k is the same constant for both the equations, show that the efficiency of the plant is given by*

$$\eta=1-\frac{T_2}{(T_1-2kQ_1)}$$

3.2. Stirling cycle : efficiency : compression ratio ; mep.

(a) *Prove that in the Carnot cycle the ratios of expansion are same and hence derive the efficiency for the Carnot cycle. On what factors in actual practice the two temperature in the Carnot cycle depend ?*

(b) *A hot air engine working on the Stirling cycle sucked air at 1 kgf/cm² and temperature 27°C. The pressure after constant volume heating was 10 kgf/cm² and temperature 307°C. Find (i) the ideal efficiency, (ii) the compression ratio, and (iii) the mep of the engine.*

[η same as the Carnot cycle $\eta=48.4\%$; $r=5.18$; $W=7.94$ units ; mep $=1.9$ kgf/cm²]

3.3. Ericsson cycle : efficiency ; loss in regenerator ; mep. hp.

The 4-cylinder air engine of the ship Ericsson worked on the Ericsson cycle between the temperature limits 50°C and 212°C and had the following values : Piston displacement per kg of air 1 38 m³ ; ratio of explosion 1·5 ; revolutions per minute 9 ; diameter of the four cylinders each 4 26 m ; stroke 1·83 m Calculate (a) the work done per kg of air per stroke, (b) the thermal efficiency of the engine, (c) the heat energy wasted in the regenerator assuming its efficiency 0 9, (d) the mean effective pressure, and (e) the indicated horse-power. Take $C_p=0.24$ and $C_v=0.171$ for air.

[$W=4.54$ kcal/kg ; $\eta=\frac{4.54}{17.48}=26\%$; heat equivalent wasted in regenerator $=3.89$ kcal/kg ; mep $=0.14$ kgf/cm² ; ihp $=292$]

4. Otto cycle : pressure and temperature at key points ; efficiency ; mep.

Sketch an actual P-V diagram for the Otto cycle and compare it with the theoretical diagram.

A certain quantity of air undergoes the following cycle of operations. It is compressed adiabatically from pressure 1 033 kgf/cm² and temperature 25°C, the ratio of compression being 8. Heat is now added at constant volume at the rate of 390 kcal/kg, after which expansion takes place adiabatically to its original volume and heat is then rejected at constant volume until the initial state is reached. Assuming C_p= 0·238 and C_v=0·169, determine (a) the pressure and temperature at key points, (b) the thermal efficiency of the cycle and (c) the mean effective pressure.

[P_2=19·38 kgf/cm² ; t_2=426°C ; P_3=83·4 kgf/cm² ; t_3=2,735°C ; P_4=4·443 kgf/cm² ; t_4=1,009°C ; W=95,600 kgfm ; η=57·4% ; mep=12·84 kgf/cm²]

3-5. Gas engine (Otto cycle) : actual and relative efficiency.

A gas engine working on the Otto cycle uses 12·8 m³ of gas in 50 minutes. The gas is at a temperature of 27°C and at a pressure of 13·6 cm of water above the atmospheric pressure of 74·5 cm of mercury. The gas has a calorific value of 4,470 kcal/m³ measured at 1·033 kgf/cm² and a temperature of 0°C. The compression ratio of the engine is 6·25 and it develops 33·6 ihp. Determine the actual thermal efficiency and the relative efficiency. Take γ=1·4.

[Ideal η=52% ; consumption of gas at NTP=11·58 m³ ; actual η=34·21% ; relative η=65·8%]

3-6. Otto cycle : cylinder dimensions given different indices for compression and expansion.

The following data refer to an internal combustion engine operating on the constant volume cycle : Compression ratio 6·5, shaft horsepower 60, mechanical efficiency 0·9, working cycle per minute 2000, temperature and pressure at the beginning of compression 360°K and 1 kgf/cm² ; ratio of piston stroke to bore 1·15, indices of compression and expansion 1·36 and 1·26 respectively, ratio of maximum pressure to minimum pressure 32 to 1. Find (a) the temperature at the end of

compression and at the beginning and end of expansion, (b) the indicated mean effective pressure, and (c) the piston diameter. $R=29 \cdot 27$.

[$t_2=433°C$; $t_3=1,499°C$; $t_4=816°C$; $W=46,750$ kgf m/kg ; $V_s=0.8319$ m^3 ; mep=5·46 kgf/cm^2 ; d=14·5 cm ; l=16·7 cm]

3·7. Diesel cycle : pressure at the end of compression and expansion ; mep.

Show by means of sketches on the pressure-volume and temperature-entropy fields, the state changes of the working substance for an engine operating on the Diesel cycle.

In such a cycle the compression ratio is 12, the expansion ratio is 4·8 and the pressure at the beginning of compression is 1 kgf/cm². Determine the pressure at the end of compression and at the end of expansion. Find also the mean effective pressure of the cycle. Take $C_v=0 \cdot 19$ and $C_p=0 \cdot 26$.

[$\rho=2 \cdot 5$; $P_2=30$ kgf/cm^2 ; $P_4=3 \cdot 506$ kgf/cm^2 ; $W=85 \cdot 47$ units ; mep=7·77 kgf/cm^2]

3·8. Relative efficiency of Otto and Diesel cycles.

Why is it preferred to compare the efficiency of modern high speed compression-ignition engines with ideal efficiency of the Otto cycle instead of the Diesel cycle ?

Find the efficiency ratio of an engine which consumes 0·16 kg/ihp hr of fuel-oil having a calorific value of 10,000 kcal/kg when compared with (a) the Otto cycle, and (b) the Diesel cycle, assuming that cut-off occurs at 0·06 of the stroke and the ratio of compression is 14. $\gamma=1 \cdot 4$.

By what percentage the Otto cycle is more efficient than the Diesel cycle ?

[Actual $\eta=39 \cdot 53\%$; Otto $\eta=65 \cdot 2\%$; η ratio=0·6063 ; Diesel $\eta=60 \cdot 4\%$; η ratio=0·655 ; Otto cycle more efficient by 7·36%]

3·9. Efficiency of Carnot, Otto, Atkinson and Diesel cycles.

Show the Carnot, Otto, Atkinson and Diesel cycles per kg of working agent on T-s diagram for extreme limits of temperature 2,000°K and 300°K, the pressure before compression being 1 kgf/cm² and ratio of compression 6 in Otto and Atkinson cycles and 18 in Diesel cycle. What is the efficiency of the hypothetical cycle in each case ? Take $C_v=0 \cdot 169$ and $\gamma=1 \cdot 4$.

[Carnot $\eta=0{\cdot}85$; Otto cycle $\eta=0{\cdot}5117$; Atkinson cycle, ratio of expansion$=13{\cdot}94$, $\eta=0{\cdot}581$; Diesel cycle, $\rho=2{\cdot}098$, $\eta=0{\cdot}6273$]

3.10. Dual combustion cycle : thermal efficiency ; mep.

The compression ratio of an engine working on the dual cycle is 9 : 1 and the maximum pressure is 40 kgf/cm². The temperature at the beginning of compression is 92°C and that of exhaust is 542°C. Considering the ideal cycle with air as working fluid and assuming that the pressure at the start of compression is 1 kgf/cm², find (a) the thermal efficiency, and (b) the mean effective pressure of indicator diagram. Take $R=29{\cdot}28$, $\gamma=1{\cdot}4$.

[$T_2=879°K$; $P_2=21{\cdot}67$ kgf/cm², $T_3=1{,}623°K$; $P_5=2{\cdot}232$ kgf/cm² ; $T_4=1{,}860°K$; $W=107{\cdot}3$ kcal or 45,800 kgf m ; thermal $\eta=58{\cdot}16\%$; $V_s=0{\cdot}95$ m³ ; mep$=4{\cdot}822$ kgf/cm²]

3.11. Dual combustion cycle : specific fuel consumption.

A semi-Diesel engine working on the dual combustion cycle has the pressure of 16 kgf/cm² at the end of adiabatic compression, and the combustion is such that the temperature at first rises at constant volume and then at constant pressure of 32 kgf/cm². The heat of combustion is shared equally between the two phases and the expansion is adiabatic. If the relative efficiency is 55 per cent, determine the actual specific fuel consumption in kg per ihp-hr.

Compression begins at 0·98 kgf/cm² and 37°C. Calorific value of the fuel is 10,300 kcal/kg. Take $\gamma=1{\cdot}4$, $R=29{\cdot}28$.

[$T_2=689°K$; $T_3=1{,}377°K$; $T_4=1{,}869°K$; $T_5=951°K$; Ideal $\eta=53{\cdot}4$; specific fuel consumption$=0{\cdot}209$ kg/ihp-hr.]

3.12. Joule cycle : ideal η and mep.

1 kg of air is taken through a Joule cycle. Initially the air is at 1 ata and 15°C. The compression ratio is 4 and the heat added is 400 kcal. Calculate the ideal cycle efficiency and the mean effective pressure.

[$T_2=428{\cdot}3°K$; $T_3=1262°K$; $T_4=848{\cdot}2°K$, $W=55{\cdot}6$ kcal ; $\eta=56\%$; mep$=1{\cdot}092$ kgf/cm².]

3.13. Special cycle : isothermal compression ; constant volume heating ; adiabatic expansion.

Derive an expression for the efficiency of the following cycle : "Starting with an isothermal compression, then an increase of pressure

4

Properties of Steam

4·1. Generation of Steam. The process of generation of steam is as follows :

(*a*) **Heating of water.** Feed water is pumped into boiler raising the pressure of water from atmospheric pressure P_a to boiler pressure P_b. Some work is done by the feed pump in increasing the pressure, which is given by $\left[\frac{P_b-P_a}{J}\times v_f\right]$ heat units per kg, where v_f is the specific volume of feed water. The temperature of water is then raised at constant pressure, to the saturation temperature t_s corresponding to the boiler pressure. The word *saturation* refers to heat saturation as any further addition of heat produces vapour. The heat absorbed during this process is termed as *sensible heat* because the rise in temperature due to heat can be detected by senses. The *specific enthalpy* of water h_f is the sum of sensible heat and heat equivalent to work done in raising the pressure (*i.e.* the work of introduction).

$$h_f = \text{sensible heat} + \frac{Pv_f}{J} \tag{4·1}$$

For engineering purposes sensible heat and enthalpy of water may be taken as same since the work of introduction is negligible. In steam tables h_f and v_f are directly read.

(*b*) **Evaporation of water.** Further addition of heat evaporates the water at constant temperature. The heat absorbed in complete evaporation of 1 kg of water from saturation temperature is known as *latent heat or heat of evaporation* of steam at that particular pressure. The latent heat goes on decreasing with increasing pressure as is seen in Fig. 4·1 and at *critical pressure* it is zero. The latent

heat consists of (*i*) heat expended in overcoming the external resistance to change in volume $=\frac{P}{J}(v_g-v_f)$ per kg + (*ii*) heat expended in overcoming the internal molecular resistance to change in state from saturated water to dry saturated steam *i e.*, change of internal energy $=(u_g-u_f)$

$$\text{Latent heat } h_{fg}=(u_g-u_f)+\frac{P(v_g-v_f)}{J} \qquad (4\cdot2)$$

(u_g-u_f) may be called *internal latent heat* and (P/J) (v_g-v_f) is known as *external work of evaporation*

In practice v_f, being very small, is neglected.

If all the water is not completely evaporated into steam, it will be wet containing suspended water particles. This is known as wet saturated vapour as the slightest decrease in temperature results in dropping of moisture. Wet steam being vapour, has no specific heats. The dryness fraction of wet steam is given by the ratio of mass of dry steam to the total mass of steam.

$$\text{Dryness fraction, } x=\frac{\text{mass of dry steam}}{\text{mass of total steam}}$$

$$=\frac{\text{volume of dry steam in 1 kg of wet steam, } v}{\text{specific volume of dry steam, } v_g}$$

$\therefore$ Volume of dry steam in 1 kg of wet steam $=x\times v_g$ (4·3)

Enthalpy of wet steam per kg, $h=h_f+xh_{fg}$ [4·4 (*a*)]

If steam is dry, $x=1$

$$\therefore \qquad h_g=h_f+h_{fg} \qquad [4\cdot4\ (b)]$$

When the last drop of suspended water is evaporated it is known as dry and saturated steam. Saturation here refers to heat content as any further addition of heat produces *superheated steam.*

(*c*) **Superheating of steam** If the dry and saturated steam is further heated the temperature rises at *constant pressure and heat* absorbed during the process is known as *heat of superheat.* Superheated steam may be assumed to behave as a perfect gas.

Heat of superheat per kg $=C_p(t-t_s)$ (4·5)

The value of C_p for superheated steam generally varies from 0 48 to 0 55. The enthalpy of superheated steam per kg

$$h=h_f+h_{fg}+C_p(t-t_s)=h_g+C_p(t-t_s) \qquad (4\cdot6)$$

Fig. 4·1 shows various phases from formation of ice to superheated steam as the addition of heat proceeds and Fig. 4·2 shows temperature-volume relation for heating of H_2O at atmospheric pressure.

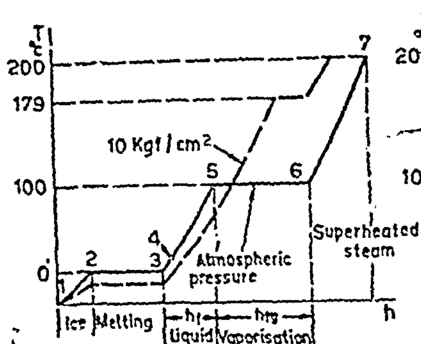


Fig. 4·1. Heat requirements for phase changes of H_2O.

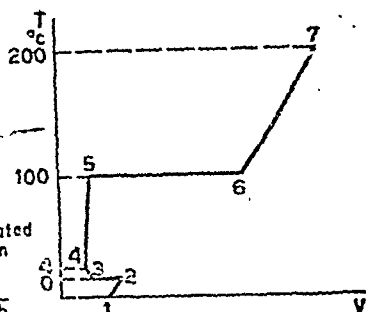


Fig. 4·2. t-V relation for heating of H_2O at atmospheric pressure.

4·2. Critical Temperature and Pressure. Critical temperature is defined as the temperature above which it is impossible to liquefy water vapour by pressure alone no matter how great is the pressure employed. The critical temperature of steam is 374·15°C. The vapour pressure corresponding to this temperature is known as *critical pressure* and for steam has a value of 225·65 kgf/cm².

4·3 Representation of Steam Raising Phenomenon. In the P-v diagram of H_2O, Fig. 4·3, the curve ac represents the water line and gives the volume of 1 kg of water at any pressure. The curve cde represents the dry steam line. The part of the diagram to the left of abc is known as the liquid region and the part enclosed by the curve $abcde$ is known as the wet region. To the right of the curve cde and bounded by the critical isothermal t_c is the superheated region.

Considering one kg of water at any pressure, say atmospheric the point b on water line represents the volume at saturation temperature, the value being 0·001 m^3. The horizontal line bd represents evaporation at constant pressure ; d is on dry steam line and the volume at this point is 1·673 m^3 ; line df in the superheated region represents superheating at constant pressure. From the diagram it is seen that the specific volume of steam decreases with rise of

pressure and at critical pressure there is no difference between the volume of liquid and dry vapour.

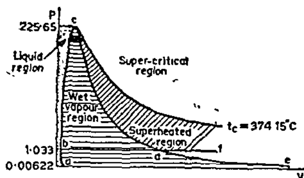


Fig. 4 3. P-v diagram for H_2O.

A similar diagram but on temperature and entropy (T-s) ordinates is more important than P-v diagram, as the former gives total heat at any point whereas the latter gives only the work done.

4·4. Internal Energy of Steam The enthalpy of steam consists of enthalpy of water h_f, the internal latent heat add the external work of evaporation. The last term is spent in doing external work and is not stored in the form of energy

∴ Internal energy of steam

= enthalpy of steam—external work of evaporation

$$u = h - \frac{Pv}{J}$$

For dry steam, $u_g = h_g - \frac{Pv_g}{J}$ (4·7)

4·5. Volume of Steam (*i*) *Wet steam.* The specific volume of wet steam is the sum of the volumes of x kg of dry vapour with a specific volume v_g and $(1-x)$ kg of liquid with a specific volume of v_f, *i.e.*

Volume of wet steam, $v = xv_g + (1-x)v_f$ [4 8 (a)]

Both v_g and v_f can directly be read from Steam Tables

The quality of steam x can vary from 0 to 1.

At lower pressure the volume of water in wet steam is very small as compared with the volume of steam and hence in practice, mentioned, the volume of water $(1-x)v_f$ is neglected ; therefore

Specific volume of wet steam, $v = xv_g$ [4·8 (b)]

and density of wet steam $= \dfrac{1}{xv_g}$ [4·9 (b)]

(*ii*) *Volume of dry saturated steam.* This is either directly read from steam tables or is found by the help of Clapeyron's equation given below.

Clapeyron's equation.

It is given by $v_g - v_f = \dfrac{J(h_g - h_f)}{T_s} \times \dfrac{dT_s}{dP}$ (4·10)

This equation is used for evaluating the specific volume at any required temperature or pressure. The value of $\dfrac{dT_s}{dP}$ in the equation is obtained from the slope of T-P curve which is plotted from experimental results.

Clapeyron's equation is derived from the Carnot theorem. The Carnot efficiency is given by $\dfrac{T_1 - T_2}{T_1}$ and for a small pressure drop dP and corresponding temperature drop dT, the Carnot efficiency is given by $\dfrac{dT}{T}$. In evaporation the heat supplied is the latent heat.

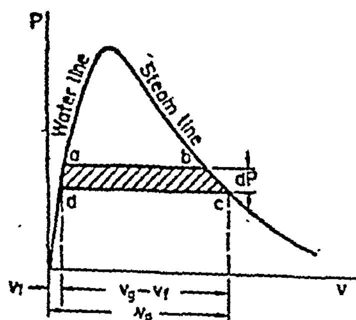


Fig. 4·4.

From P-v diagram Fig. 4·4

Work done $= J \times (h_g - h_f) \dfrac{dT_s}{T_s} = (v_g - v_f) \times dP$

$\therefore \quad v_g - v_f = \dfrac{J(h_g - h_f)}{T_s} \times \dfrac{dT_s}{dP}$ Q.E.D.

The Clapeyron's equation is even applicable to solidification and liquefication as in the Carnot cycle nothing is said about the state of working fluid. If $(v_g - v_f)$ is positive $\frac{dT_s}{dP}$ is positive and the melting point increases with temperature. If there is contraction in volume, $\frac{dT_s}{dP}$ is negative and an increase in pressure lowers the melting point.

(iii) *Volume of superheated steam.* Values of the specific volume of the superheated steam for various pressures and temperatures are obtained from steam tables.

Approximate method of finding the volume of superheated steam is by applying the gas equation, $\frac{PV}{T}$ = constant. Generally the superheating takes place at constant pressure.

$$\therefore \quad v = T \times \frac{v_g}{T_s} \qquad [4\cdot11(a)]$$

where v_g and T_s are respectively the specific volume and absolute temperature of the saturated steam and v and T are respectively the specific volume and absolute temperature of the superheated steam.

The specific volume of superheated steam is also given by

$$pv \times 10^2 = (h - 464\cdot1) \qquad [4\cdot11(b)]$$

4.6. Entropy of Steam The complete process of formation of steam is a constant pressure process and hence, the change of entropy is found by the formula $C_p \log_e \frac{T_2}{T_1}$.

The entropy of steam is generally reckoned above the freezing point of water.

Entropy of one kg of water $s_f = C_p \log_e \frac{T_s}{273}$

Assuming specific heat of water as unity

$$s_f = \log_e \frac{T_s}{273} \qquad (4\cdot12)$$

Entropy of evaporation $= \frac{dq}{T_s} = \frac{x(h_g - h_f)}{T_s} = \frac{h_{fg}}{T_s} \qquad (4\cdot13)$

$\therefore$ Total entropy of dry saturated steam

$$s_g = s_f + s_{fg}$$

$$= \log_e \frac{T_s}{273} + \frac{h_{fg}}{T_s} \qquad [4\cdot14(a)]$$

and total entropy of wet steam $s = s_f + x \times s_{fg}$

$$= \log_e \frac{T_s}{273} + \frac{x h_{fg}}{T_s} \qquad [4\cdot14(b)]$$

Increase of entropy due to superheating

$$= C_p \log_e \frac{T}{T_s}$$

where C_p is the specific heat of superheated steam.

∴ Total entropy of the superheated steam

$$s = s_g + C_p \log_e \frac{T}{T_s}$$

$$= \log_e \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_p \log_e \frac{T}{T_s} \qquad (4\cdot15)$$

4·7. Temperature-Entropy Diagram. The temperature-entropy diagram for steam is plotted from the values taken from

Fig. 4·5. Temperature-entropy diagram for steam.

steam tables. In this diagram absolute temperature is plotted as ordinate with origin as absolute zero and entropy is plotted as abscisse as shown in Fig. 4·5. In the diagram are plotted the constant volume lines in wet region and constant pressure lines in the superheated region.

In the T-s diagram the area under the curve represents the heat added as shown in Fig. 4 6.

The advantage of T-s diagram over P-v diagram is that the T-s diagram represents the whole cycle of heat addition and rejection whereas the P-v diagram represents only the work done.

An isothermal process is represented in T-s diagram by a horizontal line and a reversible adiabatic process is represented by a vertical line as shown in Fig. 4·5. A throttling process cannot be truly represented on T-s diagram as it is an irreversible process

and total entropy of wet steam $s = s_f + x \times s_{fg}$

$$= \log_e \frac{T_s}{273} + \frac{xh_{fg}}{T_s} \qquad [4 \cdot 14(b)]$$

Increase of entropy due to superheating

$$= C_p \ \log_e \frac{T}{T_s}$$

where C_p is the specific heat of superheated steam.

$\therefore$ Total entropy of the superheated steam

$$s = s_g + C_p \log_e \frac{T}{T_s}$$

$$= \log_e \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_p \ \log_e \frac{T}{T_s} \qquad (4 \cdot 15)$$

4·7. Temperature-Entropy Diagram. The temperature-entropy diagram for steam is plotted from the values taken from

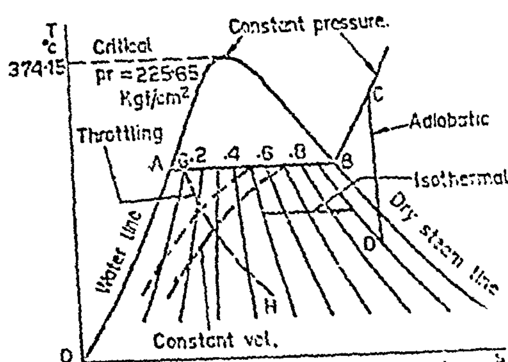


Fig. 4·5. Temperature-entropy diagram for steam.

steam tables. In this diagram absolute temperature is plotted as ordinate with origin as absolute zero and entropy is plotted as abscisse as shown in Fig. 4·5. In the diagram are plotted the constant volume lines in wet region and constant pressure lines in the superheated region.

In the T-s diagram the area under the curve represents the heat added as shown in Fig. 4 6.

The advantage of T-s diagram over P-v diagram is that the T-s diagram represents the whole cycle of heat addition and rejection whereas the P-v diagram represents only the work done.

An isothermal process is represented in T-s diagram by a horizontal line and a reversible adiabatic process is represented by a vertical line as shown in Fig. 4·5. A throttling process cannot be truly represented on T-s diagram as it is an irreversible process

It may, however, be shown by a dotted curve. The area under this curve has no significance and does not represent the heat added.

Fig. 4·6. *T-s* diagram for steam.

4·3. Heat-Entropy Chart (Mollier Chart) The total-heat entropy chart, known as Mollier chart, is very useful for solving pro-

Fig. 4·7. Mollier Chart for steam.

blems in thermodynamics. Fig. 4 7 shows complete Mollier chart, showing water line and saturation line as one curve

In commercial charts, Fig 4 8, water line is not plotted as it is not required for solving the problems ; instead the portion near the saturation line only is plotted. The constant pressure lines are the straight lines in wet region as their slope is equal to $\frac{dh}{ds}=\frac{dq}{ds}=T$ (as pressure is constant). In superheated region the constant pressure lines are curved. On Mollier chart throttling process, being a constant enthalpy process, is represented by a horizontal line and reversible adiabatic process, being a constant entropy process, is represented by a vertical line.

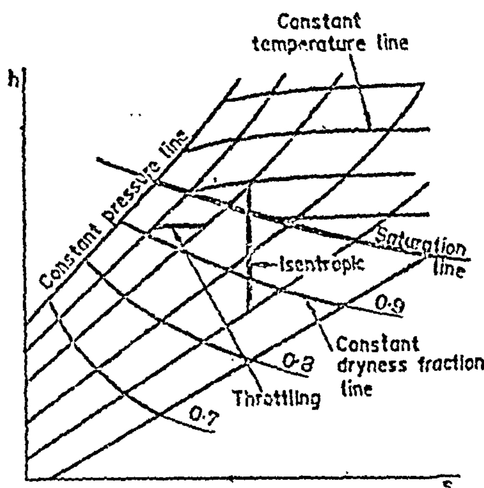


Fig. 4·8. Commercial Mollier Chart for steam.

4-9. Steam Tables. The important properties of steam required for engineering calculations are saturation temperature, specific volume, liquid enthalpy, change of phase or evaporation enthalpy, enthalpy of dry and saturated steam, internal energy and entropy at any given pressure. As steam is a vapour and not a perfect gas these values of properties cannot be calculated from the gas laws.

In steam tables these properties are tabulated for various pressures, obtained by experiment and calculation. In this book the steam properties have been taken from 'Steam and other Tables' by Mathur and Mehta.

It is found from the properties of saturated steam that with the increase of pressure there is a gradual increase of sensible heat and decrease of heat of evaporation, but the enthalpy of dry steam increases upto a certain pressure and then it again decreases.

4-10. Heating and Expansion of Vapours. As in the case of gases the vapours may also be expanded or compressed by different methods as given below.

(*i*) *Constant Volume Heating.* An example of constant volume process is the heating of steam in a closed rigid vessel. As there is no change in volume, work done in this process is zero. By the first law of thermodynamics

$$Q = W + (U_2 - U_1)$$

$$q = 0 + \left[\left(h_2 - \frac{P_2 v_2}{J}\right) - \left(h_1 - \frac{P_1 v_1}{J}\right)\right] \qquad (4\cdot16)$$

(*ii*) *Constant Pressure Expansion.* An example of constant pressure heating is the generation of steam in a boiler.

$$Q = W + (U_2 - U_1)$$

$$q = \left[\frac{P_2 v_2}{J} - \frac{P_1 v_1}{J}\right] + \left[\left(h_2 - \frac{P_2 v_2}{J}\right) - \left(h_1 - \frac{P_1 v_1}{J}\right)\right]$$

$$= h_2 - h_1 \tag{4.17}$$

In this process the heat added or subtracted is same as change in enthalpy.

(*iii*) *Constant Temperature* (*Isothermal*) *Expansion.* During evaporation or condensation (wet region) constant temperature process is the same as constant pressure process. Once the steam is in superheated region it behaves like a perfect gas obeying gas laws and hence the constant temperature expansion in this region is *hyperbolic* (Pv=constant).

(*iv*) *Hyperbolic* (Pv=*constant*) *Expansion.* The hyperbolic expansion is not isothermal in wet region but may be regarded isothermal in superheated region.

$$Q = W + (U_2 - U_1)$$

$$q = \frac{P_1 v_1}{J} \log_e r + \left[\left(h_2 - \frac{P_2 v_2}{J}\right) - \left(h_1 - \frac{P_1 v_1}{J}\right)\right]$$

$$= \frac{P_1 v_1}{J} \log_e r + (h_2 - h_1) \qquad [\text{ as } P_2 v_2 = P_1 v_1 \tag{4.18}$$

(*v*) *Expansion under a General Law of Constant* Pv^n

$$Q = W + (U_2 - U_1)$$

$$q = \frac{P_1 v_1 - P_2 v_2}{J(n-1)} + \left[\left(h_2 - \frac{P_2 v_2}{J}\right) - \left(h_1 - \frac{P_1 v_1}{J}\right)\right] \tag{4.19}$$

(*vi*) *Adiabatic or Constant Entropy Expansion* (Pv^γ=constant). In reversible adiabatic process the heat supplied is zero.

$$Q = W + (U_2 - U_1)$$

$$0 = \frac{P_1 v_1 - P_2 v_2}{J(\gamma - 1)} + \left[\left(h_2 - \frac{P_2 v_2}{J}\right) - \left(h_1 - \frac{P_1 v_1}{J}\right)\right] \tag{4.20}$$

(*viii*) *Throttling Expansion.* In a throttling or wire-drawing process, the steam is caused to flow from a higher pressure to lower pressure under conditions such that the heat interchange and work done are both zero. Thus, in this process the enthalpy at end p

is constant. This process is essentially adiabatic but irreversible causing an increase in entropy.

4.11. Measurements of Dryness Fraction. Different methods of measurements of dryness fraction are as follows :

(i) *Barrel Calorimeter.* In this method steam is mixed with water and the dryness fraction is obtained by equating the enthalpy before and enthalpy after.

(ii) *Separating Calorimeter.* It may be used when steam is wetter than about 0·95 dryness fraction.

(iii *Throttling Calorimeter.* It may be used when the dryness fraction of steam is above 0·95.

For accurate results combined separating and throttling calorimeter is used.

Combined Separating and Throttling Calorimeter. Fig. 4·9 represents schematic diagram of a combined separating and throttling calorimeter. In separating unit moisture is separated by change in the direction of flow and gravity. In throttling unit steam passes through a small opening and throttling takes place at constant enthalpy.

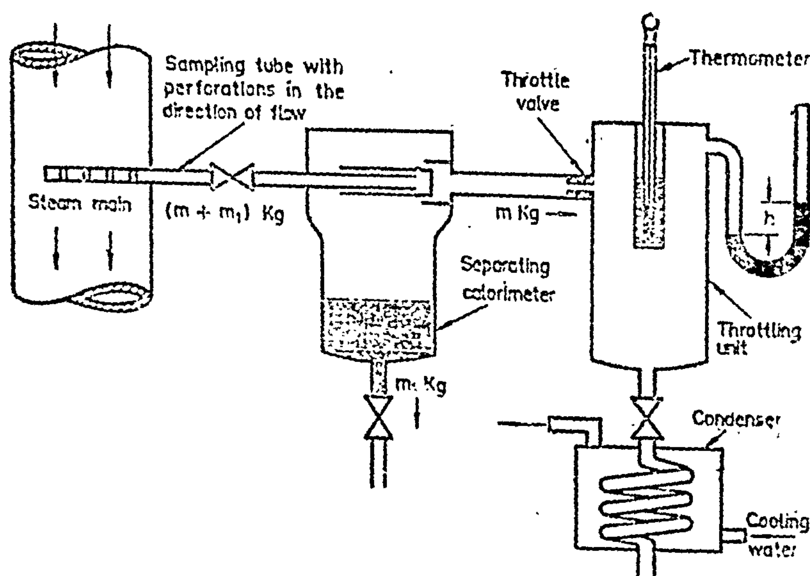


Fig. 4·9. Combined separating and throttling calorimeter.

under atmospheric pressure, some of the water would flesh into steam at lower pressure and the obtained value of x_1 and consequently the value of x_2 will be higher than the actual dryness fraction.

(*iii*) The volume of water collected from the separating unit is regarded as mass. This is not strictly true as specific volume varies with temperature.

4·12. Steam Accumulator. Steam accumulators are used for accumulation of steam where the supply or demand of steam is intermittent. The most important application is in mixed pressure turbine plants. A schematic diagram of mixed pressure turbine plant with steam accumulator is shown in Fig. 4·10. The steam accumulator consists of a big vessel, like boiler drum, containing water. When the engine exhausts more low pressure steam than the turbine requires, the surplus steam is condensed in large quantity of boiling water, thereby raising its temperature and pressure. When the engine supply is deficient the turbine demand causes a drop in pressure in the accumulator. This results in steam evaporation due to the lower saturation temperature which is available for turbine.

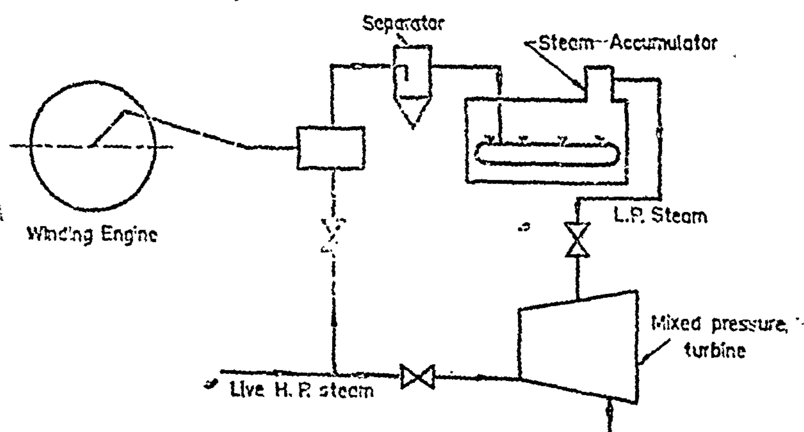


Fig. 4·10. Schematic diagram of mixed pressure turbine plant with steam accumulator.

Calculations

Let, P = initial pressure of steam in accumulator
m = initial mass of boiling water in accumulator
h = enthalpy of water in accumulator
v = specific volume of water at pressure P and temperature $0°C$

v_f=specific volume of water at boiling

h_g=specific enthalpy of entering steam

v_g=specific volume of entering steam

dm=mass of dry saturated steam added, causing an increase in pressure, dP

Initial internal energy of water in accumulator

$$U_1=m\left(h-\frac{Pv}{J}\right) \tag{4·24}$$

Energy added by steam

=internal energy of dm kg of steam+external work of evaporation of dm kg of steam given up in condensation

$$=dm\left(h_g-\frac{Pv_g}{J}\right)+dm\times\frac{P}{J}\left(v_g-v_f\right) \tag{4·25}$$

Final internal energy of water in accumulator

$$=(m+dm)\left[(h+dh)-\frac{(P+dP)v}{J}\right] \tag{4·26}$$

$$\therefore \quad m\left(h-\frac{Pv}{J}\right)+dm\times\left(h_g-\frac{Pv_g}{J}+\frac{P}{J}(v_g-v_f)\right)$$

$$=(m+dm)\left[(h+dh)-\frac{(P+dP)v}{J}\right] \tag{4·27}$$

Neglecting the volume of water v and v_f, being very small, we get

$$mh+dmh_g=(m+dm)(h+dh)$$

Neglecting second order terms

$$dm\,(h_g-h)=m\,dh$$

$$\therefore \quad dm(h_{fg})=m\,dh$$

$$\therefore \quad \int_{m_1}^{m_2}\frac{dm}{m}=\int\frac{dh}{h_{fg}}$$

$$\text{or} \quad \log_e\frac{m_{end}}{m_{start}}=\int\frac{dh}{h_{fg}} \tag{4·28}$$

IMPORTANT POINTS

1. The properties of steam given in steam tables are per k[illegible] and must be multiplied by the given mass.

under atmospheric pressure, some of the water would flesh into steam at lower pressure and the obtained value of x_1 and consequently the value of x_2 will be higher than the actual dryness fraction.

(iii) The volume of water collected from the separating unit is regarded as mass. This is not strictly true as specific volume varies with temperature.

4·12. Steam Accumulator. Steam accumulators are used for accumulation of steam where the supply or demand of steam is intermittent. The most important application is in mixed pressure turbine plants. A schematic diagram of mixed pressure turbine plant with steam accumulator is shown in Fig. 4·10. The steam accumulator consists of a big vessel, like boiler drum, containing water. When the engine exhausts more low pressure steam than the turbine requires, the surplus steam is condensed in large quantity of boiling water, thereby raising its temperature and pressure. When the engine supply is deficient the turbine demand causes a drop in pressure in the accumulator. This results in steam evaporation due to the lower saturation temperature which is available for turbine.

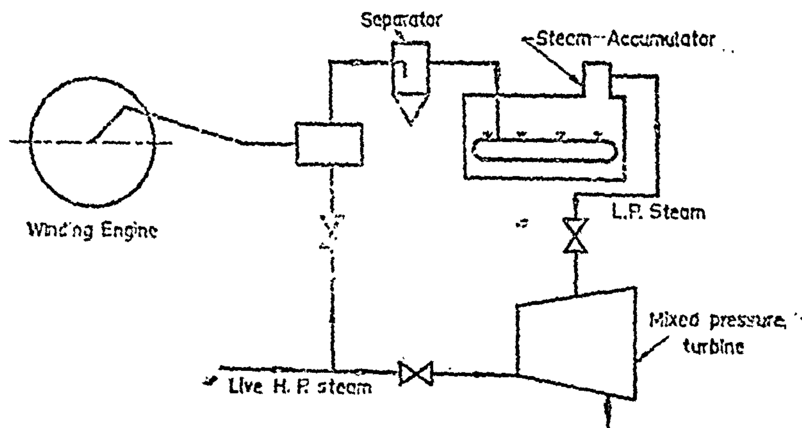


Fig. 4·10. Schematic diagram of mixed pressure turbine plant with steam accumulator.

Calculations

Let, P = initial pressure of steam in accumulator
m = initial mass of boiling water in accumulator
h = enthalpy of water in accumulator
v = specific volume of water at pressure P and temperature θ°C

Internal energy, $u = h - \frac{Pv}{J} = 542 \cdot 8 - 34 \cdot 8$ $= 508$ kcal/kg

[$u = u_f + xu_{fg} = 181 \cdot 0 + 0 \cdot 75(616 \cdot 9 - 181 \cdot 0)$ $= 508$ kcal/kg]

(b) $h_s = h_f + h_{fg} = 181 \cdot 3 + 482 \cdot 0$ $= 663 \cdot 3$ kcal/kg Ans

External work $= \frac{Pv_s}{J} = \frac{10 \times 10^4 \times 0 \cdot 198}{427}$ $= 46 \cdot 4$ kcal/kg

∴ Internal energy, $u_s = h_s - \frac{Pv_s}{J}$

$= 663 \cdot 3 - 46 \cdot 4 = 616 \cdot 9$ kcal/kg Ans

[Internal energy can also be directly read from steam tables]

(c) From steam tables :

At 10 kgf/cm² and 200°C, $h = 675 \cdot 4$ kcal, $v = 0 \cdot 2103$ m³

At 10 kgf/cm² and 250°C, $h = 702 \cdot 4$ kcal, $v = 0 \cdot 2374$ m³

∴ Total heat of 1 kg of steam at 10 kgf/cm² and 230°C temperature by interpolation

$h = 691 \cdot 6$ kcal Ans.

Volume of 1 kg of steam at 10 kgf/cm² and 230°C temperature

$v = 0 \cdot 2266$ m³

External work of evaporation per kg of superheated steam

$= \frac{P \times v}{J} = \frac{10 \times 10^4 \times 0 \cdot 2266}{427} = 53 \cdot 1$ kcal

∴ Internal energy, $u = h - \frac{Pv}{J} = 691 \cdot 6 - 53 \cdot 1 = 638 \cdot 5$ kcal/kg

Ans

[Internal energy can also be directly interpolated]

Note—The problem illustrates the use of steam tables.

4.2. Heat added ; increase in internal energy ; entropy.

0 095 m³ of water is boiling at 30°C. It is heated until the pressure is 8 kgf/cm² and dryness 0 9. Using steam tables, calculate (i) the quantity of heat added before evaporation begins at 8 kgf/cm², (ii) the increase in internal energy of the water upto the beginning of evaporation, (iii) the increase in internal energy and the external work done during the evaporation stage, and (iv) the entropy of one kg of wet steam

2. The properties of steam are tabulated for dry and saturated steam and do not apply to wet steam. For example, h_g is the enthalpy of one kg of dry and saturated steam. If it is desired to determine the enthalpy of wet steam it should always be calculated $h_f + xh_{fg}$.

3. Similarly, v_g is the volume of one kg of dry and saturated steam. Volume of one kg of wet steam is $x \times v_g$. (The volume of water particles present is assumed negligible).

4. Again, s_g is the total entropy of one kg of dry and saturated steam. Entropy of one kg of wet steam is given by $[s_f + xs_{fg}]$.

5. Enthalpy and entropy of superheated steam need not be calculated, but should be directly obtained from the steam tables for superheated steam, except when the value of C_p for superheated steam is given.

6. The specific heat of superheated steam, if required in problem, can be obtained by taking the heat of superheat per kg from the tables for superheated steam and equating to $C_p(t-t_s)$.

7. Whenever properties of steam are required for a pressure not given in steam tables it should be interpolated in the usual way.

ILLUSTRATIVE EXAMPLES

4·1. Enthalpy and internal energy of steam.

Define enthalpy, external work of evaporation and internal energy of steam.

Find the enthalpy and internal energy of 1 kg of steam at a pressure of 10 kgf/cm² (a) when the dryness fraction of steam is 0·75, (b) when steam is dry and saturared, and (c) when steam is superheated to 230°C. Neglect the volume of water.

(a) Sensible heat of steam per kg at 10 kgf/cm², $h_f = 181{\cdot}3$ kcal

Heat of evaporatior of steam per kg at 10 kgf/cm²

$$h_{fg} = 482{\cdot}0 \text{ kcal}$$

Enthalpy of steam at 10 kgf/cm² and dryness fraction 0·75

$$h = h_f + xh_{fg} = 181{\cdot}3 + 0{\cdot}75 \times 482{\cdot}0 = \underline{542{\cdot}8 \text{ kcal}} \quad \text{Ans.}$$

External work per kg of wet steam during evaporation

$$= \frac{P \times xv_g}{J} = \frac{10 \times 10^4 \times 0{\cdot}75 \times 0{\cdot}198}{427} = 34{\cdot}8 \text{ kcal}$$

Internal energy, $u=h-\frac{Pv}{J}=512.8-34.8$ $=508$ kcal/kg

$[u=u_f+xu_{fg}=181.0+0.75(616.9-181.0)$ $=508$ kcal/kg]

(b) $h_g=h_f+h_{fg}=181.3+482.0=663.3$ kcal/kg Ans

External work$=\frac{Pv_g}{J}=\frac{10\times10^4\times0.198}{427}=46.4$ kcal/kg

$\therefore$ Internal energy, $u_g=h_g-\frac{Pv_g}{J}$

$=663.3-46.4=616.9$ kcal/kg Ans

[Internal energy can also be directly read from steam tables]

(c) From steam tables :

At 10 kgf/cm² and 200°C, $h=675.4$ kcal, $v=0.2103$ m³

At 10 kgf/cm² and 250°C, $h=702.4$ kcal, $v=0.2374$ m³

$\therefore$ Total heat of 1 kg of steam at 10 kgf/cm² and 230°C temperature by interpolation

$h=691.6$ kcal Ans.

Volume of 1 kg of steam at 10 kgf/cm² and 230°C temperature

$v=0.2266$ m³

External work of evaporation per kg of superheated steam

$=\frac{P\times v}{J}=\frac{10\times10^4\times0.2266}{427}=53.1$ kcal

$\therefore$ Internal energy, $u=h-\frac{Pv}{J}=691.6-53.1=638.5$ kcal/kg

Ans

[Internal energy can also be directly interpolated]

Note—The problem illustrates the use of steam tables.

4.2. Heat added ; increase in internal energy ; entropy

0.005 m³ of water is boiling at 30°C. It is heated until the pressure is 8 kgf/cm² and dryness 0.9. Using steam tables, calculate (i) the quantity of heat added before evaporation begins at 8 kgf/cm², (ii) the increase in internal energy of the water upto the beginning of evaporation, (iii) the increase in internal energy and the external work done during the evaporation stage, and (iv) the entropy of one kg of wet steam

(i) When water is boiling at 30°C, from steam tables, pressure = 0·04325 kgf/cm², specific volume of water $v_f = 0{\cdot}001004$ m³ and $h_f = 30$ kcal/kg.

$\therefore$ Mass of boiling water $= \dfrac{V}{v_f} = \dfrac{0{\cdot}005}{0{\cdot}001004} = 4{\cdot}98$ kg

When water is at a pressure of 8 kgf/cm², $t_s = 169{\cdot}6°C$,

$v_f = 0{\cdot}001114$ m³, $v_g = 0{\cdot}2448$ m³,
$h_f = 171{\cdot}4$ kcal, $h_{fg} = 489{\cdot}8$ kcal.

Heat added,

$$Q_{2\text{-}1} = m(h_2 - h_1) = 4{\cdot}98\ (171{\cdot}4 - 30) = 704{\cdot}2 \text{ kcal} \qquad \textbf{Ans.}$$

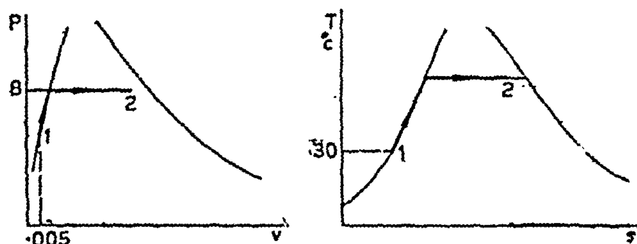


Fig. 4·11.

(ii) Internal energy

$$u_1 = h_1 - \frac{P_1 v_1}{J}$$

$$= 30 - \frac{0{\cdot}0433 \times 10^4 \times 0{\cdot}001004}{427} = 30 \text{ kcal (say)}$$

$$u_2 = h_2 - \frac{P_2 v_2}{J} = 171{\cdot}4 - \frac{8 \times 10^4 \times 0{\cdot}001114}{427} = 171{\cdot}2 \text{ kcal}$$

[The value of u_2 can be directly read in steam tables]

$\therefore$ Increase in internal energy

$$U_2 - U_1 = m(u_2 - u_1) = 4{\cdot}98(171{\cdot}2 - 30) = 703{\cdot}1 \text{ kcal} \qquad \textbf{Ans.}$$

(iii) For the steam at 8 kgf/cm² and 0·9 dry

$$u_3 = u_f + x u_{fg} = 171{\cdot}2 + 0{\cdot}9(615{\cdot}3 - 171{\cdot}2) = 570{\cdot}9 \text{ kcal}$$

$\therefore$ Increase in internal energy during evaporation

$$U_3 - U_2 = m(u_3 - u_2) = 4{\cdot}98(570{\cdot}9 - 171{\cdot}2) = 1{,}991 \text{ kcal} \qquad \textbf{Ans.}$$

External work done during evaporation,

$$= \frac{mP_3}{J}(v_3 - v_2)$$

$$= \frac{4{\cdot}98 \times 8 \times 10^4}{427}\left(0{\cdot}9 \times 0{\cdot}2448 - 0{\cdot}001114\right)$$

$$= 204{\cdot}6 \text{ kcal} \qquad \textbf{Ans.}$$

(iv) Entropy of 1 kg of steam at 8 kgf/cm² and dryness 0·9

$$s = s_f + x S_{fg} = 0{\cdot}487 + 0{\cdot}9 \times 1{\cdot}106 = 1{\cdot}4824 \qquad \textbf{Ans.}$$

(b) *First method.*

Considering separator alone, $x_1 = \dfrac{40-2 \cdot 2}{40} = 0 \cdot 945$

Considering throttling calorimeter alone

$$\text{Temperature of superheat} = 120 - 101 \cdot 8 = 18 \cdot 2^\circ C$$

$$\text{Enthalpy of superheated steam} = 639 \cdot 7 + 0 \cdot 5 \times 18 \cdot 2 = 648 \cdot 8 \text{ kcal}$$

$$\text{Enthalpy before throttling} = \text{Enthalpy after throttling}$$

$$200 \cdot 7 + x_2 \times 446 = 648 \cdot 8 \qquad \therefore \quad x_2 = 0 \cdot 9618$$

$\therefore$ Dryness fraction, $x = x_1 \times x_2 = 0 \cdot 945 \times 0 \cdot 9618 = \underline{0 \cdot 909}$ Ans.

Second method. Dryness fraction x can also be found out directly without first finding x_1, by writing the enthalpy balance equation.

Enthalpy at point 1 = Enthalpy separated at point 2 + Enthalpy at point 3

$$40(h_{f1} + x h_{fg1}) = 2 \cdot 2 \; h_{f2} + 37 \cdot 8 \; h_3$$

or $$40(200 \cdot 7 + x \times 466) = 2 \cdot 2 \times 200 \cdot 7 + 37 \cdot 8 \times 648 \cdot 8$$

$\therefore$ $$x = 0 \cdot 909, \text{ as before}$$

(c) Let the cooling water required to remove superheat and latent heat be m kg.

$$m \times 12 = 37 \cdot 8 \times (648 \cdot 8 - 101 \cdot 9) \qquad (h_f = 101 \cdot 9 \text{ kcal}$$

$\therefore$ $$\underline{m = 1{,}723 \text{ kg}}$$ Ans.

4.4. Clapeyron's equation.

(a) *State the Clapeyron's equation. How is it derived ?*

(b) *Using the Clapeyron's equation find the specific volume of steam at 0·15 kgf/cm² from the relevant data at 0·14 and 0·16 kgf/cm².*

(c) *From the Clapeyron's equation show that the gain of internal energy when 1 kg of water is just completely evaporated at constant pressure P (saturation temperature T° absolute) is given by*

$$u_g - u_f = (h_g - h_f)\left[1 - \frac{P}{T} \times \frac{dT}{dP}\right]$$

(a) See text.

(b) Clapeyron's equation, $v_g - v_f = \dfrac{J(h_g - h_f)}{T} \times \dfrac{dT}{dP}$

(*b*) *First method.*

Considering separator alone, $x_1 = \frac{40-2\cdot2}{40} = 0\cdot945$

Considering throttling calorimeter alone

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or $$40(200\cdot7 + x \times 466) = 2\cdot2 \times 200\cdot7 + 37\cdot8 \times 648\cdot8$$

$\therefore$ $x = 0\cdot909$, as before

(c) Let the cooling water required to remove superheat and latent heat be m kg.

$$m \times 12 = 37\cdot8 \times (648\cdot8 - 101\cdot9) \qquad [h_f = 101\cdot9 \text{ kcal}$$

$\therefore$ $\underline{m = 1{,}723 \text{ kg}}$ Ans.

4-4. Clapeyron's equation.

(*a*) *State the Clapeyron's equation. How is it derived ?*

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$$u_g - u_f = (h_g - h_f)\left[1 - \frac{P}{T} \times \frac{dT}{dP}\right]$$

(*a*) See text.

(*b*) Clapeyron's equation, $v_g - v_f = \frac{J(h_g - h_f)}{T} \times \frac{dT}{dP}$

At 0·14 kgf/cm², $10{\cdot}88-0{\cdot}001013=\frac{427\times567{\cdot}7}{325{\cdot}2}\times\frac{dT}{dP}$

$$\therefore\ \frac{dT}{dP}=0{\cdot}01461$$

At 0·16 kgf/cm², $9{\cdot}604-0{\cdot}001014=\frac{427\times566{\cdot}2}{327{\cdot}9}\times\frac{dT}{dP}$

$$\therefore\ \frac{dT}{dP}=0{\cdot}01302$$

$\therefore$ Mean value of $\frac{dT}{dP}=\frac{0{\cdot}01461+0{\cdot}01302}{2}=0{\cdot}013815$

At 0·15 kgf/cm², $v_g-0{\cdot}001014=\frac{427\times566{\cdot}9}{326{\cdot}6}\times0{\cdot}013815$

$\therefore$ Specific volume, $\underline{v_g=10{\cdot}23\text{ m}^3}$ Ans.

(The tabulated value in steam tables is 10·20 m³)

(c) Internal energy of 1 kg of water at saturation temperature,

$$u_f=h_f-\frac{Pv_f}{J}$$

Internal energy of 1 kg of dry saturated steam,

$$u_g=h_g-\frac{Pv_g}{J}$$

or $$u_g-u_f=(h_g-h_f)-\frac{P}{J}(v_g-v_f)$$

Substituting the value of v_g-v_f from the Clapeyron's equation

$$\underline{u_g-u_f}=(h_g-h_f)-P\left\{\frac{J(h_g-h_f)}{JT}\times\frac{dT}{dP}\right\}$$

$$=\underline{\left(h_g-h_f\right)\left(1-\frac{P}{T}\times\frac{dT}{dP}\right)}\qquad \text{Q.E.D.}$$

4-5. Constant volume cooling : t ; Q ; S_2-S_1.

A closed vessel of 0·2 m³ contains steam at 10 kgf/cm² and temperature 250°C. If the vessel is cooled so that the pressure falls to 3·5 kgf/cm², determine the final temperature, the heat transferred, and the change of entropy.

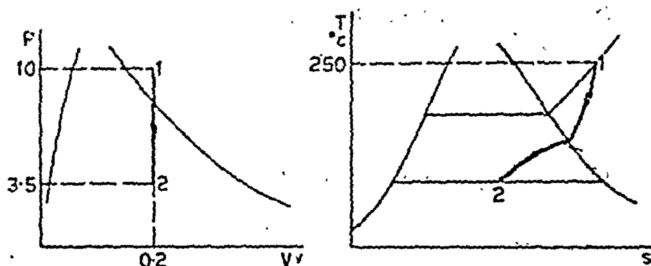


Fig. 4·13.

From steam tables at 10 kgf/cm² and temperature 250°C

$$v=0{\cdot}2374 \text{ m}^3,\ u=646{\cdot}8 \text{ kcal}$$

$$h=702{\cdot}4 \text{ kcal},\ s=1{\cdot}655$$

$$\text{Mass of steam in the vessel}=\frac{0{\cdot}2}{0{\cdot}2374}=0{\cdot}8424 \text{ kg}$$

The vapour must be wet in its final state because mv_g at 3·5 kgf/cm² is $0{\cdot}8424\times0{\cdot}5338=0{\cdot}4497$ m³, which is greater than 0·2 m³. The temperature must, therefore, be the saturation temperature corresponding to P_2, *i.e.*, $t_s=138{\cdot}2°C.$ Ans.

and final dryness fraction, $x_2=\dfrac{V}{mv_{g2}}=\dfrac{0{\cdot}2}{0{\cdot}4497}=0{\cdot}4446$ Ans.

[The obtained value of x_2 is neglecting the volume of water].

Final internal energy,

$$u_2=u_{f2}+x_2u_{fg2}$$

$$=138{\cdot}8+0{\cdot}4446(608{\cdot}6-138{\cdot}8)=347{\cdot}7 \text{ kcal}$$

Heat transferred, $Q=m(u_2-u_1)$

$$=0{\cdot}8424\ (347{\cdot}7-646{\cdot}8)=-252 \text{ kcal}$$ Ans.

[—sign indicates loss of heat]

Final entropy, $s_2=s_{f2}+x_2\, s_{fg2}=0{\cdot}411+0{\cdot}4446\times1{\cdot}248=0{\cdot}9659$

Change of entropy, $S_2-S_1=0{\cdot}8424\ (0{\cdot}9659-1{\cdot}655)$

$$=-0{\cdot}5805$$ Ans.

4·6. Constant pressure heating : dryness fraction and ΔU.

A vertical cylinder of 17 cm diameter is closed at the bottom and contains a layer of water 1·7 cm deep at a temperature of 150°C. A closed fitting frictionless piston rests on the water surface and is loaded with a constant force of 1,020 kgf in addition to pressure of atmosphere.

If 150 kcal of heat is supplied to water, find the dryness friction of resulting steam and the change in internal energy.

Calculate the work done in the above problem and check the result.

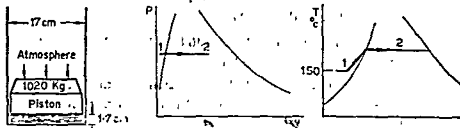


Fig. 4·14.

Pressure of vapour $=\frac{\text{force}}{\text{area}}+$atmospheric pressure

$$=\frac{1,020}{(\pi/4)\times 17^2}+1=5\ 5\ \text{kgf/cm}^2$$

Volume of water $=\frac{\pi}{4}\times 17^2\times 1{\cdot}7=386$ cc

Mass of water $=\frac{386}{1,000}=0{\cdot}386$ kg

By the application of heat, vapour is generated at constant pressure. If x is the dryness fraction of steam produced

Heat required per kg, $h=h_f+x\times h_{fg}-t=155\ 9+x\times 501{\cdot}5-150$

or $150=0{\cdot}386\,(155\ 9+x\times 501{\cdot}5-150)$ $\therefore$ $\underline{x=0\ 7635}$ Ans.

Initial internal energy of water,

$$U_1=mh_1-\frac{P_1V_1}{J}$$

$$=0{\cdot}386\times 150-\frac{5\ 5\times 10^4\times 386\times 10^{-6}}{427}=57{\cdot}85\ \text{kcal}$$

Final internal energy of vapour $U_2=mh_2-\frac{P_2V_2}{J}$ $\quad [V_2=mxv_{g2}$

$$=0{\cdot}386\,(155{\cdot}9+0{\cdot}7635\times 501{\cdot}5)-\frac{5{\cdot}5\times 10^4\times 0{\cdot}386\times 0{\cdot}7635\times 0\ 349}{427}$$

$=194{\cdot}7$ kcal

$\therefore$ Change in internal energy

$\underline{U_2-U_1}=194{\cdot}7-57\ 85=\underline{136\ 85\ \text{kcal}}$ Ans.

Work done

$$W=\frac{P}{J}(V_2-V_1)$$

$$=\frac{5\cdot5\times10^4(0\cdot386\times0\cdot7635\times0\cdot349-386\times10^{-6})}{427}$$

$=\underline{13\cdot2 \text{ kcal}}$ Ans.

Check : $Q=W+(u_2-u_1)=13\cdot2+136\cdot85=150\cdot05$ kcal, which is nearly the same as given in the problem.

Note. As there is a loaded sliding piston, heating results in generation of steam at constant pressure. Internal energy naturally increases because of heat supply.

4·7. Isothermal compression : ΔU ; S$_2$—S$_1$: Q ; W.

A cylinder contains steam at 1 kgf/cm² and temperature 154°C. The steam is compressed reversibly and isothermally to a state where the specific volume is 0·28 m³/kg. Find the change of internal energy, change of entropy, heat tranferred and work done per kg of steam.

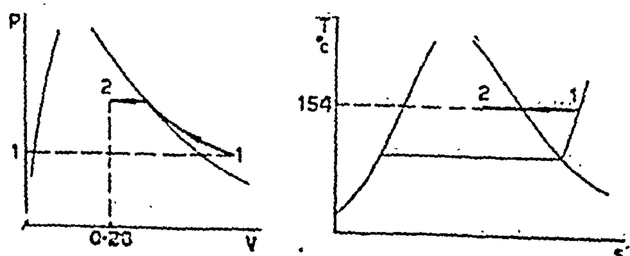


Fig. 4·15.

From superheat table, by interpolation

$$u_1=616\cdot7+\frac{(634\cdot8-616\cdot7)\times4}{50}=618\cdot1 \text{ kcal}$$

Since vapour is compressed isothermally at temperature 154°C, corresponding pressure will be 5·4 kgf/cm² and will be in wet state as $v_{g2} > v_2$.

$\therefore$ Final dryness fraction, $x_2=\dfrac{0\cdot28}{0\cdot355}=0\cdot7889$

$$u_2=u_{f2}+x_2u_{fg2}=155\cdot0+0\cdot7889\times457\cdot2=515\cdot6 \text{ kcal}$$

Change of internal energy $\underline{u_2-u_1}=515\cdot6-618\cdot1$

$=\underline{-102\cdot5 \text{ kcal/kg}}$ Ans.

By interpolation, $s_1=1\cdot975+(2\cdot214-1\cdot975)\times\dfrac{4}{50}=1\cdot994$

and $$s_2 = 0.450 + 0.7880 \times 1.75 = 1.377$$

Change of entropy, $s_2 - s_1 = 1.377 - 1.994 = -0.617$ units. Ans.

Heat transferred $q = T(s_2 - s_1)$

$$= 427(-0.617) = -263.5 \text{ kcal} \quad \text{Ans.}$$

Work done, $w = q - (u_2 - u_1)$

$$= -263.5 - (-102.5) = -161 \text{ kcal/kg} \quad \text{Ans.}$$

4-8. Polytropic expansion : x_2 ; W ; Q ; $S_2 - S_1$.

0·8 kg of steam at a pressure of 15 kgf/cm² and 250°C expands to 1·5 kgf/cm². Assuming that the steam expands according to the law $PV^{1.25}$ = constant, calculate the final dryness fraction, work done, heat transferred and change of entropy during the expansion.

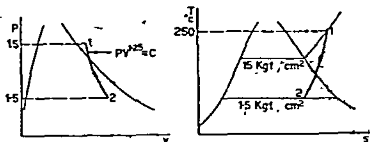


Fig. 4·16.

From steam tables, $v_1 = 0.155$ m³/kg, $u_1 = 643.5$ kcal/kg

$$h_1 = 698.0 \text{ kcal/kg}, \; s_1 = 1.604$$

Final specific volume, $v_2 = v_1 \left(\dfrac{P_1}{P_2} \right)^{\frac{1}{n}}$

$$= 0.155 \left(10 \right)^{\frac{1}{1.25}} = 0.978 \text{ m}^3\text{/kg}$$

$\therefore$ Final df, $x_2 = \dfrac{v_2}{v_{g2}} = \dfrac{0.978}{1.180} = 0.829$ Ans.

Work done, $W = m \times \dfrac{P_1 v_1 - P_2 v_2}{(n-1) J}$

$$= 0.8 \, \frac{10^4(15 \times 0.155 - 1.5 \times 0.978)}{(1.25 - 1) \times 427}$$

$$= 64.3 \text{ kcal} \quad \text{Ans.}$$

$$u_2 = u_{f2} + x_2 u_{fg2} = 111.0 + 0.829(601.6 - 111.0) = 517.7 \text{ kcal}$$

Heat transferred,

$$Q=m(u_2-u_1)+W$$
$$=0{\cdot}8(517{\cdot}7-643{\cdot}5)+64{\cdot}3=-36{\cdot}34 \text{ kcal} \quad \text{Ans.}$$

$$s_2=s_{f2}+x_2 s_{fg2}=0{\cdot}341+0{\cdot}829\times 1{\cdot}386=1{\cdot}490$$

Change of entropy, $S_2-S_1=m(s_2-s_1)$

$$=0{\cdot}8(1{\cdot}490-1{\cdot}604)=-0{\cdot}0912 \quad \text{Ans.}$$

4.9. Heat interchange in polytropic expansion.

In a steam engine cylinder the pressure and volume at cut-off are 8 kgf/cm² and 0·25 m³ respectively and dryness fraction is 0·8. The pressure and volume at release are 2 kgf/cm² and 0·75 m³ respectively. Assuming that the steam expands according to the law PV^n= constant, calculate the heat passing through the cylinder walls during expansion.

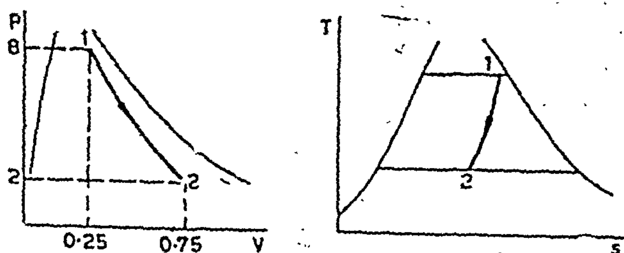


Fig. 4·17.

From steam tables, v_g at 8 kgf/cm²=0·2448 m³

Mass of steam in the cylinder $=\dfrac{V_1}{x v_{g1}}$

$$=\frac{0{\cdot}25}{0{\cdot}8\times 0{\cdot}2448}=1{\cdot}276 \text{ kg}$$

$$P_1V_1^n=P_2V_2^n, \qquad 8\times 0{\cdot}25^n=2\times 0{\cdot}75^n \qquad \therefore\ n=1{\cdot}262$$

$$W=\frac{P_1V_1-P_2V_2}{J\,(n-1)}=\frac{8\times 10^4\times 0{\cdot}25-2\times 10^4\times 0{\cdot}75}{427(1{\cdot}262-1)}=44{\cdot}7 \text{ kcal}$$

Let x_2 be the dryness fraction at the end of expansion.

$$m x_2 v_{g2}=V_2, \qquad 1{\cdot}276\times x_2\times 0{\cdot}9019=0{\cdot}75 \qquad \therefore\ x_2=0{\cdot}652$$

From steam tables

$$H_1=1{\cdot}276\ (171{\cdot}4+0{\cdot}8\times 489{\cdot}8)=717{\cdot}3 \text{ kcal}$$
$$H_2=1{\cdot}276\ (119{\cdot}9+0{\cdot}652\times 526{\cdot}4)=590{\cdot}5 \text{ kcal}$$

$$U_1=H_1-\frac{P_1V_1}{J}=717\cdot3-\frac{8\times10^4\times0\cdot25}{427}=670\cdot5\text{ kcal}$$

$$U_2=H_2-\frac{P_2V_2}{J}=590\cdot5-\frac{2\times10^4\times0\cdot75}{427}=555\cdot4\text{ kcal}$$

$$\underline{Q}=W+(U_2-U_1)=44\cdot7+(555\cdot4-670\cdot5)=\underline{-70\cdot4\text{ kcal}} \quad \text{Ans.}$$

[—ve sign indicates that heat is flowing out of the steam]

4.10. Hyperbolic and isentropic expansion : x_{end} ; Q ; n.

Explain the term entropy and show by means of sketches how lines of constant volume and constant quality can be drawn on a T-s chart of water and steam. 0·1 m³ of steam at a pressure of 30 kgf/cm² and dryness fraction 0·85 expands to 4·2 kgf/cm². Calculate the dryness of steam at the end of expansion, work done and heat flow from or to the cylinder (a) if the expansion is hyperbolic, and (b) the expansion is isentropic. If the latter is represented by the equation $PV^n=C$, find the value of 'n' which satisfies the initial and final conditions. Neglect the volume of water.

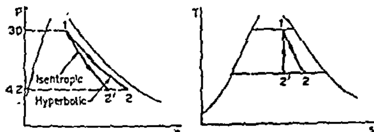


Fig. 4·18.

For theory—see text.

(a) *For hyperbolic expansion*

Mass of steam, $m=\frac{V_1}{x v_{s1}}=\frac{0\cdot1}{0\cdot85\times0\cdot06798}=1\cdot73$ kg

$P_1V_1=P_2V_2$, $\quad 30\times0\cdot1=4\cdot2\times V_2 \quad \therefore \quad V_2=0\cdot7143$ m³

$V_2=mx_2v_{s2}$, $\quad 0\cdot7143=1\cdot73\times x_2\times0\cdot4497 \quad \therefore \quad \underline{x_2=0\cdot9175}$ Ans.

Internal energy, $U=H-\frac{PV}{J}$

$$\therefore \quad U_1=1\cdot73\,(239\cdot6+0\cdot85\times429\cdot9)-\frac{30\times10^4\times0\cdot1}{427}$$

$$=976\cdot5\text{ kcal}$$

Heat transferred,

$$Q=m(u_2-u_1)+W$$

$$=0{\cdot}8(517{\cdot}7-643{\cdot}5)+64{\cdot}3=-36{\cdot}34 \text{ kcal} \quad \text{Ans.}$$

$$s_2=s_{f2}+x_2 s_{fg2}=0{\cdot}341+0{\cdot}829\times 1{\cdot}386=1{\cdot}490$$

Change of entropy, $S_2-S_1=m(s_2-s_1)$

$$=0{\cdot}8(1{\cdot}490-1{\cdot}604)=-0{\cdot}0912 \quad \text{Ans.}$$

4.9. Heat interchange in polytropic expansion.

In a steam engine cylinder the pressure and volume at cut-off are 8 kgf/cm² and 0·25 m³ respectively and dryness fraction is 0·8. The pressure and volume at release are 2 kgf/cm² and 0·75 m³ respectively. Assuming that the steam expands according to the law PV^n= constant, calculate the heat passing through the cylinder walls during expansion.

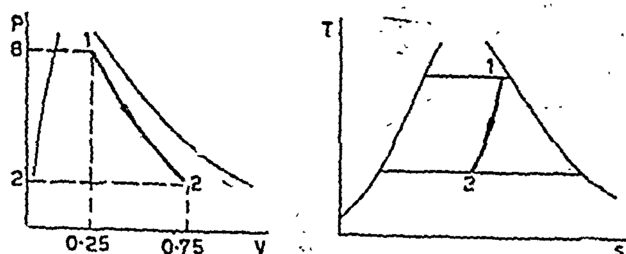


Fig. 4·17.

From steam tables, v_g at 8 kgf/cm² = 0·2448 m³

Mass of steam in the cylinder $=\dfrac{V_1}{x v_{g1}}$

$$=\frac{0{\cdot}25}{0{\cdot}8\times 0{\cdot}2448}=1{\cdot}276 \text{ kg}$$

$P_1V_1^n=P_2V_2^n$, $\quad 8\times 0{\cdot}25^n=2\times 0{\cdot}75^n \quad \therefore\ n=1{\cdot}262$

$$W=\frac{P_1V_1-P_2V_2}{J\,(n-1)}=\frac{8\times 10^4\times 0{\cdot}25-2\times 10^4\times 0{\cdot}75}{427(1{\cdot}262-1)}=44{\cdot}7 \text{ kcal}$$

Let x_2 be the dryness fraction at the end of expansion.

$m x_2 v_{g2}=V_2$, $\quad 1{\cdot}276\times x_2\times 0{\cdot}9019=0{\cdot}75 \quad \therefore\ x_2=0{\cdot}652$

From steam tables

$$H_1=1{\cdot}276\ (171{\cdot}4+0{\cdot}8\times 489{\cdot}8)=717{\cdot}3 \text{ kcal}$$

$$H_2=1{\cdot}276\ (119{\cdot}9+0{\cdot}652\times 526{\cdot}4)=590{\cdot}5 \text{ kcal}$$

$$U_1 = H_1 - \frac{P_1V_1}{J} = 717{\cdot}3 - \frac{8\times10^4\times0{\cdot}25}{427} = 670{\cdot}5 \text{ kcal}$$

$$U_2 = H_2 - \frac{P_2V_2}{J} = 590{\cdot}5 - \frac{2\times10^4\times0{\cdot}75}{427} = 555{\cdot}4 \text{ kcal}$$

$$Q = W + (U_2 - U_1) = 44{\cdot}7 + (555{\cdot}4 - 670{\cdot}5) = -70{\cdot}4 \text{ kcal} \quad \text{Ans.}$$

[–ve sign indicates that heat is flowing out of the steam]

4.10. Hyperbolic and isentropic expansion : x_{end} ; Q ; n.

Explain the term entropy and show by means of sketches how lines of constant volume and constant quality can be drawn on a T-s chart of water and steam. 0·1 m³ of steam at a pressure of 30 kgf/cm² and dryness fraction 0·85 expands to 4·2 kgf/cm². Calculate the dryness of steam at the end of expansion, work done and heat flow from or to the cylinder (a) if the expansion is hyperbolic, and (b) the expansion is isentropic. If the latter is represented by the equation $PV^n = C$, find the value of 'n' which satisfies the initial and final conditions. Neglect the volume of water.

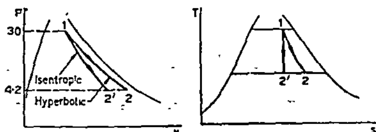


Fig. 4.18.

For theory—see text

(a) For hyperbolic expansion

Mass of steam, $m = \frac{V_1}{xv_{g1}} = \frac{0{\cdot}1}{0{\cdot}85\times0{\cdot}06798} = 1{\cdot}73$ kg

$P_1V_1 = P_2V_2, \quad 30\times0{\cdot}1 = 4{\cdot}2\times V_2 \quad \therefore \; V_2 = 0{\cdot}7143$ m³

$V_2 = mx_2v_{g2}, \quad 0{\cdot}7143 = 1{\cdot}73\times x_2\times0{\cdot}4497 \quad \therefore \; \underline{x_2 = 0{\cdot}9175}$ Ans

Internal energy, $U = H - \frac{PV}{J}$

$$\therefore \quad U_1 = 1{\cdot}73\,(239{\cdot}6 + 0{\cdot}85\times429{\cdot}9) - \frac{30\times10^4\times0{\cdot}1}{427}$$

$$= 976{\cdot}5 \text{ kcal}$$

and $U_2 = 1{\cdot}73\ (145{\cdot}5 + 0{\cdot}9175 \times 509) - \dfrac{4{\cdot}2 \times 10^4 \times 0{\cdot}7143}{427}$

$= 989{\cdot}3$ kcal

$$W = \frac{P_1 V_1 \log_e r}{J}$$

$$= \frac{30 \times 10^4 \times 0{\cdot}1}{427} \log_e \frac{0{\cdot}7143}{0{\cdot}1} = \underline{138{\cdot}2 \text{ kcal}}$$ Ans.

$\therefore$ Heat flow, $Q = W + (U_2 - U_1)$

$= 138{\cdot}2 + (989{\cdot}3 - 976{\cdot}5) = \underline{151 \text{ kcal}}$ Ans.

(Heat is flowing into the steam)

(*b*) *For isentropic expansion* $s_1 = s_2$

$0{\cdot}629 + 0{\cdot}85 \times 0{\cdot}850 = 0{\cdot}427 + x_2 \times 1{\cdot}218 \quad \therefore \quad \underline{x_2 = 0{\cdot}759}$ Ans.

In isentropic process heat flow is zero.

$V_2 = m x_2 v_{g2} = 1{\cdot}73 \times 0{\cdot}759 \times 0{\cdot}4497 = 0{\cdot}5907 \text{ m}^3$

$P_1 V_1^n = P_2 V_2^n, \quad 30 \times 0{\cdot}1^n = 4{\cdot}2 \times 0{\cdot}5907^n \quad \therefore \quad \underline{n = 1{\cdot}107}$ Ans.

$$W = \frac{P_1 V_1 - P_2 V_2}{J\ (n-1)}$$

$$= \frac{10^4 (30 \times 0{\cdot}1 - 4{\cdot}2 \times 0{\cdot}5907)}{427\ (1{\cdot}107 - 1)} = \underline{113{\cdot}6 \text{ kcal}}$$ Ans.

Note. (*i*) 0·1 m³ is the volume of wet steam. The mass of steam is calculated from the volume.

(*ii*) For gases, value of 'n' in adiabatic process is equal to γ. For vapours this does not apply.

4·11. Throttling and adiabatic expansion of steam : change in enthalpy ; loss due to throttling.

A boiler supplies steam to a turbine at 18 kgf/cm² dry and saturated. From the same boiler steam is fed by throttling to a steam engine working at 10 kgf/cm². Determine (a) the quality of steam entering steam engine, and (b) the change in entropy during throttling process. Assume C_p for superheated range 0·5. (c) If this steam in engine expands adiabatically to 1·1 kgf/cm², find the change in enthalpy and internal energy of steam during the expansion. (d) Estimate the percentage loss of the Rankine work due to throttling.

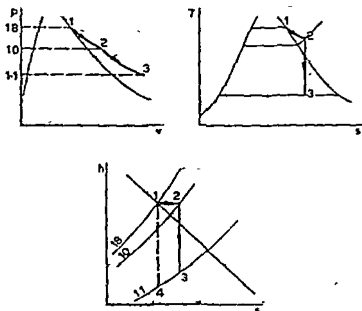


Fig. 4·19.

(a) From steam tables, $h_{g1}=667{\cdot}8$ kcal, $s_{g1}=1{\cdot}525$

$$h_2=663{\cdot}3+C_p\,(t_2-t_{s2})$$

As steam is throttled in 12, enthalpy remains constant

Hence $667{\cdot}8=663{\cdot}3+0{\cdot}5\,(t_2-176)$ $\quad\therefore\ \underline{t_2=188°K}$ Ans.

(b) $s_2=s_{g2}+C_p\,\log_e\dfrac{T_s}{T_{s2}}=1{\cdot}575+0{\cdot}5\log_e\dfrac{273+188}{273+179}=1{\cdot}5849$

$\therefore$ Change in entropy, $\underline{s_s-s_{g1}}=1{\cdot}5849-1{\cdot}525=\underline{0{\cdot}0599}$

(c) $s_2=s_3$, $1{\cdot}5849=0{\cdot}317+x_3\times1{\cdot}435$ $\quad\therefore\ x_3=0{\cdot}8837$

$h_3=h_{f3}+x_3\,h_{fg3}=101{\cdot}9+0{\cdot}8837\times537{\cdot}8=577{\cdot}2$ kcal

Change in enthalpy, $\underline{h_s-h_3}=667\ 8-577{\cdot}2=\underline{90{\cdot}6\text{ kcal}}$ Ans.

Specific volume, $\quad v_2=\dfrac{v_1}{T_{s1}}\times T_2=\dfrac{0{\cdot}198}{452}\times461=0{\cdot}2019\ \text{m}^3$

Internal energy, $\quad U=H-\dfrac{PV}{J}$

$$u_2=h_2-\frac{P_2v_2}{J}=667{\cdot}8-\frac{10\times10^4\times0{\cdot}2019}{427}=620{\cdot}5\text{ kcal}$$

$$u_3=h_3-\frac{P_3v_3}{J}=577\ 2-\frac{1{\cdot}1\times10^4\times0\ 8837\times1{\cdot}578}{427}=541{\cdot}1\text{ kcal}$$

$\therefore$ Change in internal energy$=620 \cdot 5-541 \cdot 1$

$=79 \cdot 4$ kcal (reduction) Ans.

(d) The Rankine work without throttling$=h_{g1}-h_4$

$s_1=s_4$, $1 \cdot 525=0 \cdot 317+x_4 \times 1 \cdot 435$ $\therefore$ $x=0 \cdot 8418$

$h_4=h_{f4}+x_4 h_{fg4}=101 \cdot 9+0 \cdot 8418 \times 537 \cdot 8=554 \cdot 7$ kcal

$\therefore$ Rankine work$=h_1-h_4=667 \cdot 8-554 \cdot 7=113 \cdot 1$ kcal

Percentage loss due to throttling$=\dfrac{113 \cdot 1-90 \cdot 6}{113 \cdot 1}=19 \cdot 89\%$ Ans.

Note. (i) As the value of C_p is given, enthalpy and entropy of superheated steam should be calculated and should not be taken from the steam tables.

(ii) For calculating u_2 volume of superheated steam should be taken.

4·12. Generation of steam at constant vol. and constant pr.

A boiler contains water and steam at atmospheric pressure, all air having been expelled and the stop valve closed. Find the quantity of heat required to convert 1 kg of water from this condition into dry saturated steam at 14 kgf/cm², the stop valve being still closed.

When the stop valve is open and the boiler is supplying dry saturated steam at 14 kgf/cm² how much additional heat is required per kg of steam formed from feed water at 70°C.

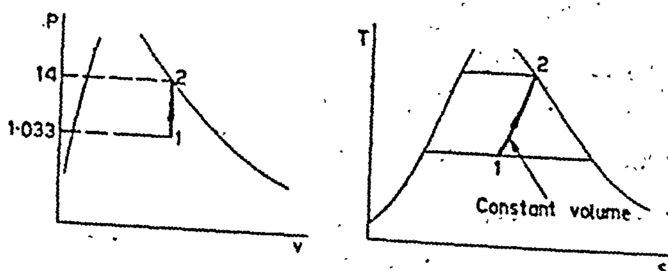


Fig. 4·20.

As the stop valve is closed under atmospheric pressure *i.e.* 1·033 kgf/cm² the steam will be at a temperature of 100°C, being in contact with water.

For 1 kg of water

$$u_{f1}=h_{f1}-\frac{P_1 v_{f1}}{J}$$

$$\therefore \quad u_{f1}=100{\cdot}1-\frac{1{\cdot}033\times10^4\times0{\cdot}001044}{427}=100\,07 \text{ kcal}$$

For 1 kg of dry at 14 kgf/cm²,

$$u_{g2}=h_{g2}-\frac{P_2 v_{g2}}{J}$$

$$=666\,2-\frac{14\times10^4\times0{\cdot}1434}{427}=619\,2 \text{ kcal}$$

[Internal energy of dry saturated steam can be directly read from the steam tables].

Since the stop valve remains closed, the operation is one of constant volume and the external work done=0

$\therefore$ Heat required $q=W+u_{g2}-v_{f1}$

$$=0+(619{\cdot}2-100\,07)=\underline{519\,1 \text{ kcal}} \qquad \text{Ans.}$$

Stop valve open. Temperature of water=70°C $\therefore$ $u_1=70$ kcal

After opening the valve the operation will be of constant pressure.

$$W=\frac{P(v_{g2}-v_{f2})}{J}=\frac{14\times10^4(0\,1434-0\,001148)}{427}=46\,7 \text{ kcal}$$

Heat required, $q=W+(u_2-u_1)=46{\cdot}7+(619\,2-70)=595{\cdot}9$ kcal

Additional heat required$=595{\cdot}9-519\,1=\underline{76\,8 \text{ kcal/kg}}$ Ans.

Second method. At constant pressure,

$$q=\triangle h=666{\cdot}2-70=596\,2 \text{ kcal}$$

which is nearly same as before.

Note. $q=\triangle h$ is applicable only in constant pressure process and in no other case. The heat required at constant pressure process is more because work is done in changing the volume of water to that of steam.

4-13. Raising of pressure in a closed vessel by addition of steam.

A closed drum contains 140 kg of steam at 1 kgf/cm², 0 5 dry. What mass of dry saturated steam at 20 kgf/cm² must be admitted to raise the pressure in the drum to 10 kgf/cm² and what is the final dryness fraction of steam? Neglect velocity of steam at entrance and all heat losses.

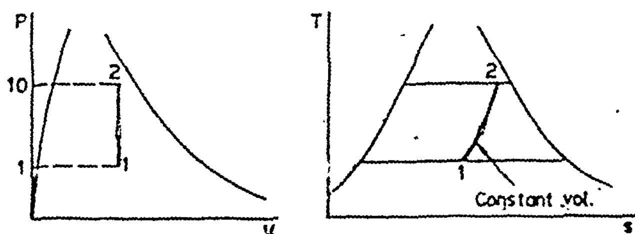


Fig. 4·21.

Volume of drum, $V=140\times0{\cdot}5\ (1{\cdot}725+0{\cdot}001043)=120{\cdot}82$ m³

Internal energy before,

$$U_1=m(u_f+xu_{fg})$$

$$=140[99{\cdot}2+0{\cdot}5(598{\cdot}4-99{\cdot}2)]=140\times348{\cdot}8 \text{ kcal}$$

Let mass of steam added be m kg.

Internal energy after, $U_2=(140+m)\ [181+x_2\ (616{\cdot}9-181)]$

$$\text{where } x_2=\frac{V}{(140+m)v_g}=\frac{120{\cdot}82}{(150+m)0{\cdot}118}=\frac{610{\cdot}1}{140+m}$$

Enthalpy of incoming steam $=U_2-U_1$

$$\therefore \quad m\times668{\cdot}8=(140+m)[181+x_2\ (616{\cdot}9-181)]-140\times348{\cdot}8$$

$$=(140+m)\left[181+\frac{610{\cdot}1}{140+m}\ 435{\cdot}9\right]-140\times348{\cdot}8$$

$\therefore \quad \underline{m=497{\cdot}4 \text{ kg}}$ Ans.

and final dryness fraction,

$\underline{x_2=0{\cdot}957}$ Ans.

4·14. Reduction of pressure in a closed vessel by injection of water.

A closed vessel of 10 m³ contains steam at 20 kgf/cm² and temperature 250°C. Water at 20°C and 50 kgf/cm² is injected for reducing the pressure in the vessel to 10 kgf/cm². Assuming no external losses, find the mass of water to be injected and the final dryness fraction of the steam in the vessel.

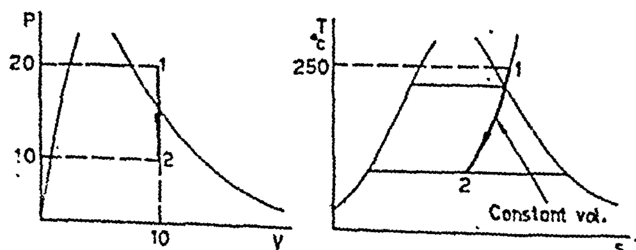


Fig. 4·22.

From steam tables, at 20 kgf/cm² and temperature 250°C

$h=693{\cdot}1$ kcal and $v=0{\cdot}1131$ m³

Mass of steam in the vessel$=\frac{V}{v}=\frac{10}{0{\cdot}1138}=87{\cdot}86$ kg

Let finally there be A kg of water and B kg of steam in the vessel

$\therefore$ Final dryness fraction, $x=\frac{B}{A+B}$.

Volume of water+volume of steam=volume of vessel

$$A\times 0\ 001126+B\times 0\ 198=10 \quad (1)$$

Also $A+B=87{\cdot}86+m$,

where m is the mass of water injected (2)

From equations (1) and (2), $A=37{\cdot}56+1{\cdot}0057\ m$ (3)

$$B=50{\cdot}30-0\ 0057\ m \quad (4)$$

Initial internal energy, $u_1=593{\cdot}1-\frac{20\times 10^4\times 0{\cdot}1138}{427}$

$=640\ 2$ kcal

[From steam tables $u_1=640\ 2$ kcal]

Enthalpy of water, $h=t+\frac{Pv_f}{J}$

$$=20+\frac{50\times 10^4\times 0{\cdot}001002}{427}=21{\cdot}17 \text{ kcal}$$

Final internal energy,

$$U_2=181{\cdot}3A+B\left(663\ 3-\frac{10\times 10^4\times 0{\cdot}198}{427}\right)$$

$$=181{\cdot}3A+616{\cdot}9B \text{ kcal}$$

As it is a constant volume process work done is zero and heat interchange is equal to the change of internal energy.

$$m\times h=U_2-mu_2$$

or $21{\cdot}17m=U_2-U_1=181\ 3A+616{\cdot}9B-640\ 2\times 87{\cdot}86$

$=181{\cdot}3(37{\cdot}56+1{\cdot}0057m)+616{\cdot}9(50\ 30-0\ 0057m)-640\ 2\times 87\ 86$

$\therefore$ Mass of water to be injected, $\underline{m=116{\cdot}5\text{ kg}}$ Ans.

Substituting the value of 'm' in equations (3) and (4)

$$\left.\begin{aligned} A &= 37{\cdot}56 + 1{\cdot}0057 \times 116{\cdot}5 = 154{\cdot}72 \\ B &= 50{\cdot}30 - 0{\cdot}0057 \times 116{\cdot}5 = 49{\cdot}64 \end{aligned}\right\} \quad \therefore \quad A+B = 204{\cdot}36$$

The total should be, $A+B=87{\cdot}16+186{\cdot}5=204{\cdot}36$, which is same as obtained above.

$\therefore$ Final dryness fraction, $x = \dfrac{B}{A+B}$

$$= \frac{49{\cdot}64}{204{\cdot}36} = 0{\cdot}243 \text{ or } \underline{24{\cdot}3\%} \quad \text{Ans.}$$

Note. Total energy per kg of water = internal energy + work of introduction.

The pressure of injection water must be greater than the steam pressure inside. By injecting the water, steam condenses ; therefore, the pressure is reduced and wet steam is obtained. Lower the pressure required, greater is the amount of water to be injected and lower is the final dryness fraction.

4-15. State of steam entering condenser ; mass of air.

State the law of partial pressures and show how it applies to the condenser of a steam plant.

The following observations were made on a condenser plant in which the temperature of condensation was measured directly by thermometers. The recorded condenser vacuum was 715 mm of mercury and the barometer read 768 mm of mercury. Temperature of condensation, 34°C. Temperature of hot well, 27·6°C. Mass of condensate per hour, 1,930 kg. Mass of cooling water per hour, 62,000 kg. Inlet temperature, 8 51°C and outlet temperature, 26 24°C.

Find the state of the steam entering the condenser and the mass of air present per m³ of condenser volume. For air, $R=29{\cdot}27$ kgf m/kg °C.

$$\text{The condenser pressure} = (768-715) \times \frac{1}{735{\cdot}5}$$
$$= 0{\cdot}07206 \text{ kgf/cm}^2$$

p.p. of steam (corresponding to 34°C), $p=0{\cdot}3542$ kgf/cm²

The latent heat corresponding to 34°C, $h_{fg}=5\ 8{\cdot}2$ kcal/kg

Hence, $m_w(t_2-t_1)=m_s(h_f+x\,h_{fg}-t)$

$62{,}000(26\ 24-8\ 51)=)1{,}930(34+x\times578\ 2-27{\cdot}6)$

$\therefore$ dryness fraction of steam at entrance, $\underline{x=0{\cdot}974}$ Ans.

p p. of air, $p=0\ 07206-0\ 0542=0\ 01786$ kgf/cm²

Mass of air per m³ of condenser volume,

$$\underline{m}=\frac{PV}{RT}=\frac{0\ 01786\times10^4\times1}{29\ 27\times(273+34)}=\underline{0{\cdot}01983\ \text{kg}}\qquad\text{Ans.}$$

4-16. Partial pressure : mass of air with steam

The temperature in a boiler before the fire is lighted is 22°C, the pressure being atmospheric. What will be the boiler pressure when the temperature has been raised to 163°C ? If 10 per cent wet steam is drawn off at this pressure, what mass of air will first come over per kg of steam ? Assume air is saturated R for air 29 27 in kg metre units.

p p of steam at 22°C=0 02694 kgf/cm²

$\therefore$ p p. of air$=1{\cdot}033-0{\cdot}02694=1{\cdot}006$ kgf/cm²

Let V=volume of steam and air space

Then, for the air $PV=mRT$, where, V, m and R are constant for the two temperatures.

$\therefore$ p p. of air at 163°$C$$=1\ 006\times\dfrac{273+163}{273+22}=1{\cdot}487$ kgf/cm²

p.p. of steam at 163°C=6·8 kgf/cm²

Hence, $\underline{\text{boiler pressure}=1{\cdot}487+6\ 8=8\ 287\ \text{kgf/cm}^2}$ Ans

Specific volume of wet steam at 163°$C$$=0\ 8\times0\ 286$
$=0\ 2574$ m³/kg

$\therefore$ Volume of air drawn with one kg of steam$=0\ 2574$ m³

$\therefore$ Mass of air, $\underline{m}=\dfrac{PV}{RT}$

$$=\frac{1{\cdot}487\times10^4\times0\ 2574}{29\ 27\times436}=\underline{0\ 3\ \text{kg}}\qquad\text{Ans.}$$

4-17. Partial pressure : heating and cooling of wet steam and air in a closed vessel.

A steam heating unit of volume 1 m³ contains 8 kg of water and steam and a quantity of air. All valves are closed when temperature in vessel is 26°C and the total pressure is 1 02 kgf/cm².

Neglecting the initial volume of water in liquid form and any change in the volume of the boiler, calculate, (a) the total pressure in the vessel when the temperature is raised to 200·4°C, (b) the mass of water in the vessel at 200·4°C, that is still in liquid form, (c) the heat given to the air alone during the heating process from 26°C to 200·4°C, and (d) the heat given by the steam alone, if the vessel is cooled from 200·4°C to 0°C. Assume that the vapour pressure of water at 0°C is negligible. Take R for air 29 27 and $C_v=0{\cdot}17$.

(a) Partial pressure of steam at temperature 26°C

$$=0{\cdot}0343 \text{ kgf/cm}^2$$

∴ Partial pressure of air at 26°C $=1{\cdot}02-0{\cdot}0343$

$$=0{\cdot}9857 \text{ kgf/cm}^2$$

$$\text{Mass of air, } m=\frac{PV}{RT}=\frac{0{\cdot}9857\times10^4\times1}{29{\cdot}27(273+26)}=1{\cdot}127 \text{ kg}$$

p.p. of air when temperature is increased to 200·4°C

$$p_2=\frac{mRT}{V}=\frac{1{\cdot}127\times29{\cdot}27\times(273+200{\cdot}4)}{1\times10^4}=1{\cdot}561 \text{ kgf/cm}^2$$

p.p. of steam at 200·4°C = 16 kgf/cm²

By Dalton's law of partial pressures,

Pressure in vessel at 200·4°C $=16+1{\cdot}561=17{\cdot}561$ kgf/cm² Ans.

(b) Specific volume, v_g at 200·4°C $=0{\cdot}1261$ m³/kg

$V=mxv_g$, $\quad 1=8\times x\times0{\cdot}1261 \quad$ ∴ dryness, $x=0{\cdot}9913$

∴ Mass of water in vessel $=8(1-0{\cdot}9913)=0{\cdot}0696$ kg Ans.

(c) Heat given to the air $=mC_v\,(t_2-t_1)$

$$=1{\cdot}127\times0{\cdot}17\,(200{\cdot}4-26)$$

$$=33{\cdot}38 \text{ kcal}$$ Ans.

(d) Internal energy, $u=h-\dfrac{Pv}{J}$

$$\text{Initial } u_1=(204+0{\cdot}9913\times463{\cdot}1)-\frac{16\times10^4\times0{\cdot}9913\times0{\cdot}1261}{427}$$

$$=616{\cdot}3 \text{ kcal}$$

$$\text{At } 0°C,\ u_2=h_2-\frac{P_2v_2}{J}=0$$

(h is zero at 0°C and P is negligible)

$\therefore$ Heat given by steam $= m(u_1 - u_2)$

$= 8(616\ 3) = 4{,}930$ kcal Ans.

Note. In a clsoed vessel heating or cooling is a constant volume process. Because air is present, total pressure is the sum of partial pressures of air and steam. The heat supplied is partly taken by air and partly by steam.

4·18. Steam accumulator : mass of steam condensed.

Describe the principle of a steam accumulator.

An accumulator holds m kg of boiling water of enthalpy h per kg. Dry saturated steam, of amount dm kg, specific volume v_g m³/kg and an enthalpy h_g per kg is condensed in the boiling water, raising the pressure. Explain the meaning of the quantities in the brackets in the corresponding equation :—

$$m\left(h-\frac{pv}{J}\right)+dm\left(h_g-\frac{Pv_g}{J}\right)+dm\left(\frac{P}{J}\right)\left(v_g-v_f\right)$$
$$=(m+dm)\left[\left(h-\frac{Pv}{J}\right)+d\left(h-\frac{Pv}{J}\right)\right]$$

where v and v_f are the volume per kg of water at 0°C and boiling point respectively.

Neglecting v and v_f as small, show that

$$\log_e\frac{m_{end}}{m_{start}}=\int\frac{dh}{h_{fg}}$$

Find the mass of dry saturated steam to be condensed in 100 kg of boiling water at 4 kgf/cm² to raise the pressure to 7 kgf/cm²

For description and theory of accumulator—see text.

To solve the problem, $1/L$ and h are plotted from 4 to 7 kgf/cm², taking values from steam tables. It almost comes a straight line and hence the area may be found considering it a straight line

$$\int\frac{dh}{L}=\text{area under the curve (straight line)}$$

$$=(165{\cdot}7-143\ 7)\times\frac{0\ 002023+0\ 00196}{2}=0{\cdot}043813$$

$$\log_e\frac{M_{end}}{M_{start}}=0\ 043813$$

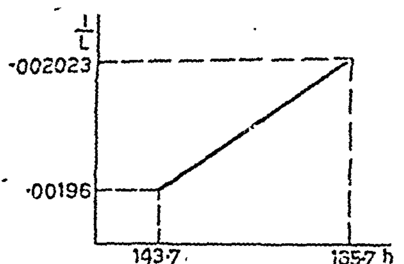


Fig. 4·23.

$$\therefore \quad \frac{M_{end}}{M_{start}} = 1·045$$

$$\therefore \quad M_{end} = 100 \times 1·045 = 104·5 \text{ kg}$$

or Mass of steam condensed = 4·5 kg **Ans.**

EXAMPLES 4

4.1. Plotting of T-s chart for steam.

What is the nature of water line in T-s chart of steam ?

Plot a T-s chart of steam for a pressure range of 1 kgf/cm² to 40 kgf/cm². Show in this diagram constant pressure superheating at 10 kgf/cm² upto 220°C, an isothermal process, a constant volume line of 0·15 m³ starting from saturation line to 0·2 dryness, adiabatic expansion from 15 kgf/cm² and 0·95 dry to 1·5 kgf/cm² and find the dryness fraction after expansion. Will the dryness fraction increase or decrease if the steam is expanded from 22 kgf/cm², 0·4 dry to 3 5 kgf/cm².

[Logarithmic ; $x_1 = 0·83$; $x_2 = 0·439$; if the initial dryness fraction is less than 0·5, dryness fraction increases by expansion ; if more than 0·5, it decreases].

4·2. Plotting of Mollier chart : adiabatic and throttling processes.

Plot a Mollier chart for steam for a pressure range of 0·035 kgf/cm² to 15 kgf/cm². Show in this chart constant dryness fraction lines, 0·6 and 0·8 dry. Show also constant temperature line of 150°C. From the chart, find (a) the adiabatic heat drop and the final condition of steam in an expansion from 10 kgf/cm², 0·9 dry to 0·75 kgf/cm², and

(b) the change of entropy and the final condition of steam in throttling from 15 kgf/cm², 0 7 dry to 8 kgf/cm².

[h_1-h_2=93 6 kcal ; x_2=0 79 ; s_1-s_3=0 0456 ; x_4=0·726].

4·3. Mollier chart : throttling process.

Sketch a T-s diagram and indicate on it .

(a) (i) the critical point ;

(ii) a constant pressure operation of generating superheated steam from the feed water at a temperature lower than the corresponding saturation temperature ;

(iii) the increase in enthalpy during the above operation ;

(iv) the ideal expansion of steam through a nozzle from a pressure p_1 to pressure p_2 ,

(v) the heat equivalent of the kinetic energy developed during the expansion through the nozzle ;

(vi) any constant volume line in the wet steam and superheated regions ; and

(vii) constant quality line for a dryness of 0 8

(b) determine the enthalpy per kg of steam at 20 kgf/cm², if its condition is (a) dry saturated (b) 15 per cent wet, and (c) superheated to 300°C.

If steam in condition (b) is passed through a reducing valve which lowers its pressure to 10 kgf/cm², find its final condition.

[668 kcal ; 600 6 kcal ; 721 3 kcal , x=0 87]

4 4 x ; u_g ; h u, given entropy

For a sample of steam at 30 kgf/cm², determine · (i) the condition of steam, (ii) the specific volume, (iii) the specific enthalpy, and (iv) the internal energy, taking datum as 0°C The steam has a specific entropy of (a) 1 372 kcal/kg deg C and, (b) 1·551 kcal/kg deg C.

[(a) x=0 8742 ; v_g=0 05945 m³ ; h=615 5 kcal , u=573 8 kcal.
(b) Degree of superheat=57·2°c ; v_g=0 0809 m³ ; h=708 5 kcal ;
u=651·4 kcal].

Entropy of water and steam.

In a boiler steam is evaporated at constant pressure of 25 kgf/cm². om fundamentals, calculate per kg (a) the liquid entropy of the water,) the entropy of wet steam when dryness fraction is 0·9, and (c) the ntropy of superheated steam when degree of superheat is 60°C. Assume average value of c_p for given range of superheat to be 0·5.

[Taking datum for entropy of steam and water at 0°C : $s_f = 0{\cdot}597$; $s_{wet\ steam} = 1{\cdot}397$; $s_{sup\ steam} = 1{\cdot}543$].

4.6. Dryness fraction by bucket calorimeter.

Describe the method of finding the dryness fraction of steam by a bucket calorimeter.

Steam is condensed in a tank containing 300 kg of water initially at 7°C. The increase in mass of water in certain time is 10·8 kg and the final temperature of water is 27°C. Calculate the dryness fraction of the entering steam, if its pressure is 14 kgf/cm² and if the heat lost from the tank during the period of condensation, is 300 kcal. The mass of the tank is 100 kg and the specific heat of the metal of the tank is 0·25

[Heat balance, heat before=heat after+loss ; $x = 0{\cdot}977$].

4.7. Separating and throttling calorimeter.

Describe with a sketch a separating and throttling calorimeter the measurement of dryness fraction of steam.

In a test to find the quality of steam in a pipe, using a co ed separating and throttling calorimeter the following results obtained ; initial pressure, 14 kgf/cm² ; pressure after throttling kgf/cm² ; temperature after throttling, 120°C ; water collected in tor, 0·45 kg ; steam condensed after throttling, 6·75 kg. Deter quality of the steam in the pipe. Take the specific heat of th heated steam as 0·5.

If the separator was removed what would be the limiting steam in the pipe which could be determined by the throttling alone, assuming the same pressure after throttling.

[$x_1 = 0{\cdot}9375$; $x_2 = 0{\cdot}9638$; $x = x_1 \times x_2 = 0{\cdot}9035$; lim of steam, $x = 0{\cdot}9423$].

4·8. Separating and throttling of steam : velocity of steam ; temperature after throttling.

Steam at a pressure of 10·5 kgf/cm² and 10 per cent wet flows along 12 cm diameter pipe at the rate of 40 kg per minute. Determine the velocity of steam in the pipe.

The above steam is passed through a separator where 3·6 kg of water is separated per minute and then it is passed through a reducing valve where it is throttled to 3 kgf/cm². Determine the temperature of steam after throttling.

[$mxv_s = AV$ ∴ $V = 10·03$ m/sec ; $40(h_f + xh_{fg}) = 3·6\ h_f + 36·4 \times h_{g2}$ ∴ $h_{g2} = 673·4$ kcal ; $t = 117·5$°C, by interpolation]

4·9. Dryness fraction by throttling calorimeter : heat loss and change in volume during passage.

An engine is supplied with steam from a boiler through a long pipe line. The steam leaving the boiler is at 15 kgf/cm² and 220°C. Some of the steam at the engine end of the pipe line is bled off and passed through throttling calorimeter, the steam temperature entering the calorimeter is 197·4°C, while the pressure and temperature of the steam after passing through the calorimeter are respectively 1·05 kgf/cm² and 120°C. Determine (a) the heat loss per kg of steam during passage through the pipe, (b) the steam dryness at the engine, and (c) the change in volume per kg of steam during passage through the pipe.

[$h_1 = 681·1$ kcal , $h_2 = 648·9 = h_2 = h_{f2} + x_2 h_{fg2}$ $x_2 = 0·962$;
constant pressure loss in pipe $= 32·2$ kcal/kg , $v_2 = x_2 v_{g2} = 0·129$ m³/kg ;
$V_1 - V_2 = 0·0146$ m³/kg]

4·10. Throttling friction heat ; diameter of pipe

Steam at 7 kgf/cm², 0·94 dry is throttled to 1·5 kgf/cm² and passed slowly at this pressure through a casing in which a turbine disc is rotated by means of external power. Neglecting friction, find (a) the dryness of the steam leaving the casing, if the flow is 0·4 kg and the horse-power absorbed by the friction of the disc is 7·5, and (b) the

meter of the exhaust pipe from casing if the steam speed is limited to 20 metres per second.

[enthalpy $= h_{g1} +$ friction $= 630{\cdot}2 + 3{\cdot}29 = 633{\cdot}49$ kcal ; $x_2 = 0\ 982$; $d = 17{\cdot}19$ cm]

4·11. Constant pressure : x_2 ; W : Q : $S_2 - S_1$.

0·5 kg of steam at 10 kgf/cm² and dryness fraction of 0·8 is heated at constant pressure until its volume is doubled. Determine the final condition of steam, work done, heat transferred and change of entropy.

[by interpolation, $t_s = 408{\cdot}6$°C ; $W = 18{\cdot}55$ kcal ; $Q = 108{\cdot}5$ kcal ; $S_2 - S_1 = 0{\cdot}2143$]

4·12. Constant volume and constant pressure cooling.

A cylinder has 1 m³ dry saturated steam under a movable piston loaded with 157 kg. The top of piston is open to atmosphere and the diameter of piston is 10 cm. Find the amount of heat given out of this steam if it is cooled at constant pressure to 91°C. If the piston is fixed, what amount of heat would be given out if the steam is cooled to the same temperature ?

[$m = 1\ 621$ kg ; $Q = m(h_{g1} - h_f - h) = 902$ kcal ; $Q = m(u_2 - u_1)$ $= 586\ 3$ kcal]

4·13. Polytropic expansion.

1 m³ of steam at 2 kgf/cm² and 40 per cent wet is compressed to 10 kgf/cm² according to the law $PV^{1{\cdot}2} = C$. Find

(*a*) *the final volume and dryness fraction,*

(*b*) *the work done in kcal during compression,*

(*c*) *the change in enthalpy per kg of steam, and*

(*d*) *the heat interchange per kg of steam.*

Neglect the volume of water.

[$m = 1{\cdot}817$ kg ; $V_2 = 0\ 2615$ m³ ; $x_2 = 0{\cdot}7148$; $W = 72$ kcal ; $h_2 - h_1 = 90{\cdot}1$ kcal ; $u_2 - u_1 = 82{\cdot}3$ kcal ; $q = 43{\cdot}3$ kcal]

4.14. Hyperbolic compression : dryness and heat interchange.

Give an example of hyperbolic expansion available in actual practice.

A cylinder contains 150 litres of steam at 4 kgf/cm² and 0·5 dry. The steam is compressed hyperbolically to 0·06 m³. Find (a) the mass of the vapour, (b) the final dryness fraction, and (c) the heat flow from or to the cylinder, indicating the direction of flow.

Neglect the volume of water present.

[$m=0.63$ kg ; $p_2=10$ kgf/cm² ; $x_2=0.4757$; $u_2-u_1=h_2-h_1$ $=11.3$ kcal/kg ; $W=-12.87$ kcal ∴ $Q=-5.35$ kcal]

4.15. Isentropic expansion : x_2 ; W.

0·9 kg of steam initially at a pressure of 15 kgf/cm² and a temperature of 250°C expands to 1·5 kgf/cm². Assuming that the steam expands isentropically, find the final condition of steam and the work done.

[$x_2=0.911$; $u_2=557.9$ kcal ; $W=77.04$ kcal]

4.16. Adiabatic and throttling processes

Steam at 20 kgf/cm² and 0·90 dry expands adiabatically to 15 kgf/cm² and is then throttled at constant enthalpy till it is just dry. Find, using steam tables, (a) the condition of steam after expansion and after throttling, (b) the adiabatic heat drop, (c) the external work done, (d) the increase in entropy per kg of steam during throttling, and (e) the mean adiabatic index of expansion.

[$x_2=0.8804$; $h_1-h_2=12.27$ kcal ; $W=10.86$ kcal ; P after throttling$=0.045$]kgf/cm² (approx.); $s_3-s_2=0.3924$; $n=1.12$]

4.17. Partial pressure ; cooling of steam air mixture

A vessel of volume 21 m³ containing a mixture of air and [illegible] ed water vapour initially at temperature 42°C and vacuum [illegible] Hg (barometer 76 cm Hg) is allowed to cool. If during [illegible] of vapour condenses, and 1·13 kg of air leaks in, determine [illegible] temperature and vacuum in the vessel. Neglect the [illegible] condensed vapour. R=29·27.

[Before cooling : $m_{air}=1\cdot191$ kg ; m_{steam} $1\cdot187$ kg. After cooling : p.p. of air$=0\cdot101$ kgf/cm^2 ; p.p. of steam$=0\cdot0727$ kgf/cm^2 ; vacuum$=12\cdot8$ cm of Hg ; temperature$=39\cdot34°C$]

4.18. Partial pressure : water injection to reduce temperature.

(a) A sealed vessel, having a volume 0·2 m³ contains 2 kg of water and a small amount of air. At a temperature of 0°C, the partial pressure of the air in the vessel is 0·8 kgf/cm² while that of water is negligible. The vessel is heated until the temperature rises to 200·4°C. Determine the heat to be supplied to the air and to the water in the vessel, and the total pressure in the vessel.

(b) How much water, at 20°C and 70 kgf/cm² should be injected to lower the temperature to 150·4°C ?

Assume $C_v=0\cdot17$ and $R=29\cdot29$ for air.

[mass of air$=0\cdot2$ kg ; p.p. after heating$=17\cdot387$ kgf/cm^2 ; $x_1=0\cdot7912$; $Q_{water}=1{,}065\cdot9$ kcal ; $Q_{air}=6\cdot814$ kcal. Total p.p.$=6\cdot141$ kgf/cm^2, $U_2=U_1+$enthalpy added ; $\therefore$ mass of water. $m=4\cdot13$ kg]

5

Theory of Simple Steam Engines

5·1. Carnot Cycle. Fig. 5·1 shows the Carnot cycle, on P-V and T-s diagrams, as applied to a wet vapour, say steam. It consists

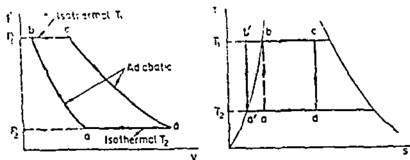


Fig. 5·1.

of two isothermal and two adiabatic processes. bc represents the isothermal evaporation of liquid at constant pressure, cd the adiabatic expansion of vapour, da the isothermal condensation of vapour at constant pressure and ab the adiabatic compression of vapour to restore it to the original state of liquid. The temperature during isothermal process bc is T_1 and during isothermal process da is T_2, T_1 being higher than T_2. Assume one kg of vapour.

Now, $$dq = T \times ds$$

$\therefore$ Heat supplied during the isothermal process, $bc = x_1 s_{fg1} T_1$

and, heat rejected during the isothermal process, $da = x_2 S_{fg2} T_2$

$\therefore$ Work done $=$ area $abcd = x_1 S_{fg1} (T_1 - T_2)$

$\therefore$ Thermal efficiency $$= \frac{x_1 S_{fg1}(T_1 - T_2)}{x_1 S_{fg1} T_1} = \frac{T_1 - T_2}{T_1} \qquad (5·1)$$

This is the same expression as for a perfect gas. The Carnot cycle, though theoretically the most efficient cycle, is not taken as standard of reference for steam plants due to the reasons explained below.

5·2. Rankine Cycle. The Rankine cycle is the ideal cycle for comparing the performance of steam plants. It is represented by the closed figure *abcd* on *P-v* and *T-s* diagrams in Fig. 5·2. Fig. 5·3 shows the schematic diagram of a steam engine or turbine plant. The various processes of the Rankine cycle are as follows :

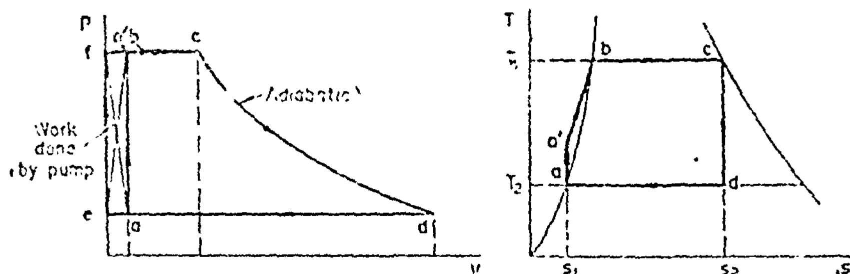


Fig. 5·2. Rankine cycle taking feed pump work into account.

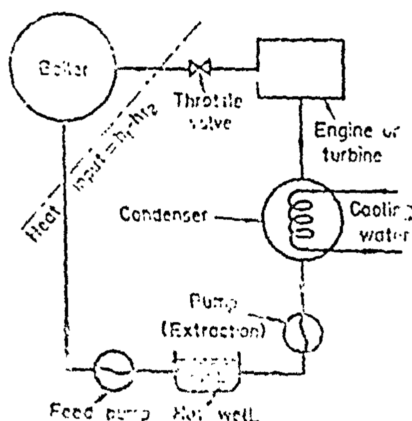


Fig. 5·3. Schematic diagram of steam engine or turbine plant.

aa'—The point *a* represents the water at condenser pressure and corresponding saturation temperature. The water is pumped into boiler by extraction and feed pumps, raising its pressure to boiler pressure by adiabatic compression *aa'*. During this process there is slight rise in temperature.

$a'b$ and bc—Heat is supplied to the boiler at constant pressure and the point b is reached, which is the saturation temperature corresponding to the boiler pressure. In P-v diagram point b nearly coincides with a' as increase in volume is negligible Further addition of heat evaporates the water and the process is represented by bc. The final condition of steam may be wet, dry or superheated depending upon the quantity of heat supplied.

cd—The steam is now expanded adiabatically to do work] in a steam engine or a turbine. This process is represented by cd.

da—The exhaust from the engine or the turbine is led into a condenser where the latent heat of the exhaust steam is removed by circulating water at constant pressure. This process is represented by da.

In the P-V diagram, work done by the pump in increasing the pressure of water from condenser pressure to boiler, is represented by the area $aa\ fe$. This is, however, very small at low pressures and is therefore generally neglected The modified P-v and T-s diagrams representing the Rankine cycle neglecting feed pump work are shown in Fig. 5 4

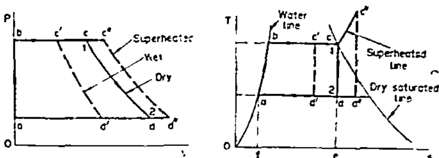


Fig. 5 4. The Rankine cycle neglecting feed pump work.

Let h_{f1}=enthalpy of water at point b,

h_{f2}=enthalpy of water at point a,

h_1=enthalpy of vapour at point c,

h_2=enthalpy of steam at point d.

Heat supplied during the processes ab and bc

$$=(h_{f1}-h_{f2})+x_1h_{fg1}=h_1-h_{f2}$$

Heat rejected during the process $da = x_2 h_{fg2}$

$\therefore$ Work done $= h_1 - h_{f2} - x_2 h_{fg2} = h_1 - h_2$

$(h_1 - h_2)$ is knows as the *heat drop* or the *Rankine area.*

$$\therefore \text{ Efficiency of the Rankine cycle} = \frac{h_1 - h_2}{h_1 - h_{f2}} \qquad [5\cdot2(a)]$$

In terms of areas on T-s diagram

$$\text{Efficiency} = \frac{\text{total heat at } c - \text{total heat at } d}{\text{total heat at } c - \text{total heat at } a}$$

$$= \frac{\text{area } cboe - \text{area } daoe}{\text{area } cboe - \text{area } aof} = \frac{\text{area } abcd}{\text{area } cbafe} \qquad [5\cdot2(b)]$$

Note. Enthalpy at any point on T-s diagram is given by the enclosed area formed by drawing a line from that point a horizontal to liquid line and a vertical to the absolute zero base line.

5·3. Rankine Cycle vs. Carnot Cycle. Rankine cycle is taken as standard of reference for the performance of steam plants because it is more practicable than the Carnot cycle because of the following reasons.

(*i*) In the Rankine cycle there is complete condensation *da* and then the pressure and temperature is raised to the initial stage *b*. In the Carnot cycle condensation is stopped at a point *e* such that the fluid is restored to its initial state *b* by adiabatic compression. This is very difficult to realise in practice.

(*ii*) If superheated steam is used the Carnot cycle would be still more difficult to realise as heat of superheat would have to be supplied at constant temperature, whereas in actual practice it is only possible at constant pressure.

Properties of a fluid that would make Rankine cycle approach Carnot cycle (*i*) Referring to T-s diagram Fig. 5·5, the Rankine cycle is represented by *abcd*. Work done is equal to the area *abcd* and heat supplied is equal to the area *hatcf*. The Carnot cycle is represented by the area *cbed* and heat supplied in it by the area *gbcf*. Therefore, the two cycles would be identical if the hatched area was zero. The hatched area represents the difference of sensible heat at *b* and *a*. The fluid therefore must have a small sensible heat or a very large latent heat, so as to render the sensible heat negligible in comparison.

Desirable properties of a fluid for use as working substance in a heat engine plant. The desirable properties of a fluid

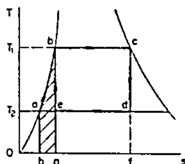


Fig 5·5.

for use as working substance in a heat engine plant are that it should be cheap, universally available, chemically stable and harmless. From thermodynamic point of view it should have the following properties

(*i*) It should have the highest saturation temperature for the moderate pressure. This would mean high efficiency due to most of the heat being added at high temperature without having to deal with high pressures, which create mechanical difficulties.

(*ii*) The fluid must have small specific heat of liquid. This will render the sensible heat negligible in comparison to heat added for boiling which is added at the upper temperature.

(*iii*) For a given output the plant size is reduced if the fluid has the high density.

(*iv*) The saturated vapour line should be very steep, so that the fluid after expansion has the high dryness fraction, without recourse to superheating.

(*v*) The saturation pressure at the exhaust temperature should be slightly higher than the atmospheric pressure. This avoids the use of condenser and the undesirabilities associated with the use of condenser.

A T-s diagram for such an ideal fluid is sketched in Fig. 5·6.

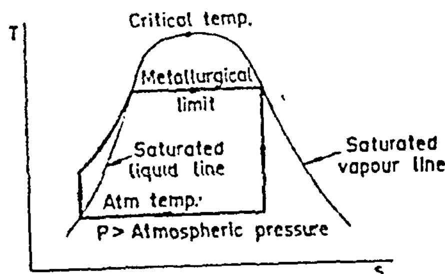


Fig. 5·6. The ideal cycle for a vapour power cycle.

Mercury is good from thermodynamic point of view. It has critical temperature and pressure of 1100°C and 1·4 kgf/cm² respectively as compared to 374°C and 225 kgf/cm² respectively of water. But mercury is costly and poisonous, whereas water is a free gift of nature and is harmless. Therefore water is universally adopted.

5·4. The Modified Rankine Cycle. In steam engines the expansion is not continued upto the point d, as the work obtained is

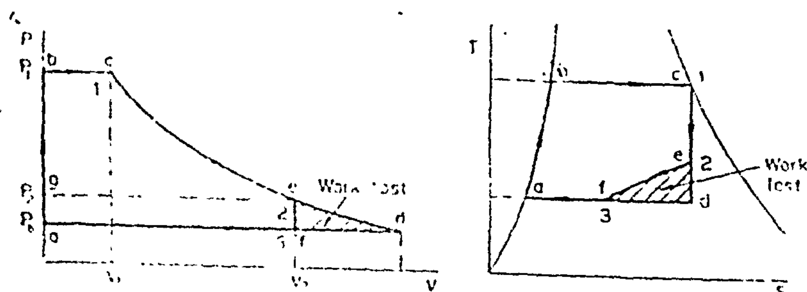


Fig. 5·7. The Modified Rankine cycle.

very small at the tail end as can be seen from Fig. 5·7. In fact it is not even sufficient to overcome the work lost in friction in tail end part of the stroke. Therefore, in actual practice, release is allowed to take place before the expansion is complete at some point e by opening the exhaust port. This causes a sudden pressure drop ef at constant volume due to steam communicating with the outside atmosphere. This considerably reduces the stroke length without any appreciable change in the work done. The cycle is then known

as *modified Rankine cycle.* In T-s diagram ef is plotted by the equation, xv_g=constant.

In P-v diagram,

Work done in modified Rankine cycle=area $gbce$+area $gefa$

∴ Efficiency of the modified Rankine cycle

$$=\frac{h_1-h_2+\left(\frac{P_2-P_3}{J}\right)v_2}{h_1-h_{f3}} \qquad [5\cdot3(a)]$$

Another expression for the efficiency of the modified Rankine cycle is

$$=\frac{\frac{P_1v_1}{J}+(u_1-u_2)-\frac{P_3v_2}{J}}{h_1-h_{f3}} \qquad [5\cdot3(b)]$$

This expression is very combersome and may not be used.

5·5. Hypothetical Indicator Diagram and Diagram Factor. The hypothetical indicator diagram of a steam engine assumes (*i*) no pressure drop, (*ii*) instantaneous opening and closing of valves, (*iii*) hyperbolic expansion, (*iv*) release and admission at the end of the strokes, and (*v*) no compression

Fig. 5·8 shows a hypothetical indicating diagram neglecting clearance. The steam is admitted into cylinder at boiler pressure

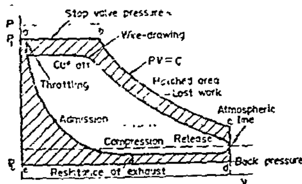


Fig. 5·8. Steam engine indicator diagram.

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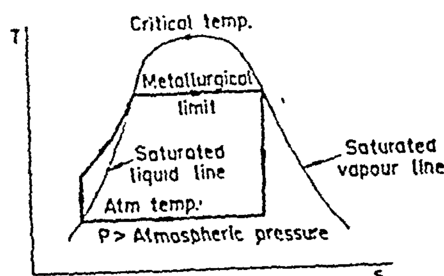


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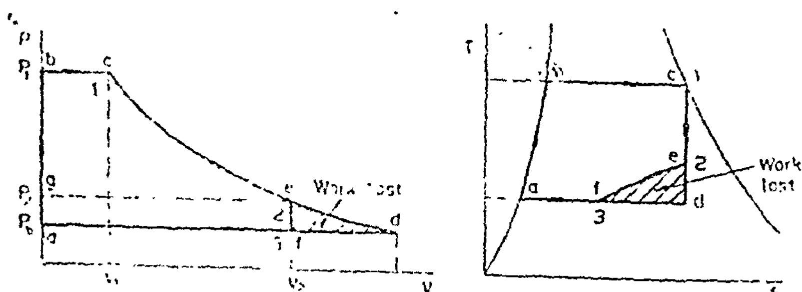


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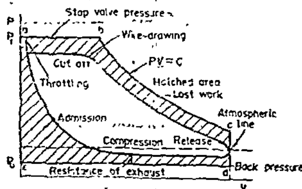


Fig. 5·8. Steam engine indicator diagram.

P_1 represented by ab. The cut-off takes place at b and bc represents hyperbolic expansion of steam The exhaust port is opened at c,

the point of release, and the steam pressure falls suddenly to back pressure, P_b. The line *de* represents exhausting of steam at constant pressure. At *e* fresh steam is admitted from boiler and pressure immediately rises to *a*.

The actual diagram differs from the hypothetical diagram due to (*i*) drop in pressure caused by throttle governing, and wire drawing at valves, (*ii*) gradual cut-off and release, (*iii*) actual expansion being different from hyperbolic due to varying interchange of heat, (*iv*) release before the end of stroke, to give sufficient time for steam to exhaust and thus prevent undue back pressure, (*v*) higher back pressure due to some invitable resistance from exhaust, (*vi*) compression, to secure greater economy in working and to relieve the bearings of inertia loads, and (*vii*) admission earlier than the end of stroke, so that beginning of the outward motion of the piston is performed under the full pressure of steam.

The area of actual indicator diagram is less than that of theoretical indicator diagram. The ratio of the two areas is known as Diagram Factor (D.F.) and is always less than one.

$$\text{D.F.}=\frac{\text{area of actual indicator diagram}}{\text{area of hypothetical indicator diagram}} \tag{5·4}$$

The hatched area represents the work lost per cycle. It can be minimized by (*i*) using corliss or drop valves having rapid closing and opening which would avoid wire drawing (*ii*) steam jacketing to make the expansion nearly hyperbolic and to avoid condensation, and (*iii*) liberal exhaust port opening to reduce back pressure.

5·6. Work Done and Mean Effective Pressure P_m. Fig. 5·9. (*a*) and (*b*) respectively show the hypothetical indicator diagrams for the steam engines with clearance and neglecting clearance.

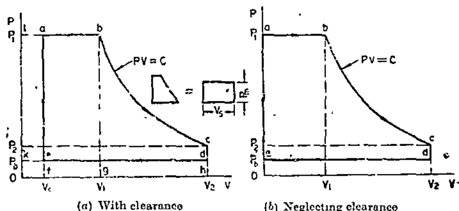
In Fig. 5·9 (*a*).

Hypothetical work done per stroke

$$=\text{area of hypothetical indicator diagram}$$
$$=\text{area } abcde$$
$$=\text{area } olbg+\text{area } gbch-\text{area } kdho-\text{area } klae$$
$$=P_1V_1+P_1V_1\log_e r-P_bV_2-(P_1-P_b)V_c \quad [r=V_2/V_1$$
$$=P_1V_1(1+\log_e r)-P_bV_2-(P_1-P_b)V_c \text{ kg m} \qquad [5·6(a)]$$

In Fig. 5·9(b), neglecting clearance *i e.* $V_c=0$,

Work done per stroke $=P_1V_1(1+\log_e r)-P_bV_2$ kgf m [5 6(b)]



(*a*) With clearance (*b*) Neglecting clearance

Fig 5·9. Hypothetical indicator diagrams.

Theoretical mean effective pressure, neglecting clearance,

$$P_m=\frac{P_1(1+\log_e r)}{r}-P_b \tag{5 7}$$

Actual mean effective pressure

= theoretical mean effective pressure × diagram factor

Actual work done = theoretical work done × diagram factor

If L = length of stroke in m, A = area of piston in cm², N = r p.m. and P_m = mean effective pressure in kgf/cm², then

$$\text{ihp}=\frac{\text{work done per stroke}\times 2N}{75\times 60} \quad [5\ 8(a)]$$

(as steam engines are always double-acting)

$$\therefore \quad \text{ihp}=\frac{P_m\times L\times A\times 2N}{75\times 60}=\frac{2P_mLAN}{75\times 60} \quad [5\ 8(b)]$$

The cylinder dimensions can be evaluated from equation [5 8(b)] for a given horse-power and revolutions per minute.

5·7. Mass of Steam in Cylinder. The total mass of steam in cylinder at cut-off is the sum of cushion-steam left in clearance volume plus fresh steam admitted per stroke

$$m=m_{cushion}+\frac{\text{steam per minute}}{2N} \tag{5 9}$$

To get the mass of cushion steam indicator diagram is calibrated. On compression curve steam is assumed to be dry and

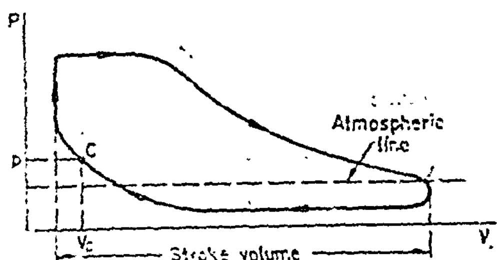


Fig. 5·10.

saturated. By taking any point on compression curve as c, the volume at this point, V_c, and its corresponding pressure is noted as shown in Fig. 5·10. From steam tables specific volume, v_g, at this pressure is used.

$$\text{Then,} \qquad m_{cushion} = \frac{V_c}{v_g} \qquad (5\cdot10)$$

5·8. The Saturation Curve and Missing Quantity. In steam engines condensation of steam occurs at valve faces, valve

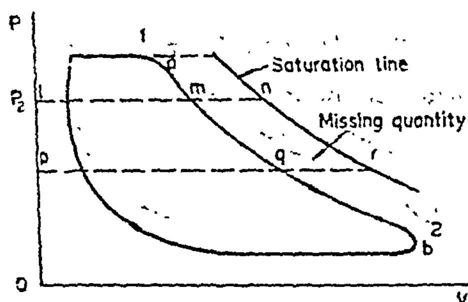


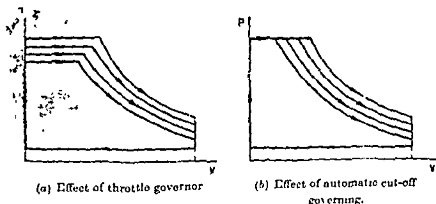
Fig. 5·11. Missing quantity.

ports and cylinder walls. It is because of the reason that the fresh steam comes in contact with the cylinder surfaces made cold by the exhaust of the previous stroke. Due to the condensation of the steam in the cylinder, the actual volume occupied by steam is less than the theoretical. If an expansion curve is drawn assuming steam to be dry and saturated at every point throughout the stroke, it gives the theoretical value of volume occupied by steam at any

point. This curve is known as *saturation curve* and is shown in Fig. 5·11. The difference in the theoretical and actual volume at any point is known as *missing quantity*. In Fig. 5·10 at pressure p, mn is the missing quantity. *Though missing quantity is mainly due to* the condensation of steam in cylinder, small amount may be due to leakage past the valves and the piston. Dryness fraction neglecting leakage$=\frac{l_m}{l_n}$. Missing quantity is reduced during later half of the stroke due to re-evaporation.

5·9. Governing of Simple Steam Engine. Simple steam engines may be governed either by *throttle* governing or *cut-off* governing.

(a) Throttle governing. In throttle governing initial steam pressure is reduced due to throttling as shown in Fig. 5·12 *(a)*. This type of governing is wasteful in consumption of steam. In throttled governed engines steam consumption is directly proportional to the indicated or brake horse-power *Hence, for these engines* a graph between consumption of steam and indicated horse-power will be straight line as shown in Fig. 5 13 This is known as *Willan's line* and the principle is known as *Willans' law*.



(a) Effect of throttle governor (b) Effect of automatic cut-off governing.

Fig. 5 12 Governing of simple steam engine

(b) Cut-off governing. In cut-off governing, the governor changes the point of cut-off but does not change the inlet pressure as shown in Fig. 5·12 *(b)* Cut off governing is more efficient and economical, the steam consumption being about 75 per cent of throttled governed engine. But it requires a special valve gear *i e.*

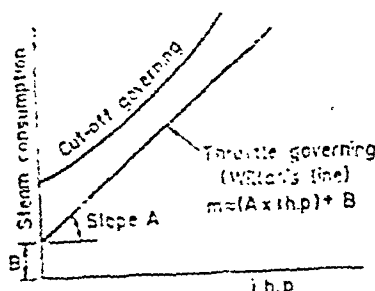


Fig. 5·13. Consumption of steam vs. power developed.

Mayer's expansion valve or a link motion Fig. 5·13 shows consumption versus power developed for a cut off governed engine.

5·10. Efficiencies. (*a*) *Mechanical efficiency.* It is the ratio of the power obtained at shaft (*bhp*) to the indicated power (*ihp*). Thus it accounts for the loss in friction.

$$\text{Mechanical efficiency} = \frac{bhp}{ihp} = \frac{(T_1 - T_2)2\pi r \times N/75 \times 60}{(P_m lA2N)} \qquad (5\cdot9)$$

(*b*) *Thermal efficiency.* It is the ratio of the indicated work done to the energy supplied to engine cylinder in steam.

$$\text{Thermal efficiency} = \frac{ihp \times 75 \times 60}{m(h_1 - h_{f2}) \times J} \qquad (5\cdot10)$$

where m = mass of steam per indicated hp

h_1 = total heat of steam supplied

h_{f2} = heat of water returned to hot well.

Note. Thermal efficiency may also be written as indicated thermal efficiency.

(*c*) *Brake thermal efficiency.* It is the ratio of the brake or shaft work obtained to the energy supplied to engine cylinder in steam.

$$\text{Brake thermal efficiency} = \frac{bhp \times 75 \times 60}{m(h_1 - h_{f2}) \times J} \qquad (5\cdot11)$$

$$= \text{Thermal efficiency} \times \text{mechanical efficiency.}$$

(*d*) *Overall efficiency.* It is the ratio of the brake work obtained to the energy supplied by the fuel for the generation of steam in boiler.

$$\text{Overall efficiency} = \frac{bhp \times 75 \times 60}{m \times CV \times J} \quad (5\cdot12)$$

where m_f = mass of fuel of calorific value CV.

(*e*) *Rankine efficiency.* It is the ratio of the Rankine work between the same temperature limits to the energy supplied with steam in boiler.

$$\text{Rankine efficiency} = \frac{h_1 - h_2}{h_1 - h_{f2}} \quad (5\cdot13)$$

(*f*) *Relative efficiency or efficiency ratio.* It is the ratio of the thermal efficiency to the corresponding Rankine efficiency.

$$\text{Relative efficiency} = \frac{\text{Thermal efficiency}}{\text{Rankine efficiency}} \quad (5\cdot14)$$

5.11. Heat Balance Fig. 5.19 shows the schematic arrangement for complete testing of a steam engine. The heat balance for

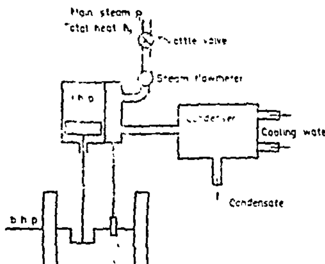


Fig. 5·11

an engine is prepared on minute basis taking datum 0°C or condensate temperature and includes the items shown in the table below.

Cr. *Heat units*	%	*Dr.* *Heat units*	%
Heat supplied in steam		ihp fhp=(ihp−bhp)	
		1. Heat equivalent to bhp	
		2. Heat rejected in condenser	
		3. Heat rejected in hot well	
		4. Heat lost in radiation, friction, etc. (by difference).	

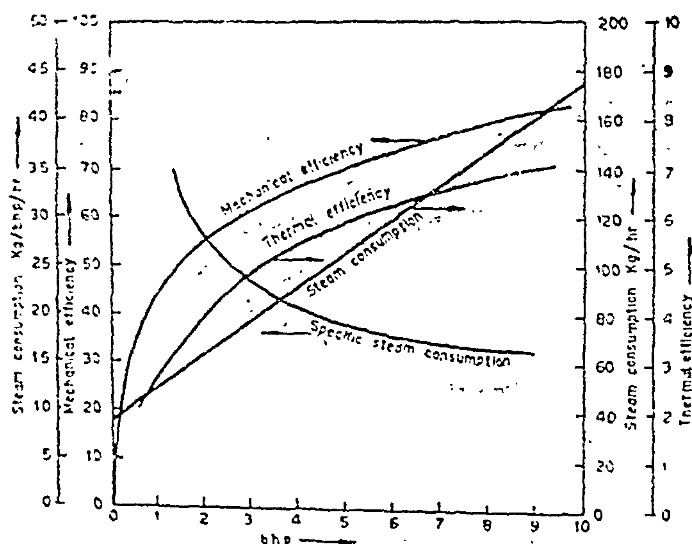


Fig. 5·15. Performance curves.

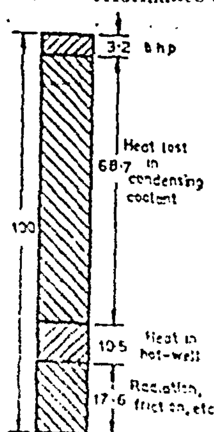


Fig. 5·16. Diagrammatic representation of heat balance.

Fig. 5·15 shows the important performance curves for steam engines and Fig. 5·16, is a diagrammatic representation of the heat balance.

IMPORTANT POINTS

(a) *The Rankine Cycle.*

1. In all problems on the Rankine Cycle work done by pump should be *neglected unless otherwise specified.*

2. Note that the work done in the Rankine Cycle is equal to (h_1-h_2) and not (u_1-u_2) as it is a flow process. (u_1-u_2) is equal to *the work done under the expansion curve only.*

3. In the Rankine cycle available heat is $(h_1-h_{f_2})$. If feed water temperature is given, h_{f_2} should be taken as the liquid heat at *this given temperature.*

4. Unless otherwise specified adiabatic expansion means fully resisted or frictionless adiabatic in which entropy remains constant.

(b) *Steam Engine.*

5. All steam engines are double-acting ; therefore number of working strokes are $2\times$r.p.m. The mean piston speed per minute is given by the length of stroke$\times$number of strokes per minute$=2LN$.

ILLUSTRATIVE EXAMPLES

5·1. Rankine efficiency : increase in efficiency due to superheating.

Explain why the increase in efficiency obtained by using superheated steam in reciprocating engines is more than the gain which would be expected from thermodynamic considerations.

A steam engine uses steam at 10 kgf/cm² and 0·9 dry and exhausts at 1·1 kgf/cm². (a) Determine its Rankine cycle efficiency. (b) By what percentage the efficiency increases if steam has a temperature of 200°C before entering the cylinder. Use steam tables.

The actual increase in efficiency by using superheated steam is more because of decrease in condensation, less heat transfer and less re-evaporation which is lost untimely in exhaust.

Cr. *Heat units*	%	*Dr.* *Heat units*	%
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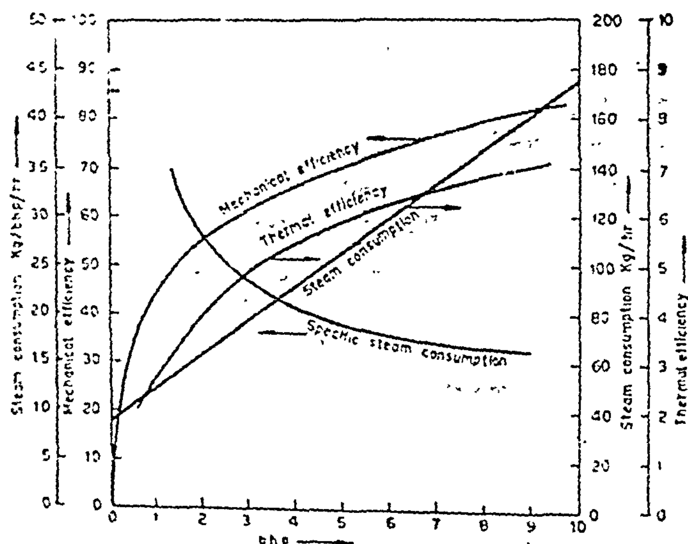


Fig. 5·15. Performance curves.

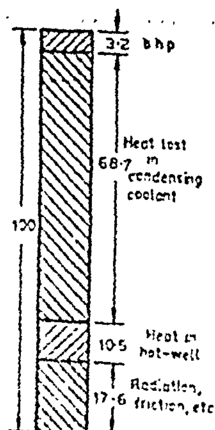


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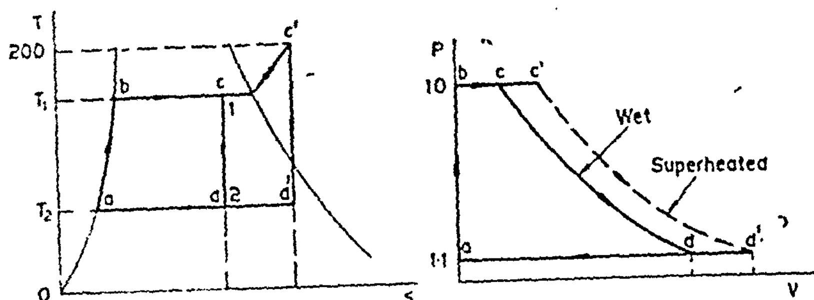


Fig. 5·17.

(*a*) *From steam tables.*

At 10 kgf/cm² and 200°C; $h_f = 181·3$; $h_{fg} = 482·0$; $s_f = 0·509$;

$s_{fg} = 1·066$ $h = 675·4$; $s = 1·601$

At 1·1 kgf/cm² : $h_f = 101·9$; $h_{fg} = 537·8$; $s_f = 0·317$: $s_{fg} = 1·435$

Entropy at c = Entropy at d

$0·509 + 0·9 \times 1·066 = 0·317 + x_2 \times 1·435$ ∴ $x_2 = 0·8024$

$h_1 = h_{f1} + x_1 h_{fg1} = 181·3 + 0·9 \times 432 = 615·1$ kcal

$h_2 = h_{f2} + x_2 h_{fg2} = 101·2 + 0·8024 \times 537·8 = 533·5$ kcal

Rankine efficiency $= \dfrac{h_1 - h_2}{h_1 - h_{f2}} = \dfrac{615·1 - 533·5}{615·1 - 101·9} = 15·9\%$

(*b*) Entropy at *c'* = Entropy at *d'*

$1·601 = 0·317 + x_2 \times 1·435$ ∴ $x_2 = 0·895$

$h_2' = 101·9 + 0·895 \times 537·8 = 583·3$ kcal

Rankine efficiency $= \dfrac{h_1 - h_2}{h_1 - h_{f2}} = \dfrac{675·4 - 583·3}{675·4 - 101·9} = 16·06\%$ Ans.

∴ Increase in Rankine efficiency $= \dfrac{16·06 - 15·9}{15·9} = 1·007\%$ Ans.

5·2. Rankine η : specific steam consumption ; effect of back pressure.

(*a*) *What should be the properties of working fluid so that the Rankine cycle approaches the Carnot cycle ?*

(*b*) *Explain why the Rankine cycle rather than the Carnot cycle is used as a standard of reference for the performance of steam plants.*

(*c*) *Dry and saturated steam is supplied from a boiler to a steam turbine at a pressure of 15 kgf/cm². It is expanded adiabatically in the turbine to a pressure of 1·1 kgf/cm². Determine :*

(i) *the Rankine efficiency and it's equivalent mean effective pressure*

(ii) the specific steam consumption in kg/hp-hr assuming an efficiency ratio of 0 65 ;

(iii) the Carnot efficiency for the same temperature limits of the cycle ;

(iv) if by connecting a jet condenser, exhaust pressure is reduced to 0 2 kgf/cm², find the percentage increase in the Rankine efficiency and decrease in specific steam consumption in kg/hp-hr.

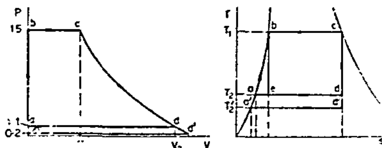


Fig. 5-18.

For theory—see text.

(i) *From steam tables*

At 15 kgf/cm² : $h_g = 666\ 7$; $s_g = 1\ 541$; $t_{sat} = 197{\cdot}4°C$

At 1·1 kgf/cm² : $h_f = 101\ 9$; $h_{fg} = 537{\cdot}8$; $s_f = 0\ 317$,
$s_{fg} = 1{\cdot}435$; $t_{sat} = 101\ 8$; $v_g = 1{\cdot}578 \times 10^4$

At 0 2 kgf/cm² : $h_f = 59{\cdot}7$; $h_{fg} = 563{\cdot}4$; $s_f = 0{\cdot}193$,
$s_{fg} = 1\ 692$; $t_{sat} = 59{\cdot}7$

Entropy at c = Entropy at d

$$1\ 541 = 0\ 317 + x_2 \times 1\ 435 \qquad \therefore \quad x_2 = 0\ 8531$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 101{\cdot}9 + 0\ 8531 \times 537\ 8 \qquad = 560\ 7 \text{ k cal}$$

$$\underline{\text{Rankine efficiency}} = \frac{h_1 - h_2}{h_1 - h_{f2}} = \frac{666\ 7 - 560\ 7}{666\ 7 - 101\ 9} = \frac{106}{564\ 8} = \underline{18{\cdot}77\%}$$

Ans

$$\underline{\text{mep}} = \frac{w}{v} = \frac{106 \times 427}{0\ 8531 \times 1\ 578 \times 10^4} = \underline{3\ 362 \text{ kgf cm}^2} \quad \text{Ans.}$$

(ii) Actual work done per kg $= 0\ 65 \times 106 = 68\ 9$ k cal

$$\underline{\text{Specific steam consumption}} = \frac{75 \times 60 \times 60}{427 \times 68\ 9} = \underline{9{\cdot}18 \text{ kg/hp hr}} \text{ Ans.}$$

(iii) Carnot efficiency between the same temperature range

$$=\frac{T_1-T_2}{T_1}=\frac{470\cdot4-374\cdot8}{470\cdot4}=\underline{20\cdot32\%} \quad \text{Ans.}$$

(iv) When back pressure is reduced to 0·2 kgf/cm²

Entropy at c=Entropy at d'

$$1\cdot541=0\cdot198+x_2\times1\cdot692 \quad \therefore \quad x_2=0\cdot7938$$

$$h_2=h_{f2}+x_2h_{fg2}=59\cdot7+0\cdot7938\times563\cdot4=506\cdot9$$

$$\text{Rankine efficiency}=\frac{666\cdot7-506\cdot9}{666\cdot7-59\cdot7}=\frac{159\cdot8}{607}=\underline{26\cdot32\%} \quad \text{Ans.}$$

∴ Percentage increase in Rankine efficiency

$$=\frac{26\cdot32-18\cdot77}{18\cdot77}=\underline{40\cdot23\%} \quad \text{Ans.}$$

$$\text{Specific steam consumption}=\frac{75\times60\times60}{427\times0\cdot65\times159\cdot8}=6\cdot09 \text{ kg/hp-hr}$$

$$\therefore \text{ Decrease in steam consumption}=\frac{9\cdot18-6\cdot09}{9\cdot18}$$

$$=\underline{33\cdot67\%} \quad \text{Ans.}$$

Note. (i) In the same temperature range the efficiency of the Carnot cycle is the highest.

(ii) By expanding the steam to a lower back pressure, efficiency is increased and steam consumption decreased. Percentage decrease in steam consumption is not equal to percentage increase in efficiency as steam in two cases has different enthalpy per kg.

5·3. The modified Rankine cycle : feed pump work.

Why the modified Rankine cycle is adopted for reciprocating steam engines ? The cylinder of a steam engine is 30 cm in diameter and piston stroke is 58 cm. The steam at admission is at 10 kgf/cm² and 300°C. It expands adiabatically to 0·7 kgf/cm² and is then released at constant volume to a condenser at 0·28 kgf/cm². Determine,

(a) the modified Rankine efficiency,

(b) the new stroke, if the same amount of steam, from the original condition is expanded adiabatically to condenser pressure,

(c) the new Rankine efficiency, and

(d) the work done in kcal by the extraction and boiler feed pump per kg of water returned to the boiler.

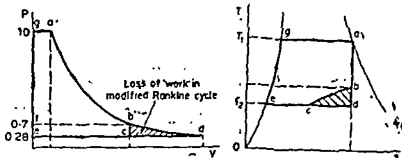


Fig. 5·19.

(a) From steam tables

At 10 kgf/cm² : $h=728$, $s=1\ 702$

At 0·7 kgf/cm² ; $h_f=89{\cdot}5$; $h_{fg}=545\ 6$, $s_f=0\ 283$,
$s_{fg}=1\ 505$, $v_g=2{\cdot}409$

At 0 28 kgf/cm² $h_f=67{\cdot}1$; $h_{fg}=559{\cdot}0$; $s_f=0{\cdot}220$;
$s_{fg}=1\ 642$; $v_f=0\ 001021$; $v_g=5{\cdot}68$

Volume of cylinder $=\frac{\pi}{4}\times\left(\frac{30}{100}\right)^2\times\frac{58}{100}=0{\cdot}041\ \text{m}^3$

Entropy at a = Entropy at b

$$1\ 702=0\ 283+x_b\times1{\cdot}505 \qquad \therefore\ x_b=0\ 943$$

$$h=h_f+xh_{fg}=89\ 5+0\ 943\times545{\cdot}6=604\ \text{kcal}$$

Work done/kg = area $gabf$ + area $bcef=(h_a-h_b)+(P_b-P_c)v$

$$=(728-604)+\frac{(0\ 7-0{\cdot}28)\times10^4\times(0\ 943\times2\ 409)}{427}=145\ 6\ \text{kcal}$$

Heat supplied/kg $=h_a-h_{fe}=728-67{\cdot}1=660\ 9$ kcal

$\therefore$ <u>Modified Rankine</u> $\eta=\frac{145{\cdot}6}{660\ 9}=\underline{22\%}$ **Ans.**

(b) Entropy at a = Entropy at d

$$1\ 702=0\ 220+x_d\times1\ 642 \qquad x_d=0\ 9026$$

Mass of steam at $b=\frac{v}{xv_g}=\frac{0\ 041}{0\ 943\times2\ 409}=0\ 01806$ kg/stroke

Volume at $d=m\times x\times v_g=0\ 01806\times0\ 9026\times5{\cdot}68=0\ 09258$ m

Considering loss in pipe, $s_1' = s_2'$

$1{\cdot}575 = 0{\cdot}191 + x_2 \times 1{\cdot}708 \quad \therefore \quad x_2 = 0{\cdot}8106$

$h_2 = h_{f2} + x_2 h_{fg2} = 57{\cdot}4 + 0{\cdot}8106 \times 564{\cdot}7 = 515{\cdot}1$

Out of 1 kg of steam supplied from boiler only 0·95 kg reaches the engine at 10 kgf/cm²

$\therefore$ W.D./kg of steam from boiler $= 0{\cdot}95(663{\cdot}3 - 515{\cdot}1)$
$= 140{\cdot}8$ kcal

Heat supplied for 1 kg of steam in boiler

= Total heat of steam – heat in engine condensate—heat in trap condensate

$= 664{\cdot}9 - 0{\cdot}95 \times 57{\cdot}4 - 0{\cdot}05 \times 181{\cdot}3 = 601{\cdot}3$ kcal

$\therefore$ Rankine $\eta = \dfrac{140{\cdot}8}{601{\cdot}3} = 23{\cdot}42\%$

Percentage reduction in output

= percentage decrease in Rankine η

$= \dfrac{25{\cdot}48 - 23{\cdot}42}{25{\cdot}48} = 8{\cdot}085\%$ Ans.

Note. When heat supplied is the same (one kg of coal is burnt in each case), percentage reduction in the Rankine efficiency is the same as percentage reduction in work done or out put.

5·5. Non-expansive working : increase in work and reduction in efficiency.

Define the term "expansive working of steam". Why is it generally adopted in steam engines? State an example where non-expansive working of steam is used.

A steam engine with expansive working admits steam at 8 kgf/cm² and 0·9 dry and exhausts at 1·1 kgf/cm². Cut-off is at 5/8th of stroke. Find (a) the percentage of work obtained by the expansive use of steam, and (b) the thermal efficiency of the cycle. Neglect clearance and assume hyperbolic expansion. (c) If this engine works non-expansively, what would be the percentage increase in work done and decrease in thermal efficiency ?

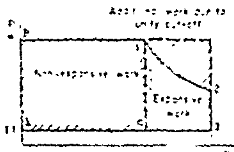


Fig. 5.22

(a) *From steam tables*

at $\quad$ 8 kgf/cm² $h = 171.1$, h_s = [illegible] ; $v_s = 0.2448$

at $\quad$ 1 kgf/cm² $h = 101.9$

Let steam used per stroke be 1 kg with expansive working

$\therefore \quad v_1 = 0.9 \times 0.2448 = 0.2203 \text{ m}^3$

and $v_2 = \dfrac{0.2203}{(5/8)} = 0.3525 \text{ m}^3$

Non expansive work/kg of steam

$$= \frac{(P_1 - P_2)v_1}{J}$$

$$= \frac{(8 - 1.1) \times 10^4 \times 0.2203}{427} = 35.63 \text{ kcal}$$

Expansive work/kg of steam

$$= \frac{1}{J}\left[P_1 v_1 \log_e r - P_2(v_2 - v_1)\right]$$

$$= \frac{10^4}{427}[8 \times 0.2203 \log_e (8/5) - 1.1(0.3525 - 0.2203)] = 16.01 \text{ kcal}$$

$\therefore \quad$ Total work/kg of steam $= 35.63 + 16.01 = 51.64$ kcal

[check Total work done/kg of steam can also be found by

$$W = \frac{1}{J}[P_1 v_1(1 + \log_e r) - P_2 v_2]$$

$$= \frac{10^4}{427}[8 \times 0.2203(1 + \log_e (8/5)) - 1.1 \times 0.3525] = 51.64 \text{ kcal}]$$

Work obtained by expansive use $= \dfrac{16.01}{51.64} = 31$ per cent $\quad$ Ans.

(b) Heat supplied/kg of steam

$= h_f + x_1 h_{fg1} - h_{f2}$
$= 171\cdot4 + 0\cdot9 \times 489\cdot8 - 101\cdot9 = 510\cdot3$ kcal

$\therefore$ Thermal efficiency $= \dfrac{51\cdot64}{510\cdot3} = 10\cdot12\ \%$ **Ans.**

(c) *Non-expansive working*

When cut-off is $\frac{5}{8}$th stroke, steam used per stroke $=1$ kg

When cut-off is unity, steam used per stroke $= \dfrac{1}{5/8} = 1\cdot6$ kg

W·D. per stroke $= (P_1 - P_2)v$
$= (8-1\cdot1) \times 10^4 \times 1\cdot6 \times 0\cdot9 \times 0\cdot2445 = 57$ kcal

$\therefore$ % increase in W.D. $= \dfrac{57-51\cdot64}{51\cdot64} = 10\cdot39\%$ **Ans.**

Thermal efficiency $= \dfrac{57}{1\cdot6 \times 510\cdot3} = 6\cdot98$

$\therefore$ % decrease in thermal $\eta = \dfrac{10\cdot12 - 6\cdot98}{10\cdot12} = 31\%$ **Ans.**

5·6. Hypothetical diagram : hp ; mep with clearance.

A double-acting steam engine has a cylinder 19 cm bore by 30 cm stroke, and cut-off occurs at 0·35 stroke. The admission pressure is 5 kgf/cm² and the exhaust pressure is 0·2 kgf/cm². If the diagram factor is 0·65, find the indicated horse-power at 180 rpm, neglecting the effect of clearance and assuming hyperbolic expansion.

If, however, the clearance volume is 20 per cent of the swept volume, calculate the mean effective pressure, the cut-off remaining at the same point of the stroke as before.

Expansion ratio, $r = \dfrac{1}{0\cdot35} = 2\cdot857$

Hypothetical mep $= \dfrac{P_1}{r}(1 + \log_e r) - P_b$

$= 5 \times 0\cdot35(1 + \log_e 2\cdot857) - 0\cdot2 = 3\cdot39$ kgf/cm²

Actual m.e.p. $= 0\cdot65 \times 3\cdot39 = 2\cdot183$ kgf/cm²

i.h.p. $= \dfrac{p_m L A N}{75 \times 60}$

$= \dfrac{2\cdot183 \times 3\cdot0 \times \pi/4 \times 19^2 \times 2 \times 180}{10 \times 75 \times 60} = 14\cdot86$ **Ans.**

Clearance volume $= 0\cdot2\ V_s$ Total volume $= 1\cdot2\ V_s$

$\therefore$ Volume at cut-off $= 0\cdot2V_s + 0\cdot35V_s = 0\cdot55V_s$

Hypothetical work done/cycle

$$=P_1(0.55V_s-0.2V_s)+P_1\times 0.55V_s\log_e\frac{1.2V_s}{0.55V_s}-P_b\times(1.2V_s-0.2V_s)$$

$$=0.779V_sP_1-V_sP_b$$

$$\text{Hypothetical m.e.p.}=\frac{\text{Work done}}{\text{Swept volume}}$$

$$=\frac{0.779V_sP_1-V_sP_b}{V_s}$$

$$=0.779\times 5-0.2=3.695 \text{ kgf/cm}^2$$

∴ Actual m.e.p. $=0.65\times 3.695=2.4$ kgf/cm² Ans.

5.7. Cylinder dimensions considering clearance.

(a) *Explain the technique of taking an indicator diagram of a steam engine. How and why does this diagram differ from the theoretical and simplified diagram assumed for the sake of mathematical treatment? How the losses can be minimised?*

(b) *Find the dimensions of a single cylinder, non-condensing steam engine to satisfy the following requirements:*

b.h.p. = 50 ; steam chest pressure 11 kgf/cm² gauge, back pressure 1·1 kgf/cm² ; cut-off at ⅝th stroke ; clearance 5 per cent of stroke ; piston speed 125 metres per min ; r.p.m. 200 ; piston rod diameter 4.5 cm ; diagram factor 0.8 ; mechanical efficiency 90 per cent.

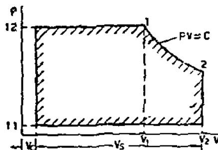


Fig. 5-24.

For theory—see text.

(b) $V_c=0.05\,V_s$

Expansion ratio, $r=\frac{V_2}{V_1}=\frac{V_s+0.05V_s}{0.625V_s+0.05V_s}=1.555$

$$\text{Hypothetical m.e.p.} = \frac{P_1V_1(1+\log_e r) - P_bV_2 - (P_1 - P_b)V_c}{V_s}$$

$$= \frac{1}{V_s}\,[12 \times 0{\cdot}675V_s(1+\log_e 1{\cdot}555) - 1{\cdot}1 \times 1{\cdot}05V_s$$
$$-(12-1{\cdot}1) \times 0{\cdot}05\, V_s]$$

$$= 10 \text{ kgf/cm}^2$$

Actual m.e.p. = Hyp. m.e.p. × D.F. = 10 × 0·8 = 8 kgf/cm²

$$\text{i.h.p.} = \frac{(\text{m.e.p.} \times \text{effective area}) \times \text{piston speed}}{75 \times 60}$$

$$\frac{50}{0{\cdot}9} = \frac{8 \times 10^4 \times A \times 125}{75 \times 60} \qquad \therefore \quad A = 0{\cdot}025 \text{ m}^2$$

In double acting engines

Cylinder area = Effective area + ½ Area of piston rod

$$\frac{\pi D^2}{4} = 0{\cdot}025 + \frac{1}{2} \times \frac{\pi}{4}\left(\frac{4{\cdot}5}{100}\right)^2$$

∴ Cylinder diameter,

$$D = 18{\cdot}4 \text{ cm} \qquad \text{Ans.}$$

$$\text{Stroke length, } L = \frac{125 \times 100}{2 \times 300} = 20{\cdot}8 \text{ cm} \qquad \text{Ans.}$$

Note. When i.h.p. is calculated as force × distance moved (piston speed per minute) it automatically takes into account that the engine is double-acting.

5·8. Total mass of steam : d.f. at cut-off and release ; Q.

A double acting steam engine uses 0·0507 kg of steam (corrected for leakage) per stroke. The cylinder diameter is 30 cm and stroke 45 cm. The clearance volume is 0·00269 m³. Compression begins at 70 per cent of the stroke, the pressure then being 1 kgf/cm² and the steam can be assumed dry at that point. Cut-off takes place at 15 per cent and release at 85 per cent of the working stroke, the corresponding pressure being 10·5 kgf/cm² and 3 kgf/cm². Estimate the total mass of the steam present during expansion and the dryness at cut-off and release.

Assuming that the expansion follows the law PV^n = constant find the heat passing through the cylinder walls per kg of steam during each expansion.

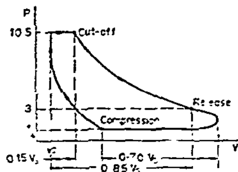


Fig 5-25.

Stroke volume, $V_s = \frac{\pi}{4}$ $(30)^2 \times 45 = 31{,}810$ c.c. or 0 03181 m³

Total volume $= 0\ 00265 + 0\ 03181 = 0\ 0345$ m³

Volume at 70% stroke $= 0\ 0345 - 0\ 7 \times 0\ 03181 = 0{\cdot}01223$ m³

As steam is dry and saturated when compression starts

Mass of cushion steam $= \dfrac{0\ 01223}{1\ 725} = 0\ 00709$ kg

Mass of steam during expansion $= 0\ 00709 + 0\ 0507$

$= 0\ 05779$ kg Ans.

Volume at cut-off $= 0{\cdot}00269 + 0{\cdot}15 \times 0{\cdot}03181 = 0\ 007462$ m³

d.f at cut-off $= \dfrac{\text{volume}}{mv_g} = \dfrac{0\ 007462}{0\ 05779 \times 0\ 189} = 0\ 683$ Ans.

Volume at release $= 0{\cdot}00269 + 0\ 85 \times 0\ 03181 = 0\ 02973$ m³

d f at release $= \dfrac{\text{volume}}{mv_g} = \dfrac{0\ 02973}{0\ 05779 \times 0\ 617} = 0\ 8337$ Ans.

For expansion curve, $PV^n = C$

$10{\cdot}5 \times (0\ 007462)^n = 3 \times (0\ 02973)^n$ $\therefore\ n = 0\ 906$

Work done/stroke $= \dfrac{P_1v_1 - P_2v_2}{n-1}$

$= \dfrac{10^4}{427}\left[\dfrac{10\ 5 \times 0\ 007462 - 3 \times 0\ 02973}{(0\ 906 - 1)}\right]$

$= 2{\cdot}704$ kcal

$\therefore$ Work done/kg of steam $= \dfrac{2\ 704}{0\ 05779} = 46\ 8$ kcal

Internal energy $u_1 = u_{f1} + x_1 u_{fg1}$

$$= 183{\cdot}2 + 0{\cdot}683(617{\cdot}2 - 183{\cdot}2) = 479{\cdot}6 \text{ kcal/kg}$$

Internal energy $u_2 = u_{f2} + x_2 u_{fg1}$

$$= 133{\cdot}3 + 0{\cdot}8337(607{\cdot}4 - 123{\cdot}3) = 528{\cdot}6 \text{ kcal/kg}$$

Heat passing through the cylinder walls,

$$q = 46{\cdot}8 + (528{\cdot}6 - 479{\cdot}6) = \underline{95{\cdot}8 \text{ kcal}}$$ Ans.

Note. For finding the work done 1 kg of steam work done per stroke is divided by the total mass of steam and not by fresh steam admitted per stroke.

5.9. Hyperbolic expansion and compression : b h p. ; indicated steam ; steam consumption.

The cylinder of a double-acting steam engine is 30 cm diameter and stroke 11 cm. The initial condition of steam is 6 kgf/cm². The steam is cut-off at 0·4 of stroke and compression starts from 0·8 return stroke, the clearance is 10 per cent of the volume swept by the piston in one stroke. The crankshaft speed is 150 revolutions per minute. Assuming hyperbolic expansion and compression, calculate (a) the ihp produced, (b) the mass of indicated steam per hour, (c) the steam consumption per i.h p. hour. Assume diagram factor 0·9.

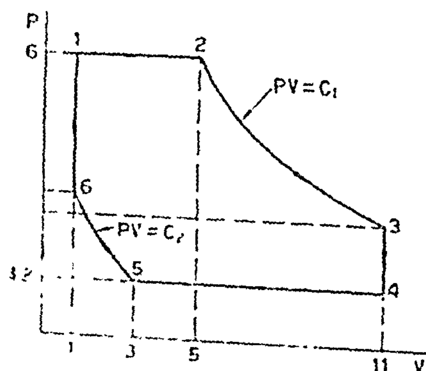


Fig. 5·26.

(*a*) Let the clearance volume be unity, $\therefore$ $V_s = 10$ and $V_4 = 11$

Volume at cut off, $V_2 = 1 + 0{\cdot}4 \times 10 = 5$

Volume at the start of compression $= 1 + 0{\cdot}2 \times 10 = 3$

Area of P-V diagram

$=P_1(V_2-V_1)+P_2V_2 \log_e r_1 - P_4(V_4-V_5)-P_5V_5 \log_e r_2$

$6(4)+6\times5 \log_e \frac{12}{5} - 1\cdot2(8) - 1\cdot2\times3 \log_e 3 = 34\cdot1$ units.

Hypothetical mep $=\frac{\text{Area of diagram}}{\text{Length of diagram}}=\frac{34\cdot1}{10}=3\cdot41$ kgf/cm²

Actual m e.p $=3\cdot41\times0\cdot9=3\cdot069$ kgf/cm²

$$\text{ihp}=\frac{P_mLAN}{75\times60}=\frac{3\cdot069\times\frac{44}{100}\times\frac{\pi}{4}(30)^2\times2\times150}{75\times60}=\underline{63\cdot65} \quad \text{Ans.}$$

$$\underline{(b)\ \text{Indicated steam per hr}}=\frac{0\cdot4V_s\times\text{number of strokes}}{0\cdot9\times\text{specific volume}}$$

$$=\frac{0\cdot4\times\{\pi/4(30)^2\times44\times10^{-6}\}\times(2\times150\times60)}{0\cdot9\times0\cdot3214}=\underline{775\cdot2\text{ kg}} \quad \text{Ans.}$$

$$\underline{(c)\ \text{Steam consumption per ihp hr}}=\frac{775\cdot2}{63\cdot65}=\underline{12\cdot18\text{ kg}} \quad \text{Ans.}$$

Note (i) This problem illustrates that it is not necessary to remember the complicated formula for indicator diagram considering compression, but the problem can be solved from the fundamentals

(ii) For calculating the m e p it is not necessary to calculate actual volumes It is simpler to solve by taking proportionate volumes.

5-10. Tractive effort and the train load of a locomotive

A steam locomotive has a two cylinder engine of 45 cm diameter by 60 cm stroke. The driving wheels are 206 cm diameter The steam supply is at 12 kgf/cm² dry and exhaust pressure is 1 3 kgf/cm² The maximum cut off is at 0 82 of the stroke, and the diagram factor for this condition is 0 78. Estimate the tractive effort at 8 km per hr with this maximum cut-off

At 80 km per hr the resistance amounts to 12 kg per 1,000 kg. Determine the total train load that can be haulded at this speed if the cut-off is then at 0 23 of the stroke and the diagram factor is 0 72.

$$\text{Actual m.e p. with 0 82 cut off}=\text{D.F.}\left\{\frac{P_1(1+\log_e r)}{r}-P_b\right\}$$

$$=0\cdot78\left[\frac{12\left(1+\log_e\frac{1}{0\cdot82}\right)}{\frac{1}{0\cdot82}}-1\cdot3\right]=8\cdot19\text{ kgf/cm}^2$$

Internal energy $u_1 = u_{f1} + x_1 u_{fg1}$

$$= 183 \cdot 2 + 0 \cdot 683(617 \cdot 2 - 183 \cdot 2) = 479 \cdot 6 \text{ kcal/kg}$$

Internal energy $u_2 = u_{f2} + x_2 u_{fg1}$

$$= 133 \cdot 3 + 0 \cdot 8337(607 \cdot 4 - 133 \cdot 3) = 528 \cdot 6 \text{ kcal/kg}$$

Heat passing through the cylinder walls,

$$q = 46 \cdot 8 + (528 \cdot 6 - 479 \cdot 6) = 95 \cdot 8 \text{ kcal} \qquad \text{Ans.}$$

Note. For finding the work done 1 kg of steam work done per stroke is divided by the total mass of steam and not by fresh steam admitted per stroke.

5·9. Hyperbolic expansion and compression : b.h p. ; indicated steam ; steam consumption.

The cylinder of a double-acting steam engine is 30 cm diameter and stroke 44 cm. The initial condition of steam is 6 kgf/cm². The steam is cut-off at 0·4 of stroke and compression starts from 0·8 return stroke, the clearance is 10 per cent of the volume swept by the piston in one stroke. The crank-shaft speed is 150 revolutions per minute. Assuming hyperbolic expansion and compression, calculate (a) the ihp produced, (b) the mass of indicated steam per hour, (c) the steam consumption per i.h p hour. Assume diagram factor 0·9.

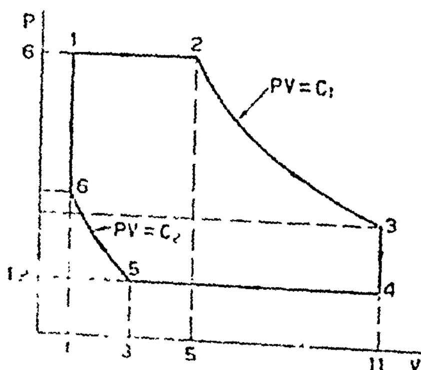


Fig. 5·26.

(a) Let the clearance volume be unity, $\therefore$ $V_4 = 10$ and $V_1 = 11$

Volume at cut off, $V_2 = 1 + 0 \cdot 4 \times 10 = 5$

Volume at the start of compression $= 1 + 0 \cdot 2 \times 10 = 3$

Area of P-V diagram

$=P_1(V_2-V_1)+P_2V_2\log_e r_1-P_3(V_4-V_5)-P_5V_5\log_e r_2$

$6(4)+6\times5\log_e\frac{11}{6}-1\cdot2(8)-1\cdot2\times3\log_e 3=34\cdot1$ units.

Hypothetical mep $=\dfrac{\text{Area of diagram}}{\text{Length of diagram}}=\dfrac{34\cdot1}{10}=3\cdot41$ kgf/cm²

Actual m e p $\quad=3\cdot41\times0\cdot9=3\cdot069$ kgf/cm²

$$\text{ihp}=\frac{P_m LAN}{75\times60}=\frac{3\cdot069\times\frac{44}{100}\times\frac{\pi}{4}(30)^2\times2\times150}{75\times60}=\underline{63\cdot65}\quad\text{Ans.}$$

(b) Indicated steam per hr. $=\dfrac{0\cdot4V_s\times\text{number of strokes}}{0\cdot9\times\text{specific volume}}$

$$=\frac{0\cdot4\times\{\pi/4(30)^2\times44\times10^{-6}\}\times(2\times150\times60)}{0\cdot9\times0\cdot3214}=\underline{775\cdot2\text{ kg}}\quad\text{Ans.}$$

(c) Steam consumption per ihp hr $=\dfrac{775\cdot2}{63\cdot65}=\underline{12\cdot18\text{ kg}}$ Ans.

Note. (*i*) This problem illustrates that it is not necessary to remember the complicated formula for indicator diagram considering compression, but the problem can be solved from the fundamentals

(*ii*) For calculating the m.e p it is not necessary to calculate actual volumes. It is simpler to solve by taking proportionate volumes.

5-10. Tractive effort and the train load of a locomotive.

A steam locomotive has a two-cylinder engine of 45 cm diameter by 60 cm stroke. The driving wheels are 200 cm diameter. The steam supply is at 12 kgf/cm² dry and exhaust pressure is 1 3 kgf/cm². The maximum cut-off is at 0 82 of the stroke, and the diagram factor for this condition is 0·78. Estimate the tractive effort at 8 km per hr with this maximum cut-off

At 80 km per hr the resistance amounts to 12 kg per 1,000 kg. Determine the total train load that can be haulded at this speed if the cut-off is then at 0 23 of the stroke and the diagram factor is 0 72.

Actual m.e p. with 0 82 cut off $=\text{D.F}\left\{\dfrac{P_1(1+\log_e r)}{r}-P_b\right\}$

$$=0\cdot78\left[\frac{12\left(1+\log_e\frac{1}{0\cdot82}\right)}{\frac{1}{0\cdot82}}-1\cdot3\right]=8\cdot19\text{ kgf/cm}^2$$

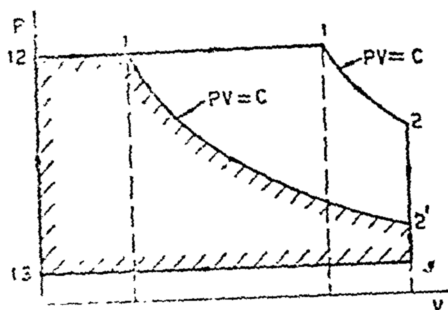


Fig. 5·27.

W.D. by driving wheels per revolution=W.D. on piston in 2 strokes.

Force × distance moved by wheels = Force on piston × length of 2 strokes

$$T\times\pi D=\left\{\left(\frac{\pi}{4}\,d^2\times P_m\right)\times 2l\right\}\times 2 \quad \text{[as there are two cylinders}$$

$$\therefore \quad T=\frac{P_m d^2 l}{D}$$

where T = tractive effort in kg, d = diameter of cylinder in cm

l = stroke of cylinder in cm, D = diameter of wheel in cm

$\therefore$ Tractive effort, $T=\dfrac{8\cdot19\times45\times45\times60}{206}$ = 4,830 kg Ans.

Actual m.e.p. with 0·23 cut-off = D.F. $\left\{\dfrac{P_1(1+\log_e r)}{r}-P_b\right\}$

$$=0\cdot72\left\{\frac{12\left(1+\log_e\dfrac{1}{0\cdot23}\right)}{\dfrac{1}{0\cdot23}}-1\cdot3\right\}=3\cdot972 \text{ kgf/cm}^2$$

$$\therefore \quad T=\frac{P_m d^2 L}{D}=\frac{3\cdot972\times45\times45\times60}{206}=2{,}343 \text{ kg}$$

Total train load $=\dfrac{2{,}343}{12}\times1{,}000$ = 1,95,300 kg Ans.

Note. The speed of the train in both cases is redundent for solving the problem. Speed will only be required for calculating the horse-power.

Assuming 100 per cent mechanical efficiency

$$\text{h.p in the first case}=\frac{4{,}830\times 8\times 1{,}000}{75\times 60\times 60}=143\cdot 1$$

$$\text{h.p. in the second case}=\frac{2{,}343\times 80\times 1{,}000}{75\times 60\times 60}=779\ 8$$

5.11. Jacketed cylinder ; expansion not hyperbolic ; comparison with the Rankine efficiency.

The steam supply to an engine with jacketed cylinders is at a pressure of 8 kgf/cm² and 0 9 dryness. Similar steam is used for the jackets and may be assumed to be just condensed there and then drained away without any reduction in temperature. The ratio of expansion for the engine is 14 and expansion follows the law $PV^{1\ 05}$=constant. The condenser pressure is 0·14 kgf/cm². Clearance and compression may be neglected.

Show that per kg of steam used in the cylinder, the work done is approximately the same as in a Rankine engine using similar steam between the same limits of pressure.

Compare the thermal efficiencies of the two engines, if 0 09 kg of steam is condensed in the jackets per kg of steam used in the cylinder.

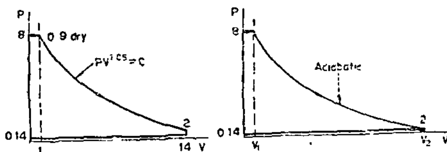


Fig. 5 28.

v_1 at 8 kgf/cm² and 0 9 dry$=0\ 9\times 0{\cdot}245=0\ 2205$ m³

$P_1v_1^n=P_2v_2^n$, $8\times 1^{1.05}=P_2\times 14^{1\ 05}$ $\quad\therefore\quad P_2=0\ 5$ kgf/cm²

Work done=area of the $P-v$ diagram

$$=\frac{1}{J}\left[P_1v_1+\frac{P_1v_1-P_2v_2}{n-1}-P_bv_2\right]$$

$$=\frac{10^4}{427}\left[8\times 0\ 2205+\frac{8\times 0\ 2205-0\ 5\times 14\times 0\ 2205}{1\ 05-1}-0\ 14\times 14\times 0\ 2205\right]$$

$$=134\ 6\ \text{kcal}$$

If the steam engine works on the Rankine cycle, process 1—2 will be adiabatic

Entropy at 1 = Entropy at 2

$0{\cdot}487+0{\cdot}9\times1{\cdot}106=0{\cdot}175+x_2\times1{\cdot}745 \quad \therefore \quad x_2=0{\cdot}7491$

The Rankine work $=h_1-h_2=(h_{f1}+x_1h_{fg1})-(h_{f2}-x_2h_{fg2})$

$=(171{\cdot}4+0{\cdot}9\times489{\cdot}8)-(52{\cdot}2+0{\cdot}7491\times567{\cdot}7)=\underline{134{\cdot}8}$ kcal

It is seen that the work done in both the cases is approximately same

In the case of jacketed engine

Heat supplied per kg of steam

$=(h_1-h_{f2})+mx(h_{fg1})$

$=(171{\cdot}4+0{\cdot}9\times489{\cdot}8-52{\cdot}2)+0{\cdot}09\times0{\cdot}9\times489{\cdot}8=599{\cdot}7$ kcal Ans.

$\therefore$ Efficiency $=\dfrac{134{\cdot}6}{599{\cdot}7}=\underline{22{\cdot}44\%}$ Ans.

Note. In the first case standard formula for work done in a steam engine cannot be applied as the expansion is not hyperbolic.

5.12. d f and missing quantity ; reevaporation.

Explain what is meant by the term missing quantity as applied to a steam engine and why is it caused ?

In the H P. cylinder of a steam engine the following particulars were obtained : cut-off ⅓rd stroke ; at a point on compression curve the pressure was found 4·1 kgf/cm² and indicated volume 0·115 m³ ; at a point on expansion line immediately after cut-off the pressure was 10·5 kgf/cm² ; pressure at ¾th stroke on expansion curve 5·6 kgf/cm² ; pressure at release 4·9 kgf/cm² ; indicated volume at release 0·5 m³ ; cylinder diameter 70 cm and stroke 131 cm ; clearance volume (1/14)th of stroke volume : dry steam admitted 1·18 kg/stroke ; working strokes 150 per minute.

Estimate the dryness fraction and the missing quantity in kg per hr (a) at cut-off (b) at ¾th stroke, and (c) at release.

Also calculate the percentage reevaporation during expansion.

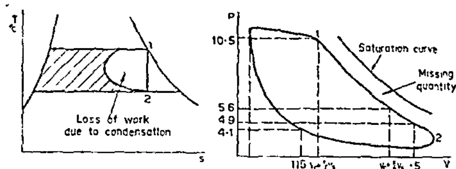


Fig. 5·29

From steam tables

v_g at 4 1 kgf/cm² = 0 46 m³, v_g at 10 5 kgf/cm² = 0 189 m³

v_g at 5 6 kgf/cm² = 0 343 m³, v_g at 4 9 kgf/cm² = 0 389 m³

Stroke volume $= \frac{\pi}{4} \times \left(\frac{70}{100}\right)^2 \times 1\ 31 = 0\ 5042$ m³

Clearance volume $= \frac{0\ 5042}{14} = 0{\cdot}036$ m³

$\therefore$ Total volume of cylinder = 0 5042 + 0 036 = 0 5402 m³

Volume at cut-off $= \frac{0\ 5042}{3} + 0\ 036 = 0\ 2041$ m³

Volume at $\frac{3}{4}$th expansion curve $= 0\ 5042 \times \frac{3}{4} + 0\ 036 = 0\ 4142$ m³

The mass of cushion steam in clearance volume is equal to the mass of 0 115 m³ steam at 4 1 kgf/cm² $= \frac{0\ 115}{0\ 46} = 0\ 25$ kg

$\therefore$ Total mass of steam during expansion = 1 18 + 0 25 = 1 43 kg

(a) Indicated dry mass of steam near cut off

$$= \frac{0\ 2041}{0\ 189} = 1{\cdot}08 \text{ kg} \qquad \text{Ans.}$$

$\therefore$ Dryness fraction at cut-off $= \frac{1\ 08}{1\ 43} = 0\ 7551$ Ans.

Missing quantity per hour = (1 43 − 1 08) × 150 × 60

= 3,150 kg

If the steam engine works on the Rankine cycle, process 1—2 will be adiabatic

$$\text{Entropy at } 1 = \text{Entropy at } 2$$

$$0{\cdot}487 + 0{\cdot}9 \times 1{\cdot}106 = 0{\cdot}175 + x_2 \times 1{\cdot}745 \qquad \therefore \quad x_2 = 0{\cdot}7491$$

$$\text{The Rankine work} = h_1 - h_2 = (h_{f1} + x_1 h_{fg1}) - (h_{f2} - x_2 h_{fg2})$$

$$= (171{\cdot}4 + 0{\cdot}9 \times 489{\cdot}8) - (52{\cdot}2 + 0{\cdot}7491 \times 567{\cdot}7) = \underline{134{\cdot}8} \text{ kcal}$$

It is seen that the work done in both the cases is approximately same

In the case of jacketed engine

Heat supplied per kg of steam

$$= (h_1 - h_{f2}) + mx(h_{fg1})$$

$$= (171{\cdot}4 + 0{\cdot}9 \times 489{\cdot}8 - 52{\cdot}2) + 0{\cdot}09 \times 0{\cdot}9 \times 489{\cdot}8 = 599{\cdot}7 \text{ kcal} \qquad \text{Ans.}$$

$$\therefore \quad \text{Efficiency} = \frac{134{\cdot}6}{599{\cdot}7} = \underline{22{\cdot}44\%} \qquad \text{Ans.}$$

Note. In the first case standard formula for work done in a steam engine cannot be applied as the expansion is not hyperbolic.

5.12. d f. and missing quantity ; reevaporation.

Explain what is meant by the term missing quantity as applied to a steam engine and why is it caused ?

In the H P. cylinder of a steam engine the following particulars were obtained : cut-off $\frac{1}{3}$rd stroke ; at a point on compression curve the pressure was found 4·1 kgf/cm² and indicated volume 0·115 m³ ; at a point on expansion line immediately after cut-off the pressure was 10·5 kgf/cm² ; pressure at $\frac{2}{3}$th stroke on expansion curve 5·6 kgf/cm² ; pressure at release 4·9 kgf/cm² ; indicated volume at release 0·5 m³ ; cylinder diameter 70 cm and stroke 131 cm ; clearance volume ($\frac{1}{12}$)th of stroke volume : dry steam admitted 1·18 kg/stroke ; working strokes 150 per minute.

Estimate the dryness fraction and the missing quantity in kg per hr (a) at cut-off (b) at $\frac{2}{3}$th stroke, and (c) at release.

Also calculate the percentage reevaporation during expansion.

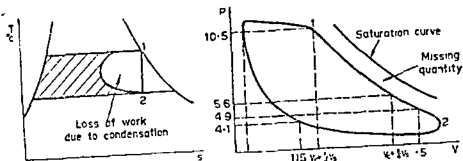


Fig. 5 29

From steam tables

v_g at 4·1 kgf/cm²=0 46 m³, v_g at 10 5 kgf/cm²=0 189 m³

v_g at 5 6 kgf/cm²=0 343 m³, v_g at 4 9 kgf/cm²=0 389 m³

$$\text{Stroke volume} = \frac{\pi}{4} \times \left(\frac{70}{100}\right)^2 \times 1\ 31 = 0\ 5042 \text{ m}^3$$

$$\text{Clearance volume} = \frac{0\ 5042}{14} = 0\ 036 \text{ m}^3$$

$\therefore$ Total volume of cylinder=0 5042+0 036=0 5402 m³

$$\text{Volume at cut-off} = \frac{0\ 5042}{3} + 0\ 036 = 0\ 2041 \text{ m}^3$$

Volume at $\frac{3}{4}$th expansion curve$=0\ 5042 \times \frac{3}{4} + 0\ 036 = 0\ 4142$ m³

The mass of cushion steam in clearance volume is equal to the mass of 0 115 m³ steam at 4 1 kgf/cm²$=\frac{0\ 115}{0\ 46}=0\ 25$ kg

$\therefore$ Total mass of steam during expansion=1 18+0 25=1 43 kg

(*a*) Indicated dry mass of steam near cut off

$$= \frac{0\ 2041}{0\ 189} = 1\ 08 \text{ kg} \qquad \text{Ans.}$$

$\therefore$ Dryness fraction at cut-off$=\frac{1\ 08}{1{\cdot}43}=0\ 7551$ Ans.

Missing quantity per hour=(1·43−1 08)×150×60

=3,150 kg Ans.

(b) Indicated dry mass of steam at $\frac{3}{4}$th stroke

$$=\frac{0.4142}{0.343}=1.208 \text{ kg}$$

$\therefore$ Dryness fraction at $\frac{3}{4}$th stroke $=\frac{1.208}{1.43}=0.8448$ Ans.

Missing quantity per hour $=(1.43-1.208)\times 150\times 60$

$=1{,}998$ kg Ans.

(c) Indicated dry mass of steam at release

$$=\frac{500\times 10^{-3}}{0.389}=1.285 \text{ kg}$$

$\therefore$ Dryness fraction at release $=\frac{1.285}{1.43}=0.8986$ Ans.

Missing quantity per hour $=(1.43-1.285)\times 150\times 60$

$=1{,}315$ kg Ans.

Percentage re evaporation

$$=\frac{\text{Missing quantity at cut-off}-\text{Missing quantity of release}}{\text{Missing quantity at cut-off}}$$

$$=\frac{3{,}150-1{,}315}{3{,}150}=58.26\%$$ Ans.

Note. The problem shows that during the later half of the stroke 58·26 per cent of the condensed steam reevaporates due to the cylinder walls becoming hotter than the steam itself. In compound steam engine this re evaporated steam does useful work in the L.P. cylinder.

5.13. Willan's line and efficiency ratio.

State the methods of governing a simple steam engine. Explain what is meant by the "Willan's Line".

The test results from a turbine governed by throttling can be represented by a line whose equation is $G=0.00096N+0.53$, where G is the kg of steam per second and N is the indicated horse-power. The maximum power is 5,000 i.h.p., and the conditions of steam supply at this load are 12·5 kgf/cm² and 240°C. Throttling is at constant heat and the condenser pressure is 0·07 kgf/cm². Assuming that

the steam pressure varies directly as the steam flow, calculate the efficiency ratio of the turbine at one-third indicated horse power.

If at no load the turbine develops 400 i h p., find the steam consumption per hour at no load and mechanical efficiency at half load, assuming friction to remain constant.

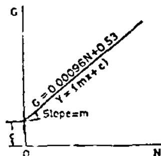


Fig 5 30

For theory—see text.

When h p is 5,000, $G = 0\ 00096 \times 5{,}000 + 0{\cdot}53 = 5\ 33$ kg/sec

When h p is $\frac{5{,}000}{3}$, $G = 0\ 00096 \times \frac{5{,}000}{3} + 0{\cdot}53 = 2\ 13$ kg/sec

Steam pressure varies directly as the steam flow

$\therefore$ New pressure $= \frac{12\ 5 \times 2{\cdot}13}{5\ 33} = 5$ kgf/cm²

Total heat before throttling (from steam tables) at 12 5 kgf/cm² and 240°C = 694·9 kcal

From steam tables (by interpolation) at 5 kgf/cm² and 694 9 kcal enthalpy, entropy = 1·714

Entropy before expansion = Entropy after expansion

$$1\ 714 = 0{\cdot}132 + x_2 \times 1\ 846 \qquad \therefore\ x_2 = 0\ 8572$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 38\ 7 + 0{\cdot}8572 \times 575{\cdot}4 = 532 \text{ kcal}$$

Rankine work/kg of steam = 694·9 − 532 = 162 9 kcal

$\therefore$ Rankine work/sec = 162 9 × 2 13 = 346 9 kcal

Actual work/sec $= \frac{5000}{3} \times \frac{75}{427} = 292\ 7$ kcal

$\underline{\eta \text{ ratio}} = \frac{\text{Indicated work done}}{\text{Rankine work}} = \frac{292\ 7}{346\ 9} = \underline{84{\cdot}4\%}$ Ans.

At 400 i.h.p., $G = 0.00096 \times 400 + 0.53 = 0.914$ kg/sec

Steam consumption/hour at no load

$$= 0.914 \times 60 \times 60 = \underline{3,290 \text{ kg}} \qquad \text{Ans.}$$

As friction is constant, friction h.p. at $\frac{1}{3}$ indicated load $= 400$

$\therefore$ At $\frac{1}{3}$ indicated load, b.h.p. $= \frac{5,000}{3} - 400 = 1,267$ h.p.

$\therefore$ Mechanical efficiency $= \frac{\text{b.h.p.}}{\text{i.h.p.}} = \frac{1,267}{1,667} = \underline{76\%}$ Ans.

5-14. Steam engine trial : mechanical and thermal η.

A small single-cylinder double-acting steam engine has a bore of 15 cm and a stroke of 23 cm. The steam is supplied at a pressure of 5 kgf/cm² at a temperature of 170°C. The length of indicator card is 6·8 cm and with a spring of 2·4 kgf/cm² per cm the area of the card obtained is 6·4 cm². The condenser pressure is 0·24 kgf/cm² and the temperature of the condensate is 43°C. The net brake load on the engine is 35·6 kg at a radius of 70 cm, the speed is 220 revolutions per minute and the steam consumption is 110 kg per hour.

(a) *Find the mechanical and thermal efficiencies.*

(b) *Estimate the mass of condenser cooling water per kg of steam if the rise in temperature is 20°C. Assume that there is no heat transfer to the environment.*

(a) $P_m = \frac{6 \cdot 4 \times 2 \cdot 4}{6 \cdot 8} = 2 \cdot 259$ kgf/cm²

$$\text{Heat equivalent to ihp} = \frac{2 \cdot 259 \times \left(\frac{23}{100}\right) \times \left(\frac{\pi}{4} \times 15^2\right) \times 2 \times 220}{60 \times 427}$$

$$= 1 \cdot 577 \text{ kcal/sec}$$

$$\text{Heat equivalent to bhp} = \frac{2\pi \times \left(\frac{70}{100}\right) \times 220 \times 35 \cdot 6}{60 \times 427}$$

$$= 1 \cdot 344 \text{ kcal/sec}$$

Mechanical efficiency $= \frac{1 \cdot 344}{1 \cdot 577} = \underline{85 \cdot 25\%}$ Ans.

Mass of steam supplied $= \frac{110}{60 \times 60} = 0.03056$ kg/sec

Heat supplied $= 0.03056(666.6 - 43) = 19.05$ kcal/sec

Indicated thermal efficiency $= \frac{1.577}{19.05} = 8.27\%$

Brake thermal efficiency $= \frac{1.344}{19.05} = 7.05\%$

(b) Heat lost to cooling water in condenser

$= 19.05 - 1.577 = 17.473$ kcal/sec.

$\therefore$ Mass of condenser cooling water per kg of steam

$m = \frac{17.473}{20} \times \frac{60 \times 60}{110} = 28.6$ kg   Ans.

5.13. Test on a simple steam engine.

What are the desirable properties of a fluid for use as working substance in a heat engine plant?

The following are the mean values of the observations made on a single cylinder double-acting steam engine when running on light load Duration of trial 20 minutes. Barometer 76 cm of Hg. Total steam used 51 kg. Gauge pressure of steam in valve chest 2 kgf/cm² The engine rejected exhaust steam into a condenser at the pressure of atmospheric which is 1.033 kgf/cm². The state of the steam in the valve chest was found to be dry and saturated. Mean engine speed 120 rpm Mean effective pressure out stroke 1.06 kgf/cm², instroke 0.984 kgf/cm². Piston diameter 21.5 cm. Stroke 30.8 cm. Piston rod diameter 3.7 cm. Net brake load (W−S) 50.5 kg. Brake wheel radius+rope radius, 59.5 cm. Cooling water flowed through the condenser at the rate of 2,850 kg/hour with a rise in temperature of 24°C Temperature of the condensate leaving the condenser 68°C. Calculate (a) the consumption in kg per ihp hr and kg per bhp hr (b) the mechanical efficiency, and (c) the brake thermal efficiency of the engine.

Draw up a heat balance sheet for the plant on minute basis taking (i) 0°C as datum, and (ii) condensate temperature as datum Sketch typical performance curves.

For theorey—see text.

(a) Steam consumption per minute$=\frac{51}{20}=2{\cdot}55$ kg

Outstroke ihp (cover side) $=\frac{1{\cdot}06\times 30{\cdot}8\times\left(\frac{\pi}{4}\times 21{\cdot}5^2\times 120\right)}{1{,}00\times 75\times 60}$

$=3{\cdot}161$

Instroke ihp (crank end) $=\frac{0{\cdot}984\times 30{\cdot}8\times \pi/4(21{\cdot}5^2-3{\cdot}7^2)\times 120}{100\times 75\times 60}$

$=2{\cdot}847$ Ans.

$\therefore$ Total ihp of the engine$=3{\cdot}161+2{\cdot}847=6{\cdot}008$ hp

Consumption of steam per ihp-hr$=\frac{2{\cdot}55\times 60}{6{\cdot}008}=25{\cdot}46$ kg

$$\text{bhp}=\frac{2\pi r_0(W-S)N}{75\times 60}=\frac{2\pi\times 59{\cdot}5\times 50{\cdot}5\times 120}{100\times 75\times 60}=5{\cdot}032$$

Consumption of steam per bhp hr$=\frac{2{\cdot}55\times 60}{5{\cdot}032}=30{\cdot}4$ kg Ans.

(b) Mechanical $\eta=\frac{\text{bhp}}{\text{ihp}}=\frac{5{\cdot}032}{6{\cdot}008}=83{\cdot}75\%$ Ans.

Brake thermal $\eta=\frac{\text{Heat equivalent to per kg of steam}}{(H_1-h_2)}$

$$=\frac{75\times 60\times 60}{427\times 30{\cdot}4(650{\cdot}7-68)}=3{\cdot}571\%$$ Ans.

Heat in steam/minute$=650{\cdot}7\times 2{\cdot}55=1{,}659$ kcal

Heat equivalent to ihp/min$=\frac{6{\cdot}008\times 75\times 60}{427}=63{\cdot}33$ kcal

Heat equivalent to bhp/min$=\frac{5{\cdot}032\times 75\times 60}{427}=53{\cdot}04$ kcal

Heat in condenser coolant per min$=\frac{2{,}850}{60}\times 24=1{,}140$ kcal

Heat in hot well/min$=2{\cdot}55-68=173{\cdot}4$ kcal

Heat balance sheet on one minute basis taking datum 0°C

Input	*kcal*	%	*Output*	*kcal*	%
Heat supply in steam	1,659	100	ihp fhp=ihp−bhp	63 33 10 29	3 818 0 62
			1. bhp	53 04	3·193
			2. Condenser coolant	1,140 00	68 73
			3. Hot well	173 40	10·54
			4. Radiation friction, etc., by difference	292 56	17 630
Total	1,659	100	by difference	1,659 00	100·008

(*ii*) Heat balance sheet on one minute basis taking condensate temperature as datum :—

Heat supplied by fuel in boiler to steam per minute

$$=2{\cdot}55\ (650\ 7-68)=1{,}486\ \text{kcal}$$

Heat balance sheet taking condensate temperature as datum :

Input	*kcal*	%	*Output*	*kcal*	%
Heat in steam	1,486	100	ihp	63 33	4 262
			bhp=ihp−bhp	10 29	0 692
			1. bhp	53 04	3 57
			2. Condenser coolant	1,140 00	76 72
			3. Radiation, friction etc, by diff.	292 16	19 71

exhaust pressure is 0·14 kgf/cm² and temperature of feed water is 50°C. Find (a) the Rankine cycle efficiency and its equivalent m.e.p., and (b) the efficiency ratio.

What is the Carnot efficiency for the above temperature range?

[h_1=767 kcal ; $s_{sup.}$=1·722 ; x_2=0·8865 ; h_2=555·4 kcal ; Rankine η=29·6% ; thermal η=16·18% ; efficiency ratio=54·7% ; Carnot η=30 2%].

5·2. The Rankine cycle : volume of steam entering condenser ; cooling water ; pipe diameter.

Sketch a schematic diagram of a condensing steam power plant.

An engine working on the Rankine cycle uses steam at 12 kgf/cm², 0·95 dry. The condenser vacuum is 583 mm of Hg (barometer 760 mm). Find (a) the Rankine efficiency, (b) the amount of condenser cooling water required per kg of steam, given that the temperature rise of cooling water is limited to 10°C, (c) the volume of steam entering the condenser per second if steam used in the cycle is 4,000 kg. per hr, and (d) the diameter of entry pipe to the condenser, if the velocity of steam is 25 m sec.

[x_2=0 78 ; Rankine efficiency=24·24% ; cooling water per kg of steam=43·8 kg ; volume of steam=5·683 m³/sec ; diameter of pipe=53·8 cm].

5·3. The Rankine cycle : overall efficiency ratio for engine and turbine.

Explain with the help of a T-s sketch why the Rankine cycle is preferable to the Carnot cycle as an ideal for comparison of performance of steam plants. How is the Rankine cycle modified for operation of reciprocating steam engine plants and why?

A reciprocating engine receives 8,000 kg of steam per hour at 20 kgf/cm² and 280°C and exhausts at 0·3 kgf/cm². It develops 1,550 i.h.p. The exhaust steam suffers a throttling drop of 0·04 kgf/cm² in passing from the engine to an exhaust turbine. The turbine has an efficiency ratio of 0·62 and it exhausts into a condenser at 0·035 kgf/cm². Find the condition of steam while (a) leaving the engine, (b) entering the turbine, and (c) entering the condenser.

Find also the h p. developed by the turbine and the overall efficiency ratio for the engine and turbine based on Rankine cycle.

[$s_1=s_2$. ∴ $x_2=0\cdot841$; $h_2=h_3$ ∴ $x_3=0\cdot8437$; $s_3=s_4$ ∴ $x_4=0\cdot7805$; i.h.p. of turbine=479 ; total wd=149 3 kcal/kg ; overall η ratio= $\dfrac{(h_1-h_2)+(h_3-h_4)}{\text{Rankine work}}=64\%$].

5.4. The modified Rankine cycle : feed pump work.

Steam is supplied to an engine at 20 kgf/cm² and 340°C. Adiabatic expansion takes place to the release point at 1 5 kgf/cm², after which there is a sudden drop of pressure at constant volume to the exhaust at 0 07 kgf/cm². Find (a) the work done per kg of steam supplied, (b) the modified Rankine cycle efficiency (c) the internal energy per kg of steam at release, (d) the work in kgf m by the extraction and boiler feed pumps per kg of water returned to the boiler, and (e) Rankine efficiency considering feed pump work.

[$x_2=0\cdot9465$; $h_2=614\cdot6$ kcal ; work done=166 2 kcal ; modified Rankine $\eta=23\cdot6\%$; $u_2=575\cdot3$ kcal/kg ; work done by pumps =200 6 kgf m ; Rankine $\eta=23\cdot5\%$].

5 5. Thermal efficiency : power developed different in two ends of the cylinder.

Assuming steam expands hyperbolically in the cylinder, estimate the i h.p. of an engine running at 150 rpm and receiving steam at 6·5 kgf/cm² pressure and exhausting at 0 28 kgf/cm² pressure, with a ratio of expansion of 10 at the back end of the cylinder and 8 at the front end. The diameter of piston and piston rod are 50 cm and 6·4 cm respectively and the length of stroke is 60 cm. What is the thermal efficiency of the engine if supplied with 6·8 kg of steam per i h p per hour of dryness 0·85 from feed water at 35°C ?

[i.h p. back end=72 1, front end=87·5, total i h p =159 6 ; thermal efficiency=19%].

5.6. Locomotive engine : cylinder diameter ; i h p, given tractive force.

The steam supplied by the boiler of a two cylinder locomotive is dry and saturated at 14 kgf/cm². The driving wheels are 200 cm diameter. Calculate the diameter of the two cylinders which have 7

stroke, so that the tractive force at 32 km/hr may be 4,500 kg when the cut-off is at 0 5 stroke.

Assume a mechanical efficiency of 80% and take diagram factor of 0·65 on a diagram which, neglects clearance, has hyperbolic expansion, and exhausts at 1·25 kgf/cm². What is the indicated horse-power at this speed ?

[$P_m = 10{\cdot}6 \times 0{\cdot}65 = 6{\cdot}89$ kgf/cm² ; $T = \frac{d^2 P_m l}{D} \times$ mech. η ; $d = 48{\cdot}6$ cm ; i h.p. = 675·6].

5·7. Non expansive working.

A double-acting steam engine develops 60 bhp at 110 rpm. The steam supply pressure is 8·4 kgf/cm² exhaust pressure 1·1 kgf/cm² and cut-off takes place at ⅓ of the stroke. Determine (a) the necessary cylinder dimensions taking $l = 1{\cdot}7\ d$, and (b) the brake thermal efficiency if the steam is taken in for whole of the stroke, the supply being dry saturated but the steam in the cylinder being 0·96 dry at the end of admission.

In both cases assume a diagram factor 0·72 and mechanical efficiency 85%.

[$P_m = 4{\cdot}776$ kgf/cm² ; $d = 31{\cdot}6$ cm ; $l = 53{\cdot}7$ cm ; work done $= 4{\cdot}4$ kcal per stroke ; $\eta = 4{\cdot}2\%$]

5·8. Indicated steam : missing quantity at cut-off and release ; percentage re-evaporation.

A double acting steam engine has a mean stroke volume for the two ends of the cylinder of 7,300 c c. At 150 rpm the steam consumption is 270 kg/hr. The mean diagram shows the following values when steam is supplied dry saturated ; length 7·8 cm, clearance length 0·86 cm. At cut-off P = 6 kgf/cm², length on card to zero volume 3 8 cm. At release P = 3 kgf/cm², length on card to zero volume 8·3 cm. At start of compression P = 1·05 kgf/cm², length of card to zero volume 1·3 cm.

Calculate the missing quantities at cut-off and release and percentage re-evaporation.

[Total steam = 0·01574 kg ; missing quantity = 0·00466, 0 00315 kg ; re-evaporation = 32·4%].

5.9. Throttle and cut-off governing.

The supply pressure of steam to a speed governed engine is 12·5 kgf/cm² and exhausts at 1 05 kgf/cm². On full load the cut-off is 45%. It may be assumed that the clearance volume is negligible and that the expansion of steam after cut-off is hyperbolic at all times. For half load conditions (a) determine the pressure in the steam chest if throttle governing is used with cut-off constant at 45%, and (b) verify that percentage cut-off is 15 65, if cut-off governing is used.

[full load mep=9 07 kgf/cm²; $\underline{P=6\ 9\ \text{kgf/cm}^2}$; mep with 0·1565 cut-off=4·535 kgf/cm², which is nearly half load mep]

5.10. Willan's law: Rankine and indicated thermal efficiencies.

What are the methods of governing a simple steam engine?

A throttle-governed steam engine running at constant speed consumes 320 kg of steam/hour when developing 21 ihp, and 565 kg/hr at 43·1 ihp. The steam supply is at 14 kgf/cm² dry saturated with a condenser pressure of 0 28 kgf/cm². Using steam tables only to obtain all steam properties and showing all calculations, determine for a output of 30 ihp and the same speed and steam condition the Rankine cycle efficiency and the indicated thermal efficiency.

[steam for 30 i.h p.=419 8 kg; x=0 8081 ; $\underline{\text{Rankine } \eta=24{\cdot}6\%}$; $\underline{\text{thermal } \eta=7{\cdot}544\%}$].

5.11. Test on a simple steam engine.

During a trial of a single cylinder steam engine the following data were obtained.

Cylinder diameter=20 cm, Stroke=1 25 d
Speed=300 rpm., Radius of brake drum=37 cm
Diameter of the rope=2·5 cm
Admission pressure=7 kgf/cm² gauge
Condenser vacuum=68 4 cm of Hg
Atmosphere=77·7 cm of Hg
Dryness fraction of steam=0·97
Brake Load=145 kg, Spring balance reading=9 5 kg
Area of indicator card=17·3 cm²
Length of the indicator card=10 cm

Spring strength $= 1{\cdot}4$ *kgf/cm²/cm*
Duration of trial $= 20$ *minutes*
Steam consumption $= 70{\cdot}6$ *kg*
Condenser cooling water $= 1{,}800$ *kg*
Temperature rise of condenser cooling water $= 15°C$
Temperature of condensate $= 38°C$

Find (a) bhp, (b) ihp, (c) mechanical efficiency, (d) indicated thermal efficiency (e) steam consumption in kg per bhp-hr., and (f) draw up a heat balance sheet on the minute basis taking condensate temperature as datum.

[bhp $= 21{\cdot}72$; ihp $= 25{\cdot}37$; mechanical $\eta = 85{\cdot}6\%$; indicated thermal $\eta = 12{\cdot}15\%$; steam consumption $= 9{\cdot}752$ kg/bhp-hr. ; heat in steam $= 2{,}200$ kcal (100%) ; heat in bhp $= 229$ kcal (10·40%) ; heat in condensate coolant $= 1{,}350$ kcal (61·37%) ; radiation, friction, etc., (by difference) $= 621$ kcal (28·23%)].

6

Compound Steam Engines

6·1. Introduction. A compound steam engine is one in which expansion of steam takes place in two or more than two cylinders in series. In such an engine part of the pressure drop takes place in first cylinder and the exhaust of first cylinder is admitted to the next cylinder where further expansion takes place and the exhaust of second cylinder is admitted to third and so on. The main advantage of compounding is reduction in condensation due to the following reasons :

(*i*) Small pressure range and consequently small temperature range in each cylinder.

(*ii*) In a steam engine, steam condenses on admission due to cylinder walls being cooler than the incoming steam, but the condensed steam partly re-evaporates during the latter part of the stroke as the cylinder walls then become hotter than the steam In a simple steam engine this heat recovered is lost immediately into exhaust

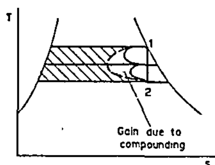


Fig. 6·1. Gain due to compounding

whereas in a compound engine this is utilised in the second cylinder. Thus the loss of condensation is restricted to low pressure cylinder only as shown in Fig. 6·1.

The other advantages of compounding are reduced stroke length, less weight, less first cost and more uniform torque resulting in lighter flywheel. Due to better mechanical balance it is less noisy and can be designed for higher speeds. It is easier to start in any extreme position. Due to less pressure differences across pistons there is reduced steam leakage and reduced forces on working parts. Reheating of steam between stages is also possible. Further, in compound engine if by chance one cylinder fails the work does not suffer totally.

Compounding increases the steam economy at rated load by 10 to 25 per cent for non-condensing and 15 to 40 per cent for, condensing operation. At fractional load economy is lesser. At light loads, sometimes it may consume even more steam. The saving of steam shown by a compound engine over a simple engine is greater at higher boiler pressures.

Compound engines are made in the range of 50 to 4,000 horse-power.

6·2. Combined Indicator Diagram. The indicator diagrams of H.P. and L.P. cylinders taken during a test of a two-cylinders compound engine have different volume scale and spring strength. The mass of the cushion steam is also different in the two cases. The two diagrams may be combined into one diagram as follows :

(*i*) Obtain the average indicator diagrams for both sides of the H.P. cylinder and both sides of the L.P. cylinder.

(*ii*) Reduce the diagrams to same pressure and volume scale.

(*iii*) If the diagram is to be plotted with clearance, set off the clearance volume from the pressure axis and then plot the average diagrams, using the corresponding clearance axis.

Plot the saturation curve ; this does not form a continuous curve because of the different masses of clearance steam in the H.P. and the L.P. cylinders and also because of condensation in the receiver. The small percentage clearance of the L.P. cylinder together with the receiver loss causes the L.P. saturation curve to fall inside the H.P. curve.

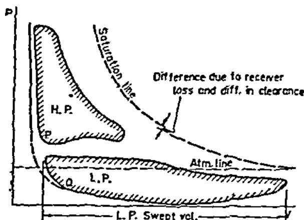


Fig. 6·2. Combined diagram with cushion steam.

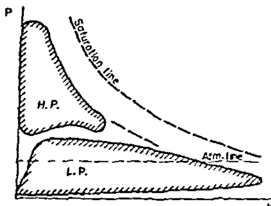


Fig. 6·3. Combined diagram without cushion steam.

(iv) *If it is desired to compare the actual indicator diagrams* with the hypothetical diagram having common expansion curve, cushion steam is to be neglected. It may be done by taking points P and Q on H P. and L P diagrams respectively (*see* Fig. 6·2) where *steam is supposed to be dry and saturated and draw compression* curve, assuming compression to be hyperbolic, and take the volumes from the compression curves. The combined diagram without cushion steam is shown in Fig 6·3.

6·3. Calculations of Cylinder Dimensions The hypothetical indicator diagram for all cylinders of a compound engine is drawn on a common scale neglecting the clearance volume, so that

the expansion curve is a continuous uniform curve as shown in Fig. 6·4.

Let P_1 = Inlet pressure of steam in H.P. cylinder.

P_2 = Release „ „ „ „

P_3 = Back „ „ „ „

= Inlet pressure of steam in L.P. cylinder

P_4 = Release „ „ „ „

P_b = Back „ „ „ „

V_1 = Volume at cut-off in H.P. cylinder

V_2 = Volume of H.P. cylinder

V_3 = Volume at cut-off in L.P. cylinder

V_4 = Volume of L.P. cylinder

r_1 = Ratio of expansion in H.P. cylinder $= \frac{V_2}{V_1}$

r_2 = Ratio of expansion in L.P. cylinder $= \frac{V_4}{V_3}$

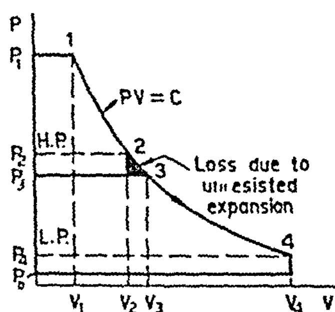


Fig. 6·4. Hypothetical indicator diagram of compound engine (neglecting clearance)

The cylinder dimensions of a compound engine may be designed on the following basis :

(i) Equal temperature drop in each cylinder for economy of steam.

(ii) Equal power developed in each cylinder for uniform turning moment.

(iii) Equal initial load on all pistons for obtaining same size of piston rod, connecting rod, etc., for all cylinders.

A compromise is generally struck between (*ii*) and (*iii*), which causes loss of work done due to unresisted expansion *i e.* expansion at constant volume during release as shown in Fig. 6·8.

Let K_1 and K_2 be the diagram factors for H.P. and L.P. cylinders respectively.

Actual indicated work in H.P. cylinder per stroke

$$=[P_1V_1(1+\log_e r_1)-P_2V_2]\times k_1 \qquad [6\cdot1(a)]$$

Actual indicated work in L.P. cylinder per stroke

$$=[P_3V_3(1+\log_e r_2)-P_4V_4]k_2 \qquad [6\cdot1(b)]$$

The value of diagram factor for H.P. and L.P. cylinders may be same or different.

Loss due to unresisted expansion

=Area of total diagram—(Area of H.P diagram + Area of L.P. diagram) (6 3)

(*a*) **For equal initial loads**

Let A_1 be the area of H.P. piston and A_2 be the area of L.P. piston.

Then $$(P_1-P_2)A_1=(P_3-P_4)A_2 \qquad (6\cdot3)$$

Mean effective pressure

Let, a=mep of H.P. cylinder

a'=mep of H.P cylinder refered to L.P. cylinder

b=mep of L P cylinder

c=mep of combined diagram

mep of H P. cylinder, $a=\frac{P_1(1+\log_e r_1)}{r_1}-P_2$ [6·4(a)]

mep of L.P cylinder, $b=\frac{P_3(1+\log_e r_2)}{r_2}-b_b$ [6 4(b)]

Generally mep of all cylinders is given referred to L.P. cylinder.

mep of H.P. cylinder referred to L.P. cylinder

$$a' = \frac{\text{m.e.p. of H.P. cylinder} \times \text{volume of H.P. cylinder}}{\text{Volume of L.P. cylinder}}$$

$$= a \times \frac{V_2}{V_4} \qquad [6\text{-}4(c)]$$

(*b*) **For equal work done in cylinders.**

mep of H.P. cylinder referred to L.P. cylinder

= m.e.p. of L.P. cylinder

$$\therefore \qquad a' = b \qquad (6\text{-}5)$$

6·4. Overall Diagram Factor. The overall diagram factor K is defined with reference to the combined diagram and neglecting

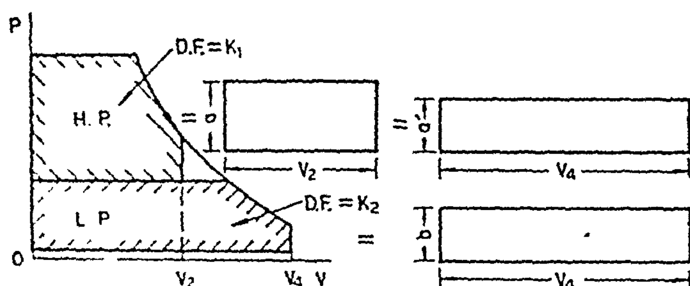


Fig. 6·5. m.e.p. of H.P. and L.P. cylinder.

the loss due to unresisted expansion. The sum of the hypothetical areas of H.P. and L.P. diagrams multiplied by their respective diagram factor is equal to the area of the combined diagram multiplied by the overall diagram factor.

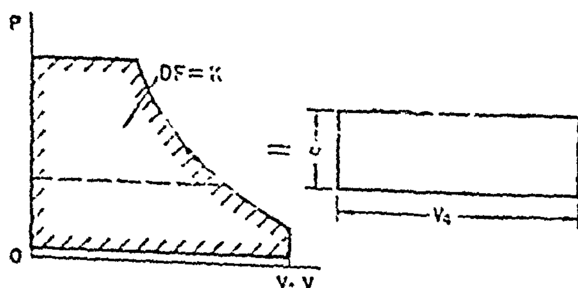


Fig. 6·6. m.e.p. of combined diagram.

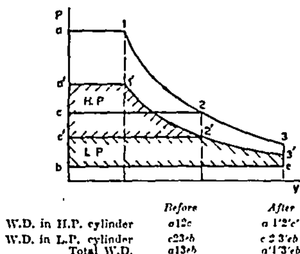
D.F. of H.P. cy. × m.e.p. of H.P. cy. referred to L.P. cy. + D.F. of L.P. cy. × m e.p. of L.P. cy.

= Overall D.F. × m e.p. of combined diagram

$$k_1 a' + k_2 b = k \times c \tag{6.6}$$

6.5. Methods of Governing. There are three methods of governing compound engines :

(1) *Throttle governing.* In this method steam is throttled by governor before it enters the H P. cylinder. Though throttling is at constant heat, this method is wasteful in steam consumption as it reduces the heat potential of steam and, therefore, the capacity of doing work per kg of steam. In throttle governing there is reduction in work in both the cylinders, greater reduction taking place in H.P. cylinder with greater throttling which disturbs the balance of work done in two cylinders. See Fig. 6·7

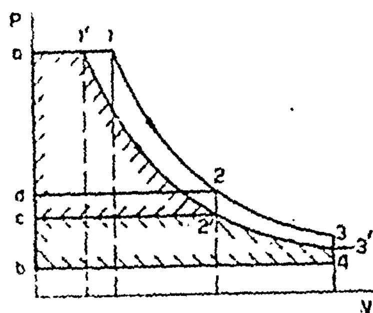


	Before	*After*
W.D. in H.P. cylinder	$a12c$	$a1'2'c'$
W.D. in L.P. cylinder	$c23eb$	$c'2'3'eb$
Total W.D.	$a13eb$	$a'1'3'eb$

Fig. 6·7. Throttle governing.

(2) *Cut-off governing in H P. cylinder* This method is more economical as there is no drop in heat potential and hence in capacity of doing work per kg of steam Cut off governing is achieved by the use of Mayer's expansion valve where cut-off is controlled independently of other operations The disadvantage of cut off in H P. cylinder is that work done in H.P cylinder remains more or less the

same and almost all reduction takes place in L.P. cylinder. (see Fig. 6·8). This disturbs the balance of work done in two cylinders.



	Before	*After*
W.D. in H.P. cy.	*a*12*d*	*a*1'2'*c*
W.D. in L.P. cy.	*d*234*b*	*c*2'3'4*b*
Total W.D.	*a*134*b*	*a*1'3'4*b*

Fig. 6·8. Cut-off governing.

(3) *Simultaneous cut-off in high pressure and low pressure cylinders.* The unbalance of work done in two cylinders can be corrected by simultaneous change of cut-off in high pressure and low pressure cylinders. Cut-off in low pressure cylinder does not change the total power produced by the engine but only changes the ratio of power produced in high pressure and low pressure cylinders.

6·6 The Uniflow Engine (straight flow). In ordinary steam engines steam entrance and exhaust take place from the same port *i.e.* the path or flow of steam is retraced. But in uniflow engines steam enters at one end and exhusts at the centre, and hence the name uniflow engine.

Fig. 6·9 shows a longitudinal and transverse section of a uniflow engine. At the commencement of inside stroke the valve *A* opens and high pressure steam enters the cylinder. Valves *A* and *B* are controlled mechanically, so that desired cut-off and expansion ratio can be achieved. At middle of the cylinder barrel there are number of exhaust ports. The steam expands till the right hand end of the piston uncovers the exhaust ports. The exhaust takes place until the ports are again covered in the return stroke which is by admission of fresh steam through valve *B*. The exhaust length available being

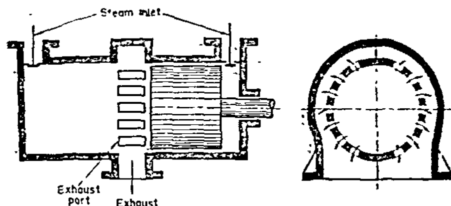


Fig. 6 9. Uniflow steam engine.

short in comparison to stroke length there is large pressure drop at exhaust

The uniflow engine, like the compound engine reduces the steam condensation. In the uniflow arrangement cool exhaust steam does not come in contact with the ends of the cylinder. It decreases the temperature range of the cylinder walls and thus prevents condensation. The uniflow engine has an advantage over the compound engine in having higher diagram factor due to absence of receiver losses *See* Fig 6 10

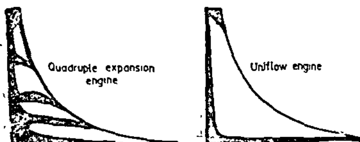


Fig. 6 10. Comparision of indicator diagrams of compound and uniflow engines

The disadvantage of the uniflow engine in comparison to compound engine is in having greater variation of torque due to one cylinder, consequently there is large flywheel and much vibrations. The construction of cylinder is also heavy to withstand the maximum pressure. The section is weakest where it has exhaust ports. The length of piston P is very long, its length being equal to the stroke minus the length of exhaust ports.

Horizontal uniflow engines are available in sizes from 75 to 1,200 h.p. The cylinder heads are steam jacketed. Vertical engines are built upto 2,400 h.p. and upto 7,000 h.p. for marine service. These engines provide better economy with respect to steam consumption.

IMPORTANT POINTS

(1) The total work done in high pressure and low pressure cylinders is not equal to the area of the combined diagram as there is loss due to unresisted expansion represented by the small triangle. Therefore, the areas of high pressure and low pressure indicator diagrams must always be calculated separately and added.

(2) However, if "overall diagram factor" is given the complete area of the diagram is to be taken because by definition of "overall diagram factor" its multiplication with the total indicator diagram area is equal to the sum of (D.F. of H.P. cy. × area of H.P. indicator diagram + D.F. of L.P. cy. × area of L.P. indicator diagram).

(3) The loss due to unresisted expansion can only be neglected if it is mentioned that "the expansion in the high pressure cylinder is carried down to the inlet pressure of low pressure cylinder" or "there is no pressure drop at release in the high pressure cylinder".

"No pressure drop in receiver" does not mean that there is no unresisted expansion. It only means that there is no further drop of pressure in receiver due to condensation.

(4) Problems on compound engines should be solved by first finding the equation of hyperbolic expansion ($PV=C$). For this purpose the volume at cut-off may be taken as unity if the total number of expansions is given or the volume of high pressure cylinder may be taken as unity if the ratio of cylinder volumes is given.

(5) The work done in each cylinder is proportional to the mean effective pressure of each cylinder referred to low pressure cylinder. Therefore, whenever the ratio of work done in two cylinders is required, the work done in each cylinder need not be calculated but the mep's referred to low pressure cylinder should be calculated and compared.

ILLUSTRATIVE EXAMPLES

6·1. Cylinder dimensions : cut-off in L.P. cylinder ; ratio of work done.

On what basis compound steam engines are designed ?

Determine the main dimensions of a horizontal compound steam engine to develop 400 bhp under the following conditions : pressure at steam chest 8 kgf/cm² gauge ; vacuum, 670 mm of Hg ; barometer reading 765 mm of Hg ; average piston speed, 180 metres per minute ; revolutions per minute 120 ; ratio of H.P. to L.P. cylinder area, 1 : 4 ; total number of expansions 14, diagram factor 0·85 ; mechanical efficiency 0·8. Also find the cut-off in L.P. cylinder and the ratio of work done in two cylinders.

Assume equal initial load on pistons.

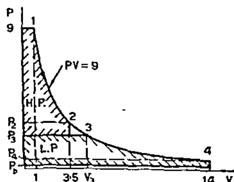


Fig. 6·11.

For theory—see text.

$$\text{ihp}=\frac{\text{bhp}}{\text{mechanical }\eta}=\frac{400}{0{\cdot}8}=500$$

Back pressure, $P_b=\dfrac{(765-670)}{735{\cdot}5}=0{\cdot}1291$ kgf/cm²

For equal initial loads

$$(P_1-P_3)A_1=(P_3-P_b)A_2$$

$$(9-P_3)\times 1=(P_3-0{\cdot}1291)\times 4 \qquad \therefore\ P_3=1{\cdot}903 \text{ kgf/cm}^2$$

Assuming volume at cut-off to be unity, equation of the expansion curve is $PV=9$.

$$P_3V_3=9 \qquad \therefore \quad V_3=\frac{9}{1\cdot903}=4\cdot73$$

$\therefore$ Cut-off in L.P. $=\frac{V_3}{V_4}=\frac{4\cdot73}{14}=0\cdot338$ Ans.

$$r_1=\frac{14}{4}=3\cdot5 \qquad \text{and } r_2=\frac{14}{4\cdot73}=2\cdot96$$

$$\text{mep of H.P. cylinder}=\left[\frac{P_1(1+\log_e r_1)}{r_1}-P_3\right]\times DF$$

$$=\left[\frac{9(1+\log_e 3\cdot5)}{3\cdot5}-1\cdot903\right]\times 0\cdot85=3\cdot281 \text{ kgf/cm}^2$$

$$\text{mep of L.P. cylinder}=\left[\frac{1\cdot903(1+\log_e 2\cdot96)}{2\cdot96}-0\cdot1291\right]\times 0\cdot85$$

$$=1\cdot03 \text{ kgf/cm}^2$$

mep of H.P. cylinder referred to L.P. cylinder

$$=\frac{3\cdot281\times 3\cdot5}{14}=0\cdot82 \text{ kgf/cm}^2$$

Total mep referred to L.P. cylinder

$$P_m=1\cdot03+0\cdot82=1\cdot85 \text{ kgf/cm}^2$$

$$\text{ihp}=\frac{(P_m\times A)\times \text{piston speed in metre per min}}{75\times 60}$$

$$500=\frac{1\cdot85\times\left(\frac{\pi}{4}d_2^2\right)\times 180}{75\times 60}$$

$\therefore$ L.P. cylinder dia, $d_2=92\cdot8$ cm Ans.

H.P. cylinder dia, $d_1=\frac{92\cdot8}{2}=46\cdot4$ cm, Ans.

$$\text{stroke}=\frac{180\times 100}{2\times 120}=75 \text{ cm}$$ Ans.

$$\frac{\text{Work done in H.P. cylinder}}{\text{Work done in L.P. cylinder}}$$

$$=\frac{\text{mep in H.P. cylinder referred to L.P. cylinder}}{\text{mep in L.P. cylinder}}$$

$$=\frac{0\cdot82}{1\cdot03}=0\cdot796:1$$ Ans.

Note. (i) Diagram factor is same for H.P. and L.P. cylinders.

(*ii*) For finding the total work done, mep's of H.P. and L.P. cylinders have been separately calculated and thus loss due to unresisted expansion has been taken into account.

(*iii*) For comparing the work mep's of H.P. and L.P. cylinders referred to L.P. cylinder have been compared.

6-2. mep : overall D.F. ; temperature drop.

Explain the advantages of compounding in a steam engine.

A double-acting compound steam engine has cylinder diameter, H.P. 30 cm, L.P. 66 cm and the common stroke is 38 cm. When running at 160 r.p.m. the engine develops 168 bhp and its mechanical efficiency is 78 per cent. Dry saturated steam is supplied at 14 kgf/cm², cut-off in H.P. cylinder is at ½ stroke and the back pressure is 0.28 kgf/cm².

Calculate the actual and hypothetical mean effective pressures referred to the L.P. cylinder and hence find the overall diagram factor.

Also compare the temperature drop in H.P. and L.P. cylinders assuming equal initial piston loads. Neglect clearance and compression effects.

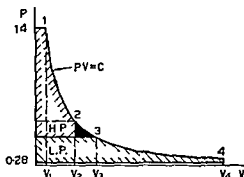


Fig. 6·12.

For theory—see text.

$$\text{ihp} = \frac{\text{bhp}}{\text{mechanical } \eta} = \frac{168}{0.78} = 215.5$$

$$\text{ihp}=\frac{P_m lAN}{75\times 60},\ 215\cdot 5=\frac{P_m\times\frac{38}{100}\times\frac{\pi}{4}\times 66^2\times 2\times 160}{75\times 60}$$

$\therefore$ $P_m=2\cdot 331$ kgf/cm² **Ans.**

$$\text{Cylinder volume ratio}=\frac{V_4}{V_2}=\frac{\left(\frac{\pi}{4}\right)d_4^2}{\left(\frac{\pi}{4}\right)d_2^2}=\frac{66\times 66}{30\times 30}=4\cdot 84$$

$$\text{Overall compression ratio}=\frac{V_4}{V_1}=\frac{4\cdot 84}{\left(\frac{1}{3}\right)}=14\cdot 52$$

Hypothetical m.e.p. of combined diagram

$$=\left[\frac{P_1(1+\log_e r)}{r}-P_b\right]$$

$$=\left[\frac{14(1+\log_e 14\cdot 52)}{14\cdot 52}-0.28\right]=3\cdot 264\ \text{kgf/cm}^2$$

$\therefore$ Overall diagram factor $=\frac{2\cdot 331}{3\cdot 264}=0\cdot 7144$ **Ans.**

For equal initial piston loads, $(P_1-P_3)A_1=(P_3-P_b)A_2$

$(14-P_3)\times 1=(P_3-0\cdot 28)\times 4\cdot 84$ $\quad\therefore\quad$ $P_3=2\cdot 616$ kgf/cm²

From steam tables,

Temperature drop in H.P. cylinder $=194\cdot 1-128\cdot 3=65\cdot 8°C$

Temperature drop in L.P. cylinder $=128\cdot 3-67\cdot 1=61\cdot 2°C$

$$\therefore\quad \frac{\text{Temperature drop in H.P. cylinder}}{\text{Temperature drop in L.P. cylinder}}=\frac{65\cdot 8}{61\cdot 2}=1\cdot 074 \quad \textbf{Ans.}$$

Note. Overall diagram factor × hypothetical mep of total diagram gives the sum of the actual mep's in H.P. and L.P. cylinder referred to L.P. cylinder.

6.3. mep and load on piston ; given equal work and initial load.

A high-speed, double-acting compound engine develops 315 ihp equally between the two cylinders when running at 400 rpm and using steam at 12 kgf/cm² initial pressure against a back pressure of 0·17 kgf/cm². If the cylinder diameters are 30 cm and 60 cm and the stroke

is 30 cm, determine the mep in each cylinder and the initial loads on the pistons, assuming them to be equal and neglecting any pressure drop between the cylinders.

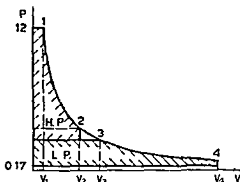


Fig. 6·13.

For theory—see text.

Volume of L.P. cylinder, $V_4=\frac{\pi}{4}\times 60^2\times 30=84{,}800$ c c

Volume of H.P. cylinder, $V_2=\frac{\pi}{4}\times 30^2\times 30=21{,}200$ c c

Let total mep of H.P and L.P. cylinders referred to L.P. cylinder be P_m

$$\text{ihp}=\frac{P_m LAN}{75\times 60},\quad 315=\frac{P_m\left(\frac{30}{100}\right)\times\frac{\pi}{4}\,60^2\times 2\times 400}{75\times 60}$$

$$\therefore\quad P_m=2\,088 \text{ kgf/cm}^2$$

As the work done in both cylinders is same, therefore, mep of L P. cylinder and mep of H.P. cylinder referred to L P. cylinder will also be same.

$\therefore$ mep of L.P. cylinder $=\frac{2\,088}{2}=1{\cdot}044$ kgf/cm² Ans.

Also mep of H.P. cylinder referred to L P cylinder

$$=1{\cdot}044 \text{ kgf/cm}^2$$

$\therefore$ mep of H P. cylinder

$$=\frac{\text{mep of H P. cylinder}\times\text{L P. cylinder volume}}{\text{H.P. cylinder volume}}$$

$$=1{\cdot}044\times\frac{4}{1}=4{\cdot}176 \text{ kgf/cm}^2$$ Ans.

For equal initial loads, $(P_1-P_3)A_1=(P_3-P_b)A_2$

$$(12-P_3)\times 1=(P_3-0{\cdot}17)4, \quad \therefore \quad P_3=2{\cdot}536 \text{ kgf/cm}^2$$

$\therefore$ Initial load on pistons $=(12-2{\cdot}536)\times\frac{\pi}{4}\times 30^2$

$=6{,}690$ kgf   Ans.

Note. No pressure drop between cylinders means there is no pressure drop in receiver ; pressure drop at release, however, is there.

6.4. Receiver loss : cylinder dia. ; LP. cylinder cut-off for equal initial load ; temp. difference at inlet and exhaust.

State the causes and effect of initial condensation in the cylinder of a reciprocating engine. What are the merits and demerits of reheating between the stages ?

A double-acting compound engine is required to give 600 ihp at 200 rpm. The supply is at 15 kgf/cm² dry and saturated ; back pressure 0·28 kgf/cm² ; ratio of cylinder volumes 4 ; overall number of expansions 6·5 ; stroke equal to the 2/3rd of the low-pressure cylinder diameter ; overall diagram factor 0·66.

Assuming hyperbolic expansion with no clearance and allowing for a receiver loss of 0·21 kgf/cm², calculate, (a) the cylinder diameters, (b) the low pressure cylinder cut-off to give equal initial piston loads, (c) the ratio of work done in two cylinders, and (d) the difference of temperatures between inlet and exhaust of the high pressure cylinder.

Reheating between the stages. The reheating of steam between the stages may be done by heating the inter-stage receiver or by retaining the steam to boiler furnace, which however involves considerable complications.

The thermodynamic gain due to reheating would be small as in the case of supply of superheated steam to a simple steam engine the temperature is high only at one point of the cycle. But practical advantage would be considerable due to avoidance of condensation and reduced rate of heat transfer through walls.

(a) Let d_1 and d_2 = diameter of H.P. and L.P. cylinders respectively

$$\frac{v_4}{v_2}=4, \quad r=\frac{v_5}{v_1}=6{\cdot}5, \quad l=\frac{2}{3}d_2$$

The hypothetical mep of combined diagram referred to L.P. cylinder

$$=\left[\frac{P_1(1+\log_e r)}{r}-p_b\right]$$

$$=\left[\frac{15(1+\log_e 6\cdot5)}{6\cdot5}-0\cdot28\right]=6\cdot348 \text{ kgf/cm}^2$$

Actual mep of combined diagram $=6\cdot348\times0\cdot66=4\cdot19$ kgf/cm²

$$\text{ihp}=\frac{P_m lAN}{75\times60}600=\frac{4\cdot19\times\left(\frac{2}{3}\times\frac{d_2}{100}\right)\times\left(\frac{\pi}{4}d_2^2\right)\times2\times200}{75\times60}$$

$\therefore$ $\underline{d_2=67\cdot52 \text{ cm}}$ Ans.

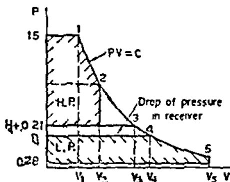


Fig 6·14.

$\frac{\pi}{4}d_1^2\times l\times4=\frac{\pi}{4}d_2^2\times l$ $\therefore$ H P cy dia $\underline{d_1=33\cdot76 \text{ cm}}$ Ans.

(b) Assuming volume of HP cylinder as unity,

$V_1=\frac{4}{6\cdot5}=0\cdot615$, $V_5=4$, $P_1V_1=15\times0\cdot615=9\cdot225$

$\therefore$ Equation of the expansion curve is, $PV=9\cdot225$

As there is drop in receiver pressure,

Back pressure of H P cy = inlet pressure of L P. cy $+0\cdot21$

For equal initial loads, $(P_1-P_3)A_1=(P_4-P_b)A_2$

$$[15-(P_4+0\cdot21)]\times1=(P_4-0\cdot28)\times4$$

$\therefore$ $P_4=3\cdot182$ kgf/cm² and $P_3=3\cdot392$ kgf/cm²

$P_4V_4=9\cdot225$ $\therefore$ $V_4=\frac{9\cdot225}{3\cdot182}=2\cdot898$

$\therefore$ $\underline{\text{L P cylinder cut-off}}=\frac{V_4}{V_5}=\frac{2\cdot898}{4}=\underline{0\cdot7245}$

(c) $\frac{\text{H.P. cy work}}{\text{L.P. cy work}}=\frac{\text{mep of H.P. cy. referred to L.P. cy.}}{\text{mep of L.P. cy.}}$

$$=\frac{\frac{1}{4}\left[\frac{15\left(1+\log_e \frac{1}{0.615}\right)}{\frac{1}{0.615}}-3.392\right]}{\frac{3.18\left(1+\log_e \frac{1}{0.7245}\right)}{\frac{1}{0.7245}}-0.28}=\underline{1.206} \quad \text{Ans.}$$

(d) Difference of temperature between inlet and exhaust of H.P. cylinder.

=saturation temp. at 15 kgf/cm^2 − saturation temp at 3·392 kgf/cm^2

$=197.4-137.1=\underline{60.3°C}$ Ans.

Note. The pressure drop in receiver is inevitable due to condensation but generally it is neglected to simplify the problem. It should not be confused with the pressure drop in constant volume release.

6·5. Triple expansion engine : cut-off ; power developed ; piston load ; loss due to incomplete expansion.

What is meant by unresisted expansion ?

A triple expansion marine engine has the following cylinder volume ratios : $\frac{L.P.}{H.P.}=7.0$; $\frac{I.P.}{H.P.}=2.8$. *Initial steam pressure is 15 kgf/cm^2 and L.P. back pressure is 0·14 kgf/cm^2. L.P. terminal pressure is 1 kgf/cm^2. Receiver pressures are fixed at 4·6 and 1·5 kgf/cm^2. Determine the cut-off for each cylinder, the relative values of initial piston loads and power developed referred to the L.P. cylinder. Determine the theoretical loss of work due to incomplete expansion in H.P. and I.P. cylinders.*

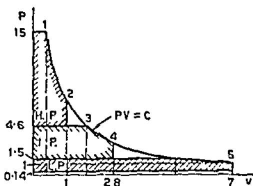


Fig. 6 15.

For theory—see text

Let H.P. cylinder volume be unity

I.P. cylinder volume 2·8 ; L.P. cylinder volume = 7

Equation of expansion curve, $PV = C = 1 \times 7 = 7$

Cut-off in H.P. $= \dfrac{7}{15} = 0.4667$ Ans.

Cut-off in I.P. $= \dfrac{7}{4.6} \times \dfrac{1}{2.8} = 0.5434$ Ans.

Cut-off in L.P. $= \dfrac{7}{1.5} \times \dfrac{1}{7} = 0.6667$ Ans.

Initial piston load $= (P_1 - P_b)\,A$

Initial loads

H.P. : I.P. : LP : : $(15-4.6)\times 1 : (4.6-1.5)\times 2.8 : (1.5-0.14)\times 7$

$= 1.092 : 0.912 : 1$

mep of H.P. cylinder referred to L.P cylinder

$$= \frac{1}{7}\left[\frac{15\left\{1+\log_e\left(\frac{1}{0.4667}\right)\right\}}{\frac{1}{0.4667}} - 4.6\right] = 1.106 \text{ kgf/cm}^2$$

mep of I.P. cylinder referred to L.P. cylinder

$$= \frac{2.8}{7}\left[\frac{4.6\left\{1+\log_e\left(\frac{1}{0.5434}\right)\right\}}{\left(\frac{1}{0.5434}\right)} - 1.5\right] = 1.01 \text{ kgf/cm}^2$$

mep of L.P cylinder

$$= \left[\frac{1.5\left\{1+\log_e\left(\frac{1}{0.6667}\right)\right\}}{\left(\frac{1}{0.6667}\right)} - 0.14\right] = 1.266 \text{ kgf/cm}^2$$

Work done is in proportion to mep of various cylinders referred to L.P. cylinder

Work done in H.P. : I.P. : L.P. : : 1·106 : 1·01 : 1·266
: : 0·874 : 0·798 :1 Ans.

Σ mep of all cylinders referred to L.P. cylinder

$$=1{\cdot}106+1{\cdot}01+1{\cdot}266=3{\cdot}382 \text{ kgf/cm}^2$$

Overall mep referred to L.P. cylinder

$$=\frac{15\left\{1+\log_e\left(\frac{7}{0{\cdot}4667}\right)\right\}}{\left(\frac{7}{0{\cdot}4667}\right)}-0{\cdot}14=3{\cdot}568 \text{ kgf/cm}^2$$

$\therefore$ Loss due to incomplete expansion in H.P. and I.P. cylinder

$$=\frac{3{\cdot}568-3{\cdot}382}{3{\cdot}568}=5{\cdot}213\%$$ Ans.

Note. The percentage loss due to unresisted expansion has been found by subtracting the sum of the mep's of H.P. I.P. and L.P. diagrams from the mep of combined diagram.

6.6. Different clearance in L.P. and H.P. cy. : mep ; hp.

How would you combine the indicator diagrams of L.P. and H.P. cylinder of a compound steam engine ?

The following particulars refer to a double-acting compound engine : H.P. cylinder : diameter 25 cm, cut-off 0·32 stroke, clearance 10 per cent of swept volume. L.P. cylinder : diameter 47 cm, cut-off 0·42 stroke, clearance 7 per cent of swept volume.

If the engine is supplied with steam at 6·5 kgf/cm² and exhausts into a condenser at 0·28 kgf/cm², estimate the mep in each cylinder and the total hp developed when running at 100 rpm. Take a diagram factor of 0·8 for the H.P. and 0.7 for the L.P. cylinder. Assume hyperbolic expansion and neglect cushioning but take account of clearance.

For theory—see text.

Let the common stroke of cylinders be s cm.

H.P. cylinder.

Stroke volume$=\frac{\pi}{4}\times25^2\times s=490{\cdot}9s$ cc

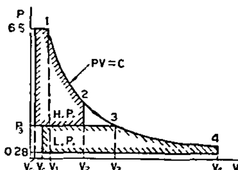


Fig. 6·10.

Clearance volume $=0{\cdot}1\times490{\cdot}9\,s=49{\cdot}09\,s$

$\therefore$ Total volume $=490{\cdot}9\,s+49{\cdot}09\,s=540s$ cc.

Volume at cut-off$=49{\cdot}09\,s+0{\cdot}32\times490{\cdot}9\,s=206{\cdot}2\,s$ cc.

L.P. cylinder.

Stroke volume$=\frac{\pi}{4}\times47^2\times s=1{,}734{\cdot}9\,s$ c c.

Clearance volume$=0{\cdot}07\times1{,}734{\cdot}9\,s=121{\cdot}4\,s$ c c.

$\therefore$ Total volume$=1{,}734{\cdot}9\,s+121{\cdot}4\,s=1{,}856\,s$ c c.

Volume at cut-off$=121{\cdot}4\,s+0{\cdot}42\times1{,}734{\cdot}9\,s=850{\cdot}1\,s$

r_1 (for H.P. cylinder)$=\frac{540\,s}{206{\cdot}2\,s}=2{\cdot}619$

r_2 (for L P. cylinder)$=\frac{1{,}856\,s}{850{\cdot}1\,s}=2{\cdot}183$

Assuming expansion curve of two cylinders to be continuous,

$P_1V_1=P_3V_3$,

$6{\cdot}5\times206{\cdot}2\,s=P_3\times850{\cdot}1\,s$

$\therefore$ $P_3=1{\cdot}577$ kgf/cm²

Actual m.e p.$=$D.F. $\left[\frac{P_1V_1(1+\log_e r)-P_bV_2-(P_1-P_b)V_c}{V_s}\right]$

Actual m.e.p for H.P. cylinder

$$=0{\cdot}8\left[\frac{6{\cdot}5\times206{\cdot}2\,s(1+\log_e 2{\cdot}619)-1{\cdot}577\times540s-(6{\cdot}5-1{\cdot}577)\times49{\cdot}09\,s}{490{\cdot}9\,s}\right]$$

$=2{\cdot}507$ kgf/cm² Ans.

Actual m.e.p. for L.P. cylinder

$$=0{\cdot}7\left[\frac{1{\cdot}577\times850{\cdot}1\,s\,(1+\log_e 2{\cdot}183)-0{\cdot}28\times1{,}856\,s-(1{\cdot}577-0{\cdot}28)\times121{\cdot}4\,s}{1734{\cdot}95}\right]$$ Ans.

$$=\underline{0{\cdot}69\ \text{kgf/cm}^2}.$$

Actual mep of H.P. cylinder referred to L.P. cylinder

$$=\frac{2{\cdot}507\times490{\cdot}9\,s}{1{,}734{\cdot}9\,s}=0{\cdot}709\ \text{kgf/cm}^2$$

$\therefore$ Total mep $=0{\cdot}69+0{\cdot}709=1{\cdot}399$ kgf/cm²

$$\text{hp.}=\frac{p_m l A N}{75\times60}$$

$$=\frac{1{\cdot}399\times10^4\times1{,}734{\cdot}9\,s\times10^{-6}\times2\times100}{75\times60}=\underline{1{\cdot}077\,s}$$ Ans.

6·7. Cut-off and throttle governing.

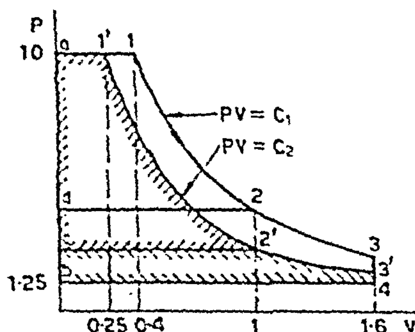
What are the methods of governing a compound steam engine ?

In a compound steam engine admission pressure to the H.P. cylinder is 10 kgf/cm² and the back pressure is 1·25 kgf/cm². The total number of expansion is 4 and there is no pressure drop at release in the high pressure cylinder. Calculate the percentage change in the power produced in (a) the H.P. cylinder, (b) the L.P. cylinder, and (c) the complete engine, if the cut-off in H.P. cylinder is reduced from 0·4 stroke to 0·25 stroke. Assume hyperbolic expansion.

If the same power reduction is obtained by throttling the steam, find the initial steam pressure to H.P. cylinder and percentage reduction in power in H.P. and L.P. cylinders.

Comment on the result.

For theory—see text.



Cut-off	H.P.	L.P.	[illegible]
0·4	a 12 d	d 234 b	a 134 b
0·25	a 1′2′ c	c 2 3′4 b	a 1′3′4 b

Fig. 6·17.

Cut off governing

Assume volume of H.P. cylinder to be unity. When cut-off is ·4 stroke, equation of the expansion curve is $PV=10\times0{\cdot}4=4$

Volume at cut-off=0·4 ; volume of L.P. cy $=4\times0\ 4=1{\cdot}6$

$$P_2V_2=4,\ \therefore\ P_2=\frac{4}{1}=4\ \text{kgf/cm}^2$$

mep of H.P. cylinder referred to L P. cylinder

$$=\frac{P_1V_1\log_e r}{V_4}$$

$$=\frac{10\times0\ 4\log_e\left(\frac{1}{0\ 4}\right)}{1{\cdot}6}=2\ 291\ \text{kgf/cm}^2$$

$$\text{mep of L.P. cylinder}=\frac{P_2(1+\log_e r)}{r}-P_b$$

$$=\frac{4(1+\log_e 1{\cdot}6)}{1{\cdot}6}-1{\cdot}25=2{\cdot}425\ \text{kgf/cm}^2$$

Total mep referred to L.P. cylinder

$$=2{\cdot}291+2{\cdot}425$$

$$=4{\cdot}716\ \text{kgf/cm}^2$$

When cut-off is 0 25 stroke, new equation of the expansion curve is

$$PV=10\times0\ 25=2\ 5$$

$$\therefore\qquad P_2=\frac{2\ 5}{1}=2{\cdot}5\ \text{kgf/cm}^2$$

mep of H.P. cylinder referred to L.P. cylinder

$$=\frac{10\times0\ 25\log_e\left(\frac{1}{0\ 25}\right)}{1{\cdot}6}=2{\cdot}166\ \text{kgf/cm}^2$$

$$\text{mep of L.P. cylinder}=\frac{(1+\log_e 1\ 6)}{1{\cdot}6}-1\ 25=1\ 047\ \text{kgf/cm}^2$$

Total mep of engine referred to L.P cylinder

$$=2\ 166+1{\cdot}047=3\ 213\ \text{kgf/cm}^2$$

As power produced is proportional to mep referred to L.P. cylinder,

(a) Reduction in H.P. cy $= \dfrac{2{\cdot}291-2{\cdot}166}{2{\cdot}291} = 5{\cdot}456\%$ Ans.

(b) Reduction in L.P. cy $= \dfrac{2{\cdot}425-1{\cdot}047}{2{\cdot}425} = 56{\cdot}82\%$ Ans.

(c) Reduction in total power $= \dfrac{4{\cdot}716-3{\cdot}213}{4{\cdot}716} = 31{\cdot}88\%$ Ans.

It is seen from calculations as well as from the indicator diagram that by reducing the cut-off in H.P. cylinder there is very little change in the work done in the H.P. cylinder and nearly whole reduction in work done is in the L.P. cylinder. The ratio of work done in $\dfrac{\text{H.P. cylinder}}{\text{L.P. cylinder}}$ changes from 1 : 0·945 to 1 : 0·4834.

Throttle governing (cut-off same *i.e.*, 0·4 of stroke).

For the same reduction in power total mep referred to L.P. cylinder should be reduced to same, *i.e.*, 3·213 kgf/cm²

$$P_m = \frac{P_1(1+\log_e r)}{r} - P_b \text{ or } 3{\cdot}213 = \frac{P_1(1+\log_e 4)}{4} - 1{\cdot}25$$

∴ Initial pressure, P_1 (after throttling) $= 7{\cdot}48$ kgf/cm² Ans.

New equation of the expansion curve, $PV = 7{\cdot}48 \times 0{\cdot}4 = 2{\cdot}992$

mep of H.P. cylinder referred to L.P. cylinder

$$= \frac{2{\cdot}992 \log_e \left(\dfrac{1}{0{\cdot}45}\right)}{1{\cdot}6} = 1{\cdot}714 \text{ kgf/cm}^2$$

mep of L.P. cylinder $= \dfrac{2{\cdot}992\,(1+\log_e 1{\cdot}6)}{1{\cdot}6} - 1{\cdot}25$

$= 1{\cdot}499$ kgf/cm²

[Total mep of engine $= 1{\cdot}714 + 1{\cdot}499 = 3{\cdot}213$ kgf/cm²]

Power reduction in H.P. cylinder

$$= \frac{2{\cdot}291-1{\cdot}714}{2{\cdot}291} = 25{\cdot}2\% \quad \text{Ans.}$$

Power reduction in L.P. cylinder $= \dfrac{2{\cdot}425-1{\cdot}499}{2{\cdot}425} = 38{\cdot}2\%$ Ans.

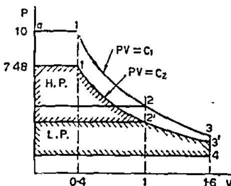


Fig. 6·18. Throttle governing.

It is seen from calculations as well as from indicator diagram that by throttle governing work is reduced in both cylinders. In the example there is greater reduction in the L P. cylinder but as the load goes on decreasing there will be greater and greater reduction in H.P. cylinder.

6·8. Trial on triple engine : percentage power developed in each cylinder

During a trial on a triple-expansion engine the following results were recorded :

Cylinder	*Diameter*	*Actual* P_m
H.P.	*30 cm*	*6 kgf/cm²*
I.P.	*50 cm*	*2 2 kgf/cm²*
L.P.	*90 cm*	*0 9 kgf/cm²*

Steam supply pressure, 15 kgf/cm², cut-off in H P. cylinder, 0·6 stroke ; Back pressure in I P. cylinder, 0 21 kgf/cm²

Calculate the actual and hypothetical P_m *referred to the L.P. cylinder and hence obtain the overall diagram factor. Calculate the percentage of the total ihp developed in each cylinder. All cylinders have the same stroke.*

For H.P. cylinder,

$$P_m \text{ referred to L.P. cylinder} = 6 \times \frac{30^2}{90^2} = 0{\cdot}667 \text{ kgf/cm}^2$$

For I.P. cylinder,

P_m referred to I.P. cylinder $= 2\cdot 2 \times \frac{50^2}{90^2}$

$= 0\cdot 679$ kgf/cm²

$\therefore$ Total P_m referred to L.P. cylinder

$= 0\cdot 667 + 0\cdot 679 + 0.9 = 2\cdot 246$ kgf/cm² Ans.

Total number of expansions through engine $= \frac{90^2}{0\cdot 6 \times 30^2} = 15$

Theoretical P_m referred to L.P. cylinder

$$= \frac{P_1}{r}(1 + \log_e r) - P_b$$

$$= \frac{15}{15}(1 + \log_e 15) - 0\cdot 21 = 3\cdot 5 \text{ kgf/cm}^2 \quad \text{Ans.}$$

Overall diagram factor $= \frac{2\cdot 246}{3\cdot 5} = 0\cdot 642$ Ans.

The percentage of the total ihp developed in each cylinder is the same as the percentage analysis of the actual mean effective pressure referred to the L.P. cylinder.

$\therefore$ Output from each cylinder,

H.P. $= \frac{0\cdot 667}{2\cdot 246} \times 100 = 29\cdot 7\%$

I.P. $= \frac{0\cdot 679}{2\cdot 246} \times 100 = 30\cdot 2\%$

L.P. $= \frac{0\cdot 9}{2\cdot 246} \times 100 = 40\cdot 1\%$ Ans.

EXAMPLES 6

6.1. Horse-power, diagram factor, given indicated areas.

The cylinder diameter of a triple-expansion engine are 41,56 and 104 cm respectively, and the stroke is 69 cm. The following data are taken from the mean indicator cards :

Cylinder	*H.P.*	*I.P.*	*L.P.*
Area of diagram in cm²	*25·0*	*27·8*	*29·8*
Length of diagram in cm	*11·3*	*11·2*	*11·3*
Scale of indicator spring (kgf/cm)	*2·4*	*1*	*0·36*

The admission pressure at the high pressure cylinder is 12 kgf/cm² and the exhaust pressure from the low pressure cylinder is 0·3 kgf/cm². The cut-off in the high-pressure cylinder takes place at 70 per cent of the stroke. The speed of the engine is 100 revolutions per min. Calculate the indicated hors-power and the diagram factor, assuming the expansion to be hyperbolic.

[actual mep of engine=2 494 kgf/cm²; hp=650; overall expansion ratio=9·19; hypothetical mep=3 9 kgf/cm²; D F.=0 64].

6.2. Cylinder dimensions; cut-off in L.P. cylinder, given equal initial load.

A double-acting compound steam engine is required to give 180 bhp at 90 rpm with a mechanical efficiency of 0 9. The supply pressure is 10 kgf/cm² and exhaust 0 17 kgf/cm². Take the H.P. cylinder cut-off at 0·45 stroke, ratio of cylinder volumes 1 0 to 3·2, common stroke equal to two-thirds of the L.P. cylinder diameter, overall diagram factor referred to L.P. cylinder 0 75.

(a) Find suitable piston diameters and stroke.

(b) Find the fraction of stroke at which cut off should occur in the L.P. cylinder for approximately equal initial loads on both pistons, and state the resulting steam receiver pressure.

Assume hyperbolic expansion and neglect clearance and compression effects.

[Overall expansion ratio=7 1; actual mep=3 kgf/cm²; d_1=38 2 cm; d_2=68 4 cm; l=45 6 cm; P_3=2·51 kgf/cm²; cut-off in L P. cylinder=0 561]

6.3. Cylinder diameter; L.P. cut-off; temperature range.

What are the causes of cylinder condensation? State the methods of reducing the same.

The following data were obtained during a trial of a horizontal compound steam engine:

ihp developed=500; Pressure in steam chest=11 kgf/cm²

Vacuum = 66 cm Hg, (Atmosphere = 76 cm) ; No. of expansions = 12

Diagram factor = 0·8 ; Piston speed = 220 m/min

Cut-off in H.P. cylinder = ½ stroke.

Assuming equal initial load on pistons, determine (a) the L.P. cut-off, (b) the temperature range in two cylinders, and (c) the cylinder diameters.

Also compare the power developed in the two cylinders.

[P_3 = 2·311 kgf/cm² ; cut-off in L.P. cy = 0·3967 ; temperature range : H.P. = 58·9°C, L.P. = 72·7°C ; mep of H.P. cy ref. to L.P. cy = 1·077 kgf/cm² ; mep of L.P. cy = 1·302 kgf/cm² ; d_2 = 74 cm ; d_1 = 37 cm ; Power developed H.P. : L.P. : 0·8273 : 1].

6.4. Mechanical efficiency ; cut-off in L.P. cylinder, given equal initial load.

The following data refer to a two-stage double-acting steam engine coupled to a reciprocating pump : diameter of H.P. cylinder 35 cm ; diameter of L.P. cylinder 61 cm ; stroke 50 cm ; rpm 50 ; steam inlet pressure 6·5 kgf/cm² gauge ; exhaust pressure 0·35 kgf/cm² ; diagram factor of H.P. cylinder 0·75 ; diagram factor for L.P. cylinder 0·7 ; cut-off in H.P. cylinder 0·5. Assuming equal initial loads and hyperbolic expansion, determine cut-off in L.P. cylinder, mechanical efficiency of the engine, and ratio of work done in the two cylinders.

[P_3 = 2·125 kgf/cm² ; cut off in L.P. cy = 0·5813 ; actual mep of H.P. cy ref. to L.P. cy = 1·043 kgf/cm² ; actual mep of L.P. cy = 1·089 kgf/cm² ; ihp = 69·2 ; mech η = 72·3% ; ratio of W.D. in two cy H.P. : L.P. : : 0·958 : 1].

6.5. Cylinder diameters for equal work.

A two cylinder compound double-acting steam engine is required to give 80 bhp at 360 rpm when supplied with steam at 18 kgf/cm² and exhausting to a condenser at 0·17 kgf/cm² ; cut-off is to be 0·4 in both cylinders, which are to have a common stroke of 25 cm and to do equal work. Take the mechanical efficiency as 0·85 and the diagram factor of each cylinder as 0·80 and calculate suitable cylinder bores.

Neglect clearance and assume hyperbolic expansion.

[equating mep of both cylinders referred to L.P. cylinder, P_3=1·749 kgf/cm²; actual mep of the engine referred to L.P. cylinder=1·874 kgf/cm²; d_1=12·45 cm; d_2=40 cm].

6·6. Cylinder dimensions for equal power.

A compound steam engine develops 125 ihp at 110 rpm when the steam supply pressure is 7·5 kgf/cm² and the condenser pressure is 0 21 kgf/cm². If the overall expansion ratio is 15 and the expansion is hyperbolic with a diagram factor of 0·7, find the dimensions of two cylinders, if both of them produce equal power. The stroke in both the cylinders is same as the diameter of L.P. cylinder. Neglect any losses in the pipes between the two cylinders.

[actual mep of engine ref. to L.P. cylinder=1·151 kgf/cm²; stroke length=diameter. of L.P. cylinder=65·6 cm; dia. of H.P. cylinder=38 5 cm].

6·7. Triple expansion engine : cylinder volumes for equal power; initial load.

A triple expansion engine is supplied with steam at 14 kgf/cm² and the condenser pressure is 0 28 kgf/cm². The overall ratio of expansion is 14. There is no pressure drop at release in the high pressure and low pressure cylinder. Assuming hyperbolic expansion and neglecting clearance, determine the ratios of cylinder volumes, taking high pressure cylinder volume as unity, in order that equal power may be developed in the three cylinders.

With this arrangement what would be the ratios of initial steam forces on the three pistons?

[Work in H.P. cylinder and I.P. cylinder=$P_1V_1 \log_e r_1$; $r_1=r_2$=3 065, cylinder volume ratios=1 : 3 065 · 4 567; P_2=4 567 kgf/cm²; P_3=1·49 kgf/cm²; ratio of initial loads=1 : 1 : 0 586]

6·8. Heat balance sheet of a compound engine.

A tandem-compound steam engine is fitted with jackets which drain through a steam trap direct to the hot well. The cylinder dimensions and mean effective pressures, from a test of the engine are :

	High pressure		*Low pressure*	
	Front end :	*Back end*	*Front end* :	*Back end*
Piston area, cm^2	*480*	*450*	*1920*	*1890*
mep kgf/cm^2	*2·81*	*2·67*	*0 772*	*0·844*

The stroke is 92 cm.

The steam supply is at 10·5 kgf/cm^2 *dry saturated and the hot well is at 48°C. The speed is 198 rpm and the engine uses 1,225 kg of steam per hour. The circulating water from the condenser is 23,400 kg/hr and is raised from 11°C to 36°C.*

Draw up a heat balance for the engine, the time basis being one minute and determine the thermal efficiency. Find also the thermal efficiency of an engine working on the Rankine cycle between the same limits of supply and exhaust.

[Total ihp=227 9 ; heat supplied=13.500 kcal ; heat equivalent to ihp=2,400 kcal ; heat to exhaust=10.725 kcal (circulating water+hot well) ; difference=375 kcal ; thermal η=17·78% ; Rankine η=19·16%]

7

Fuels and Combustion

7·1. Combustion Equation. The following chemical equations apply to combustion problems in boilers and engines:



(1) $$C+O_2=CO_2 \qquad (7{\cdot}1)$$

By mass, 12 kg+32 kg=44 kg or $1+\frac{8}{3}=\frac{11}{3}$

By volume, 1 mol+1 mol=1 mol



(2) $$2C+O_2=2CO \qquad (7{\cdot}2)$$

By mass, 24 kg+32 kg=56 kg or $1+\frac{4}{3}=\frac{7}{3}$

By volume, 2 mol+1 mol=2 mol



(3) $$2CO+O_2=2CO_2 \qquad (7{\cdot}3)$$

By mass, 56 kg+32 kg=88 kg or $1+\frac{4}{7}=\frac{11}{7}$

By volume, 2 mol+1 mol=2 mol



(4) $$2H_2+O_2=2H_2O \qquad (7{\cdot}4)$$

By mass, 4 kg+32 kg =36 kg or 1+8=9

By volume, 2 mol+1 mol=2 mol



(5) $$S+O_2=SO_2 \qquad (7{\cdot}5)$$

By mass, 32 kg+32 kg=64 kg or 1+1=2

By volume 1 mol+1 mol=1 mol.



(6) $$CH_4+2O_2=CO_2+2H_2O \qquad (7{\cdot}6)$$

By mass, 16 kg+64 kg=44 kg+36 kg or $1+4=\frac{11}{4}+\frac{9}{4}$

By volume 1 mol+2 mol=1 mol+2 mol

Note. Contraction in volume takes place in the equatio (2), (3), (4) and (5).

	High pressure Front end	High pressure Back end	Low pressure Front end	Low pressure Back end
Piston area, cm^2	480	450	1920	1890
mep kgf/cm^2	2·81	2·67	0 772	0·844

The stroke is 92 cm.

The steam supply is at 10·5 kgf/cm² dry saturated and the hot well is at 48°C. The speed is 198 rpm and the engine uses 1,225 kg of steam per hour. The circulating water from the condenser is 23,400 kg/hr and is raised from 11°C to 36°C.

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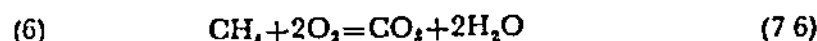
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By volume 1 mol+2 mol=1 mol+2 mol

Note. Contraction in volume takes place in the equations (1), (2), (3), (4) and (5).

The results of the above equations are summarised in the following table.

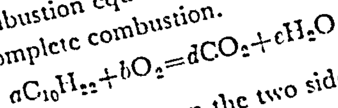
Table 7.1

Mass of oxygen required and products of combustion

Constituents, (a)	Oxygen required, kg (b)	Products of combustion, kg (c)=(a)+(b)		
		CO_2	H_2O	SO_2
C	$\frac{8}{3}$	$\frac{11}{3}$	—	—
CO	$\frac{4}{7}$	$\frac{11}{7}$	—	—
H_2	8	—	9	—
S	1	—	—	2
CH_4	4	$\frac{11}{4}$	$\frac{9}{4}$	—

(7) *Combustion equation for any hydrocarbon.*

Let a combustion equation for any hydrocarbon $C_{10}H_{22}$ be required, with complete combustion.



$$aC_{10}H_{22}+bO_2=dCO_2+eH_2O$$

Equating C, H and O on the two sides of the equation,

$$a\times10=d \quad \therefore \quad d=10a$$

$$a\times22=e\times2 \quad \therefore \quad e=11a$$

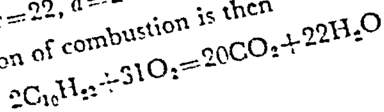
$$b\times2=d\times2+e\times1=10a\times2+11a\times1=31a \quad \therefore \quad b=\frac{31}{2}a$$

Taking $a=1$, we get, $e=11$, $d=10$ and $b=\frac{31}{2}$

To avoid fraction take $a=2$

Then, $e=22$, $d=20$ and $b=31$

$\therefore$ The equation of combustion is then



$$2C_{10}H_{22}+31O_2=20CO_2+22H_2O$$

7.2 Minimum Air Required for Complete Combustion. From the previous article it is seen that an adequate supply of oxygen is essential for complete combustion and this is obtained from atmospheric air along with nitrogen. For all practical purposes we can consider air to contain oxygen by mass 23 per cent and by volume 21 per cent and nitrogen by mass 77 per cent and by volume 79 per cent. For calculating the amount of oxygen required, the oxygen content of the fuel itself must be considered as it also takes part in combustion. Thus, net oxygen required will be less. The calculations of air required are given in the Table 2 for a fuel having the following analysis by mass. C=88.1%, H_2=3.6%, O_2=2.6%, N_2=0.8%, S=0.9%, H_2O=0.8% and ash 3.2%

Table 7.2

Mass of Air Required for a Fuel

Mass of constituents per kg of fuel (a)	*Oxygen required per kg of constituents* (b)	*Oxygen required per kg of fuel* (c) = (a) × (b)
C=0.881	8/3	[illegible]
H_2=0.036	8	0.288
O_2=0.026	—	−0.026
N_2=0.008	—	
S=0.009	1	0.009
H_2O=0.008	—	
ash=0.032	—	
1.000	—	Σ(c)=2.627

∴ Mass of air required (theoretical)

$$= \frac{2.627 \times 100}{23} = 11.42 \text{ kg}$$

Excess air. In practice excess air is supplied for the complete combustion of the fuel because all air does not come into intimate contact with the particles of fuel. The quantity of excess air depends on the type of fuel and method of combustion used and may be from [illegible] to [illegible] per cent. Excess air ensures better comb[illegible]

same time cools the furnace and increases heat loss in the exhaust gases.

7.3. Analysis of Flue Gases by Orsat Apparatus. The volumetric analysis of dry products of combustion from a boiler or an engine is done by means of Orsat apparatus (Fig. 7·1). It consists of a graduated eudiometer tube connected to an aspirator-bottle containing water and three double absorption flasks of CO_2, O_2 CO respectively. The front flasks are packed with glass tubes to increase the wetted surface.

The first flask next to eudiometer tube contains 40 per cent of KOH solution which would absorb twenty times its own volume of CO_2. The second flask contains pyrogallic acid and KOH. This mixture may be made by mixing 5 gm of powdered pyrogallic acid with 100 cc of 40 per cent KOH solution which would absorb twice its own volume of O_2. The third flask contains a solution of 5 gm of copper oxide in 100 cc of commercial HCl, which would absorb CO equal to its own volume.

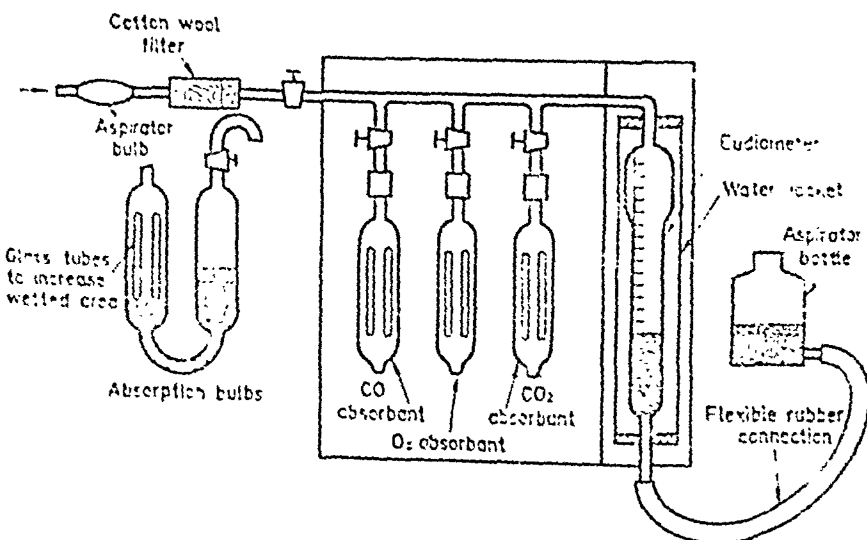


Fig. 7·1. Orsat apparatus

Gas to be analysed may be collected directly into eudiometer tube after removing air, or it may be collected in jars on salt water (pure water may absorb some gases). After 100 c c. of gas is taken into

eudiometer tube it is forced into the first flask by raising the bottle whence CO_2 is absorbed. The liquid in the flask is then levelled back by slowly lowering the aspirator bottle, and the reading of eudiometer tube is taken. The difference in reading gives percentage of CO_2 by volume. The process is repeated with second and third flasks to determine O_2 and CO percentages.

Additional flasks may be provided for the determination of hydrocarbons and hydrogen. The balance is assumed N_2.

The flue gases will have H_2O vapour due to presence of hydrogen in fuel. But in Orsat apparatus only dry flue gas will be collected as H_2O vapour will condense at room temperature. Therefore the percentage CO_2 obtained by Orsat analysis will be slightly more (due to the absence of H_2O).

7.4. Conversion of Volumetric Analysis to Analysis by Mass. Let the volumetric analysis of dry flue gases, as measured by Orsat apparatus, burning the fuel mentioned in article 7·2, be CO_2=7 79%, CO=0 535%, O_2=11·6% and N_2=80·075%. The volumetric analysis can be converted to mass analysis as in Table 7 3.

Table 7·3

Coversion of Volumetric Analysis into Mass Analysis

Volume of constituents per mol of d/g	*Mol wt.*	*Proportional mass of constituents*	*Mass per kg of flue gases*	*Mass of carbon per kg of flue gases*
(a)	(b)	(c)=(a)×(b)	(d)=(c)/Σ(c)	(e)
N_2=80 075	28	28×80 075=2,424	0 75461,75 46%	
CO=0 535	28	28×0·535=15	0 00505, 0 505%	$\frac{0\ 00505\times12}{28}$=0 002164
CO_2=7·790	44	44×7·790=343	0 11551, 11 55%	$\frac{0\ 11550\times12}{44}$=0 03150
O_2=11 600	32	32×11 600=371	0 12491, 12·49%	
Total 100 000		Σ(c) 2,971	Σ(d)1 00005,100 005	0 033664

The principle behind conversion of mass analysis into volume analysis and vice-versa should be clearly understood 22 41 m^3 (one mol) of all gases at N T.P. (760 mm of water and 0°C) weigh equal

to their molecular weight. Therefore, dividing by molecular weight we get mol or proportional volume. Similarly, by multiplying mol (or volume) by molecular weight we get mass.

7·5. Mass of Dry Flue Gases per Kg of Fuel Burnt. In flue gases only CO_2 and CO contain carbon. From equations (7·1) and (7·2) we know that 44 kg of CO_2 contains 12 kg of carbon, and 28 kg of CO contains 12 kg of carbon. From this carbon content per kg of flue gases is found as in table 7·3. Knowing total carbon per kg of fuel (*i.e.* carbon content of fuel), the mass of dry flue gases can be found as follows ;

Mass of dry flue gases per kg of fuel

$$= \frac{\text{mass of carbon per kg of fuel}}{\text{mass of carbon per kg of gases}} \qquad (7{\cdot}8)$$

$$= \frac{0{\cdot}884}{0{\cdot}033664} = 26{\cdot}26 \text{ kg}$$

7·6. Mass of Excess Air Supplied. Minimum O_2 required is on the basis of complete combustion, *i.e.* whole C should be burnt to CO_2 and no CO should be present in the flue gases. If in a case, CO and O_2 are present in flue gas analysis, whole of this O_2 is not "excess". Excess O_2 is found by subtracting the amount of O_2 required for burning CO to CO_2.

Again taking previous problem

Mass of flue gases per kg of fuel $= 26{\cdot}26$ kg

From table 7·3

Mass of CO per kg of fuel $= 26{\cdot}26 \times 0{\cdot}00505 = 0{\cdot}1326$ kg

Mass of unused O_2 per kg of fuel $= 26{\cdot}26 \times 0{\cdot}1249 = 3{\cdot}27987$ kg

Mass of O_2 required for burning of CO

$$= 0{\cdot}1326 \times \tfrac{4}{7} = 0{\cdot}07577 \text{ kg}$$

Excess O_2 per kg of fuel $= 3{\cdot}27987 - 0{\cdot}07577 = 3{\cdot}2041$ kg

$$\therefore \quad \text{Excess air} = \frac{3{\cdot}2041 \times 100}{23} = 13{\cdot}93 \text{ kg/kg of fuel}$$

$$\text{or Percentage excess air} = \frac{13{\cdot}93}{11{\cdot}42} = 122 \text{ per cent}$$

7·7. Heat Carried Away by Flue Gases. To calculate the heat carried away by flue gases the mean specific heat of the flue

gases is obtained from its mass analysis as shown in table 7·4. The mass analysis of the flue gases is taken from Table 7-2.

Table 7·4

Specific Heat of Dry Flue Gases

Constituents	*Mass/kg of dfg* (a)	C_p (b)	*Heat per kg of dfg* (c)=(a)×(b)
CO_2	0·1155	0·216	0 0249
CO	0 005	0 245	0·0012
O_2	0 1249	0 2175	0 0297
N_2	0 7546	0·2438	0·1846
	1·0000		C_p=0·2404

Heat carried away by dfg/kg of fuel burnt

$=mC_p(t_1-t_o)$

where t_1= temperature of flue gases leaving boiler

t_o=temperature of boiler house

Heat carried away by steam in flue gas/kg of fuel burnt

$=m_1(h_1-h_2)$

where m_1=mass of H_2O formed/kg of fuel

h_1=enthalpy of superheated steam at temperature t_1 and p p of 0 07 kgf/cm^2

h_2=enthalpy of water at boiler house temperature

∴ Total heat carried away by flue gases=heat carried by dfg+heat carried by H_2O formed

$$=mC_p(t_1-t_o)+m_1(h_1-h_2) \quad (7\ 9)$$

7·8. Calorific Value of a Fuel The combustion of fuel is accompanied by the evolution of a large amount of heat The number of heat units evolved by the complete combustion of one kg of a fuel is called the calorific value of fuel. The calorifie value is measured in two ways :—

Higher calorific value (H.C.V.). The higher or gross calorific value is the total heat liberated by the unit quantity of fuel when

the products of combustion are cooled to room temperature. It can be determined accurately by experiment only. Dulong has given an approximate method of calculating higher calorific value as follows :—

$$\text{H.C.V. of coal} = 8{,}100C + 33{,}980(H - O/8) + 22{,}105 \text{ in kcal}$$

where 33,980 kcal is the higher C.V. of hydrogen. In this equation of H.C.V. it is assumed that oxygen present in the fuel is combined with hydrogen in the fuel in the ratio 8 : 1 so that useful hydrogen available for combustion is only $(H - O/8)$.

Lower calorific value (L.C.V.). The lower calorific value is defined as

H.C.V. – heat taken away by H_2O, due to combustion and surface moisture.

The evaporation of H_2O is assumed at partial pressure corresponding to saturation temperature of 20°C. H_2O is therefore supposed to take latent heat at this pressure ; this heat is 586 kcal

$$\text{L.C.V.} = \text{H.C.V.} - \text{Mass of } H_2O \times 586, \text{ in kcal} \qquad (7{\cdot}10)$$

This definition has been standardised but it does not take into account actual temperature of flue gases and therefore does not give actual heat loss due to steam formation.

7·9. Coal Analysis and Sampling of Coal. The analysis of coal is represented in two ways (*i*) proximate analysis and (*ii*) ultimate analysis.

Proximate analysis. It is the determination of percentages of moisture (3-30%), volatile matter (3-50%), fixed carbon (16-92%) and ash (2-30%) in the fuel. Proximate analysis is easy and quick to determine compared with an ultimate analysis and is usually done for commercial purposes.

Ultimate analysis. It is a more precise chemical test for finding the compositions with respect to the element, like, carbon (50-95%), hydrogen (2-5%), oxygen (2-40%), nitrogen (0·5-3%), sulphur (0·5-7%) and ash (2-30%). The chemical composition of fuel is very useful in combustion calculations and in finding the composition of the flue gases.

Both the analysis may be done on fuel as obtained or on "dry" fuel basis

Sampling of coal for boiler trial. Samples of coal are taken continuously during the trial to obtain (2-3 cwt) of coal. It is reduced to 10-30 lb by coning and quartering and is then finely ground. For moisture content determination about 1-2 gm is weighed and placed in a drying oven at 105°C-110°C for one hour, cooled in a desicator and re-weighed.

For calorific value determination a small quantity of the final sample is dried and compressed in a pellet. The remainder of sample can be used for proximate or ultimate analysis.

7·10 Bomb Calorimeter. For measuring calorific value of solid and liquid fuels the apparatus generally used is Bomb calori-

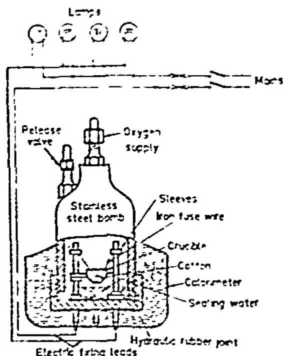


Fig. 7·2. Bomb Calorimeter

meter (Fig. 7·2). It consists of a stainless steel 'bomb' tested for a pressure of 100 kgf/cm². The fuel briquette of about 3/4 gm mass is placed in a crucible inside it. The bomb has electric connections for causing spark, a connection for oxygen supply and a release valve for

burnt gases. The bomb is kept in calorimeter which contains 2,500 c c of water stirred by an electric motor. Radiation is reduced by having a felt-lagged water jacket around the calorimeter. Oxygen at a pressure of 25 atmosphere is filled in the calorimeter. Water in the calorimeter is stirred and when temperature becomes constant, fuel is ignited and readings are taken at regular intervals for about 15 minutes, by a precise thermometer known as "Beckmann thermometer".

Correction for radiation is made by plotting temperature on a time basis as shown in Fig. 7·3.

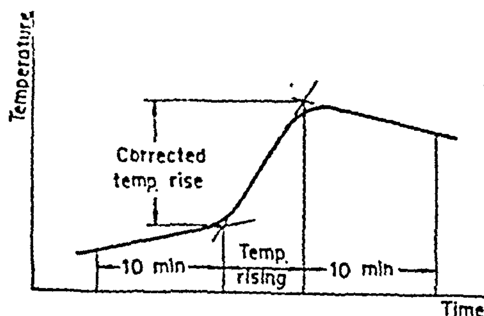


Fig. 7·3. Correction for radiation.

7·11. Gas Producers. A gas producer converts solid fuels into combustibles gaseous fuels. It is done by blowing restricted

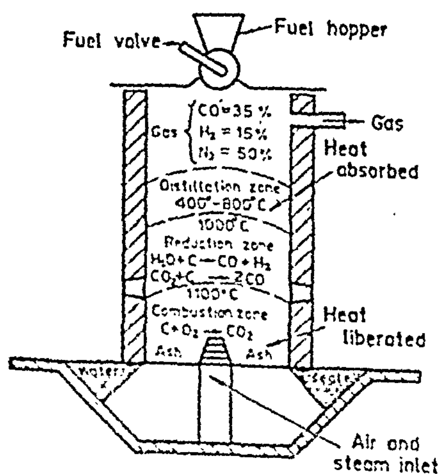


Fig. 7·4. Gas producer.

supply of air or steam and air through a deep bed of incandescent carbon. In the simplest form a gas producer is a vertical cylindrical vessel, brick lined or water jacketed with a grate at bottom to support the fuel bed (Fig. 7·4). It is provided with an inlet for blast (either air or mixture of steam and air) through grate, an inlet at top for feeding the fuel and an outlet at top for gaseous fuel.

Gas producers in which the steam-air mixture is forced by external means are known as *pressure producers*. Gas producers in which the steam air mixture is drawn by the suction stroke of the engine are known as *suction producers*. Pressure producers are meant for larger plants whereas suction producers are used for plants of 30 to 40 hp and rarely going beyond 500 h p.

(*a*) *When charge is air only*. When no CO_2 is formed the reaction is as follows :



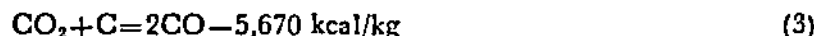
$$2C+O_2=2CO+2{,}430 \text{ kcal/kg} \quad (1)$$

Alternatively CO_2 may be formed according to the equation



$$C+O_2=CO_2+8{,}100 \text{ kcal/kg} \quad (2)$$

In the presence of incandescent carbon this CO_2 is reduced to CO, according to the equation



$$CO_2+C=2CO-5{,}670 \text{ kcal/kg} \quad (3)$$

The net result of reactions (2) and (3) is same as (1).

The calorific value of CO per kg of carbon is thus, 8,100-2,430 =5,670 kcal per kg of carbon

$$\therefore \text{ Efficiency of producer} = \frac{5{,}670}{8{,}100}=70\%$$

Volume of gas per kg of carbon. From eq 1 when 1 kg of carbon is burnt, O_2 required $= \frac{22{\cdot}41}{24} = 0\,9338 \text{ m}^3$

and CO formed is $= \frac{2\times 22{\cdot}41}{24} = 1\,8676 \text{ m}^3$

N_2 in gas (associated with O_2 in air)

$$= \frac{0\,9338\times 79}{21} = 3{\cdot}5129 \text{ m}^3$$

$\therefore$ Analysis of gas produced will be, CO

$$= \frac{1{\cdot}8676}{1{\cdot}8676+3\,5129} = 34{\cdot}7\%$$

and $$N_2 = \frac{3{\cdot}5129}{1{\cdot}8676+3{\cdot}5129} = \underline{65{\cdot}3\%}$$

Calorific value of this gas $= \dfrac{5{,}670}{1{\cdot}8676+3{\cdot}5129} = \underline{1{,}054 \text{ kcal/m}^3}$

(*b*) *When charge is steam and air.* In previous case when air alone is drawn, CO is formed with surplus heat of 2,430 kcal per kg of carbon burnt. This tends to produce very high temperature rise of the producer plant. This rise in temperature may cause fusing of the ash and therefore stoppage of blast, and in extreme cases, fusing of the refractory lining and fire bars. In order to absorb this surplus heat water or steam is usually supplied along with air. The water reacts with carbon and liberates hydrogen (which absorbs excess heat produced by burning C to CO) and incidentally, it results in richer producer gas (i.e. CO and H_2) and higher thermal efficiency of the plant.

Combustion equations. Fig. 7·4 shows the various combustion zones in a gas producer when steam and air is drawn.

(*i*) *Ash or clinker zone.* This increases the temperature of steam and air blast.

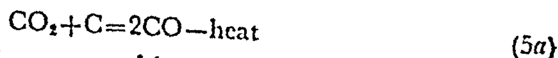
(*ii*) *Combustion zone.* In this zone temperature is high enough and C burns to CO_2, liberating heat.



$$C + O_2 = CO_2 + \text{heat} \qquad (4a)$$

$$1 \text{ kg of C} + \frac{8}{3} \text{ kg of } O_2 = \frac{11}{3} \text{ kg of } CO_2 + 8{,}100 \text{ kcal} \qquad (4b)$$

(*iii*) *Reduction zone.* CO_2 of combustion zone comes in contact with carbon forming CO

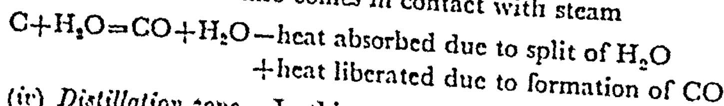


$$CO_2 + C = 2CO - \text{heat} \qquad (5a)$$

$$1 \text{ kg of } CO_2 + \frac{3}{11} \text{ kg of C} = \frac{14}{11} \text{ kg of CO} - \text{heat} \qquad (5b)$$

$$\frac{11}{3} \text{ kg of } CO_2 + 1 \text{ kg of C} = \frac{14}{3} \text{ kg of CO} - \text{heat} \qquad (5c)$$

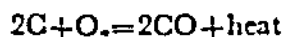
In this zone carbon also comes in contact with steam



$$C + H_2O = CO + H_2O - \text{heat absorbed due to split of } H_2O$$
$$+ \text{heat liberated due to formation of CO}$$

(*iv*) *Distillation zone.* In this zone the coming charge (coal) is heated by outgoing gases, thus utilizing its heat which also distils the fuel.

Theoretical mass of steam and air required. Combining equations



$$2C+O_2=2CO+\text{heat}$$

$$1\text{kg of C}+\frac{4}{3}\text{kg of }O_2=\frac{7}{3}\text{kg of CO}+2{,}430\text{ kcal} \quad (7)$$

Repeating Eq. (6), $C+H_2O=CO+H_2$

$$1\text{kg of C}+\frac{3}{2}\text{kg of steam}$$

$$=\frac{7}{3}\text{kg of CO}+\frac{1}{6}\text{kg of }H_2+2{,}430-\frac{1}{6}\times 35{,}400$$

$$=\frac{7}{3}\text{kg of CO}+\frac{1}{6}\text{kg of }H_2-3{,}470\text{ kcal} \quad (8)$$

From the Eq. (7) and (8) it is seen that the heat is liberated in the former case whereas heat is required in the latter case. To balance the heat given in Eq (7) to that required in Eq (8), multiply Eq. (7) by $\frac{3{,}470}{2{,}430}=1{\cdot}428$.

$$1{\cdot}428\text{ kg of C}+1{\cdot}904\text{ kg of }O_2=3\ 332\text{ kg of CO}+3{,}470\text{ kcal} \quad (9)$$

Combining Eq. (8) and (9)

$$2{,}428\text{ kg of C}+1{\cdot}5\text{ kg of steam}+1\ 904\text{ kg of }O_2$$
$$=5\ 6653\text{ kg of CO}+0\ 1667\text{ kg of }H_2 \quad (10)$$

$$1\text{ kg of C}+0\ 6178\text{ kg of steam}+0\ 7843\text{ kg of }O_2$$
$$=2{\cdot}3334\text{ kg of CO}+0{\cdot}0687\text{ kg of }H_2 \quad (11)$$

Mass of steam required per kg of carbon = 0 6178 kg

Mass of air required per kg of carbon

$$=\frac{0{\cdot}7843\times 100}{23}=3\ 41\text{ kg}$$

N_2 associated with oxygen in air $=\frac{3\ 41\times 77}{100}=2\ 6257$ kg

From Eq. (11), the yield per kg of carbon is

$CO=2{\cdot}3334$ kg ; $H_2=0\ 0687$ kg ; $N_2=2\ 6257$ kg

Volumetric composition of gas. This is found as in Table below :

Constituents of gas per kg of carbon burnt (a)	*Density of constituents* (b)	*Volume in m³* (c)=(a)/(b)	*Percentage volumetric composition* $(d)=\frac{(c)}{\Sigma(c)}$
$CO = 2.3334$	$\frac{28}{22.41}$	1·867	39·3
$H_2 = 0.0687$	$\frac{2}{22.41}$	0·77	16·2
$N_2 = 2.6257$	$\frac{28}{22.41}$	2·103	44·5
Total		4·74	100·00

From the above table it is seen that yield in m³/kg of carbon burnt is 4·74 m³.

$$\underline{\text{The calorific value of the gas}} = \frac{8,100}{4.74} = \underline{1,710 \text{ kcal/m}^3}$$

Important Combustion Formulae. Though it is advisable to solve problems from fundamentals, the following formulae are useful for checking the results. These formulae can also be used when time available is short.

Let m = Mass of air supplied per kg of fuel

C = Carbon content per kg of fuel

H_2 = Hydrogen content per kg of fuel

N_2 = Percentage of N_2 by volume in dry flue gas

CO_2 = ,, CO_2 ,, ,,

CO = ,, CO ,, ,,

O_2 = ,, O_2 ,, ,,

(i) Air used per kg of fuel $= \dfrac{N_2 \times C}{33(CO_2 + CO)}$ (7·10)

(ii) Carbon burnt to CO per kg of fuel $= \dfrac{CO \times C}{CO_2 + CO}$ (7·11)

(*iii*) Mass of excess air per kg of fuel

$$=\frac{79\ O_2\times C}{21\times 33(CO+CO_2)} \qquad (7{\cdot}12)$$

(*iv*) Volume of excess air per 100 m^3 of gas

$$=\frac{\text{Minimum volume of dry products}\times O_2}{(21-O_2)} \qquad (7{\cdot}13)$$

(*v*) Percentage of CO_2 by volume in dry flue gases

$$=\frac{80{,}000\times C}{3(11m-800H)} \qquad [7{\cdot}14(a)]$$

For ordinary coal, percentage of CO_2 is approximately

$$=\frac{200}{m} \qquad [7{\cdot}14(b)]$$

The formula [7·14(*b*)] may be used as an approximate check only.

IMPORTANT POINTS

1. Some fuels have got O_2 as one of the constituents. In such cases minimum O_2 required for complete combustion
= Theoretical O_2 required $-$ O_2 present in the fuel.

2. Minimum O_2 required is on the basis of complete combustion, *i.e* whole C should be burnt to CO_2 and no CO should be present in the flue gas. If in a case CO and O_2 are present in flue gas analysis, whole of this O_2 is not excess O_2. Excess O_2 is found by subtracting the amount of O_2 required for burning CO to CO_2.

3. Excess air per kg of fuel

$$=\frac{100}{23}\ [\text{excess } O_2 \text{ per kg of dfg}\times \text{mass of dfg per kg of fuel}].$$

4. The amount of dry flue gases formed by combustion of 1 kg of fuel may be found by different methods. The method generally used is to calculate carbon in 1 kg of flue gas and divide the carbon content in 1 kg of fuel by carbon in 1 kg of flue gas.

The amount of dry flue gas can also be calculated from N_2 or O_2 content.

5. Gaseous fuels generally have CO_2 and N_2 in the fuel In the exhaust gas analysis calculation, this CO_2 should be added to the CO_2 formed by combustion and N_2 should be added to the N_2 in the air.

6. The method of determining experimentally the exhaust gas analysis excludes H_2O and SO_2 vapour. Hence the "volumetric analysis" given is always for "dry products of combustion". When finding volumetric analysis by calculation, it should also be found on "dry basis", unless otherwise mentioned.

7. The steam in the products of combustion is at partial pressure and therefore the heat taken by steam should be calculated by first determining the partial pressure of steam, if sufficient data is given. If partial pressure is not given or cannot be calculated, the approximate methods of finding the heat taken away by steam in the products of combustion are as follows :

(a) By assuming the partial pressure of steam as 0·07 kgf/cm².

In both cases the specific heat of superheated steam may be taken as 0·48.

(b) When the steam is superheated, total heat for a particular temperature changes very little for pressure of 1 kgf/cm² and less ; therefore, calculations may be done on the basis of steam pressure as 1 kgf/cm².

8. When writing or calculating the percentage analysis it is golden rule to add and check whether the total is 100. It will detect mistakes in calculation, if any.

9. The N_2 percentage by volume in exhaust gases is about 80% for solid fuels, and about 84% for liquid fuels. This data is useful in checking the results.

ILLUSTRATIVE EXAMPLES

7-1. Gravimetric analysis of wet and dry exhaust gases : partial pressure of steam.

Define 'proximate' and 'ultimate analysis' of coal. How sampling of coal is done in a boiler trial ?

A fuel oil contains 85·5% of carbon, 12·3% of hydrogen and 2·2% incombustible residue. If 25 kg of air per kg of fuel is supplied, determine the wet and dry gravimetric analysis of the exhaust gases. Assume air contains 23% of oxygen by mass.

If vapour of combustion is regarded as perfect gas, estimate its partial pressure of steam, given that the total pressure of exhaust gases is 1 kgf/cm².

For theory—see text.

Mass of constituents per kg of fuel (a)	Combustion equation (b)	Mass of O_2 required per kg of fuel (c)	Products of combustion $(d)=(a)+(c)$	
			CO_2	H_2O
C=0·855	$C+O_2=CO_2$ $12+32=44$	$0{\cdot}855\times\frac{32}{12}=2{\cdot}280$	3·135	
H_2=0·123	$2H_2+O_2=2H_2O$	$0{\cdot}123\times8=0{\cdot}984$		1·107
ash=0·022	$1+8=9$			
Total=1·000		Total O_2=3·264		

Since air supplied is 25 kg

$$\text{Excess } O_2=\frac{23}{100}\times25-3{\cdot}264=2{\cdot}486 \text{ kg}$$

and
$$N_2 \text{ supplied}=\frac{77}{100}\times25=19{\cdot}25 \text{ kg}$$

Constituents of wet exhaust gases in kg (a)	% gravimetric analysis of wet exhaust gases $(b)=\frac{(a)}{\Sigma(a)}$	Mol wt M (c)	Proportional volumes or mol (d) $=(a)/(c)$	Constituents of dry exhaust gases in kg (e)	% gravimetric analysis of dry exhaust gases $(f)=\frac{(e)}{\Sigma(e)}$
CO_2= 3·135	12·09	44	0·274	CO_2= 3·135	12·59
H_2O= 1·107	4·20 Ans.	18	0·234		Ans.
O_2= 2·486	9·59	32	0·299	O_2= 2·486	10·00
N_2=19·25	74·12	28	0·645	N_2=19·25	77·41
Total 25·978	100·00		3·452	24·871	100·00

$$\text{Partial pressure of steam} = \frac{H_2O \text{ mol}}{\text{Total mol}} \times \text{Total pressure}$$

$$= \frac{0{\cdot}234}{3{\cdot}452} \times 1 = \underline{0{\cdot}0678 \text{ kgf/cm}^2} \qquad \text{Ans.}$$

2. Orsat apparatus : percentage of CO_2.

Describe the apparatus that is commonly used for the analysis of dry exhaust gases. Discuss the probable sources of error in such an analysis.

A boiler burns coal of the following composition : $C=88\%$, $H_2=3{\cdot}8\%$, $O_2=2{\cdot}2\%$ and the remainder ash. On a particular occasion the percentage of CO_2 passing up the chimney was 10%. The temperature of chimney gases was then 250°C. If a sample of this gas is analysed by the Orsat apparatus at room temperature, what percentage of CO_2 would you expect, assuming complete combustion of the fuel.

For theory—see text.

Mass of constituents per kg of fuel (a)	Mass of product of combustion (b)		Mol wt (c)		Proportional volume (d)=(b)/(c)	
	CO_2	H_2O	CO_2	H_2O	CO_2	H_2O
C=0·880	$0{\cdot}88 \times \frac{44}{12}$	$0{\cdot}038 \times 1$	44	18	0·733	0 01
H_2=0·038						
O_2=0·022						

$$\text{Total exhaust gases} = CO_2 + H_2O + O_2 + N_2$$

$$\text{Actual percentage of } CO_2 = \frac{CO_2 \times 100}{CO_2 + H_2O + O_2 + N_2}$$

$$10 = \frac{0{\cdot}0733 \times 100}{0{\cdot}0733 + 0{\cdot}019 + O_2 + N_2}$$

$$\therefore \quad O_2 + N_2 = 0{\cdot}641$$

In Orsat apparatus only dfg is measured

$$\therefore \quad \underline{\text{Percentage of } CO_2} = \frac{CO_2 \times 100}{CO_2 + O_2 + N_2}$$

$$= \frac{0{\cdot}0733 \times 100}{0{\cdot}0733 + 0{\cdot}641} = \underline{10{\cdot}28\%}$$

7-3. Gravimetric and volumetric analysis ; $\triangle U$.

The gravimetric analysis of a fuel oil shows 86 2 per cent carbon and 13 8 per cent hydrogen. If 50 per cent excess air is supplied find the percentage analysis of the products (a) by mass, and (b) by volume.

Also find with the aid of the values tabled below, the change in internal energy of 1 kg of the products when cooled from 2,100°C to 900°C. Atomic weights of carbon, oxygen, nitrogen, and hydrogen are 12, 16, 14 and 1 respectively Air contains 23 2 per cent of oxygen by mass.

T deg °K	*Internal energies, kcal/kg*			
	CO_2	H_2O	N_2	O_2
2,100	*488*	*940*	*426*	*405*
900	*162*	*324*	*166*	*152*

(a) Theoretical mass of oxygen required per kg of fuel for complete combustion

$$=0.862\times\tfrac{8}{3}+0.138\times8=3\ 403\text{ kg}$$

Excess $O_2=3.403\times0.5=1.7015$ kg

Constituent of exhaust gases per kg of fuel

$CO_2=0.862\times\frac{11}{3}=3.161$ kg, $H_2O=0\ 138\times9=1\ 242$ kg

O_2(excess) $=1.702$ kg $\quad N_2=(3\ 403\times1\ 5)\times\frac{76\ 8}{23\ 2}=16\ 9$ kg

Constituents of exhaust gases in kg	% gravimetric analysis	Mol Wt	Proportional volumes	% Volumetric analysis of wet exhaust gases
(a)	(b) $=\frac{(a)}{\Sigma(a)}$	(c)	(d)=(b)/(c)	(e)$=\frac{(d)}{\Sigma(d)}$
$CO_2=$ 3·161	13 7	44	0·3114	8 97
$H_2O=$ 1 242	5 4	18	0 3	8 60
$O_2=$ 1·702	7 4	32	0 2313	6 67
$N_2=$16 9	73 4	28	2 6214	75 7
Total=23 005	99 9		3 4641	100 00

(*b*)

Constituents of wet exhaust gases in kg	*Change in internal energy, per kg of constituents*	*Change in internal energy per kg of exhaust gases*
(*a*)	(*b*)	(*c*)=(*a*)×(*b*)
CO_2=0·137	488−162=326	44·66
H_2O=0·054	940−324=616	33·26
O_2=0·074	436−166=270	19·98
N_2=0·734	405−152=253	18·57
	Change in internal energy=116·74 kcal	Ans.

7-4. Percentage excess air ; C_p, given fuel and dfg analysis.

The coal supplied to a boiler furnace has the following ultimate analysis :

Carbon 82 per cent, hydrogen 5·4 per cent, oxygen 7 per cent, nitrogen 0·5 per cent, sulphur 0·6 per cent, moisture 1 per cent and the rest is ash. The volumetric analysis of the dry flue gases was CO_2 9·6 per cent, CO 1·2 per cent, N_2 81·4 per cent and O_2 7·8 per cent. Determine,

(*a*) *percentage of excess air supplied to the boiler furnace,*

(*b*) *mean specific heat of the dry flue gases.*

Given the specific heats at constant pressure CO_2=0·21, O_2=0·218, N_2=0·244 and CO=0·248.

(*a*) Mass of air required per kg of fuel for complete combustion

$$=\frac{100}{23}\{0\cdot82\times\frac{8}{3}+0\cdot054\times8+0\cdot06\times1-0\cdot07\}=11\cdot11 \text{ kg}$$

Volume of constituents per mol of dfg	*Mol Wt*	*Proportional mass of constituents*	*Mass per kg of flue gas*	*Mass of carbon per kg of flue gas*
(a)	(b)	(c)=(a)×(b)	(d)=(c)/Σ(c)	(e)
CO_2= 9 6	44	9·6×44=422 4	0 14160	$\frac{0{\cdot}1416\times12}{44}$=0 03862
CO= 1·2	28	1·2×28=33 6	0 01126	$\frac{0{\cdot}01126\times12}{28}$=0 00482
N_2= 81·4	28	81·4×28=2,279	0 76350	
O_2= 7 8	32	7·8×32=249·6	0·08364	
Total=100·0		2,984 6	1 00000	0 04344

Mass of dry flue gases per kg of fuel

$$= \frac{\text{Mass of carbon per kg of flue gases}}{\text{Mass of carbon per kg of fuel}}$$

$$= \frac{0{\cdot}82}{0\ 04344} = 18{\cdot}88 \text{ kg}$$

From table above

Mass of CO per kg of fuel = 18 88 × 0 01126 = 0 2126 kg

Mass of unused O_2 per kg of fuel = 18 88 × 0 08364 = 1 5791 kg

Mass of O_2 required for burning of CO = $\frac{0\ 2126\times4}{7}$ = 0 1215 kg

Excess O_2 per kg of fuel = 1·5791 − 0·1215 = 1 4576 kg

$\therefore$ *Exces air* = $\frac{1{\cdot}4576\times100}{23}$ = 6·3374 kg

or Percentage excess air = $\frac{6\ 3374}{11{\cdot}11}$ = 57 05% Ans.

(*b*) Mean specific heat of dry flue gases

Constituent	*Mass per kg of dry flue gas*	*Specific heat of constituents at constant pressure*	*Heat per kg of dry flue gas*
(*a*)	(*b*)	(*c*)	(*d*)=(*b*)×(*c*)
CO_2	0·14160	0·21	0·0297
CO	0·01126	0·248	0·0028
N_2	0·76350	0·244	0·1863
O_2	0·08364	0·218	0·0182
Total	1·00000		0·2370

Mean value of specific heat of dry flue gas $C_p = 0{\cdot}237$ Ans.

7-5. Bomb calorimeter : H.C.V. and L.C.V.

Describe the method of determining the calorific value of a solid fuel by means of a Bomb calorimeter. How correction for cooling is obtained in the above method.

A Bomb calorimeter was used to determine the calorific value of a sample of coal whose hydrogen content is 4 per cent and the following results were recorded :

Mass of sample of coal $= 1$ *gm*
Mass of water in calorimeter $= 2{,}500$ *gm*
Water equivalent of apparatus $= 744$ *gm*
Initial temperature of water $= 17{\cdot}48°C$
Maximum observed temperature of water $= 20{\cdot}07°C$
Cooling correction $= +0{\cdot}015°C$

Determine from these results the "higher" and the "lower" calorific value of the fuel.

Corrected rise in temperature $= (20{\cdot}07 - 17{\cdot}48) + 0{\cdot}015 = 2{\cdot}605°C$

Heat to calorimeter and water $= (2{,}500 + 744) \times 2{\cdot}605 = 8{,}460$ cal

Higher calorific value, $\underline{H.C.V. = 8{,}460 \text{ cal/gm}}$ **Ans.**

$$\text{Mass of steam formed} = 0{\cdot}04 \times 9 = 0{\cdot}36 \text{ gm}$$

Lower calorific value, $\underline{L.C.V.} = H.C.V. - \text{mass of steam} \times 585$

$$= 8{,}460 - 0{\cdot}36 \times 585$$

$$= \underline{8{,}249 \text{ cal/gm}}$$ **Ans.**

7·6. Volumetric analysis : L.C.V.

A gaseous fuel has the following analysis by volume—H_2, 52 ; CH_4, 20 ; CO, 16 ; CO_2, 3 ; O_2, 2 ; and N_2, 7 per cent. Determine :

(a) the volume of air required for complete combustion of 1 m^3 of gas ;

(b) the volumetric analysis of the "dry" products of combustion with 30 per cent excess air ;

(c) the lower calorific value of the gas at 1 kgf/cm^2 and 0°C, allowing 585 kcal for the latent heat of one kg of water of combustion.

The higher calorific values of H_2, CO and CH_4 are 6,800, 6,570 and 21,000 kcal per m^3 at 1 kgf/cm^2 and 0°C, respectively.

Air contains 20·9 per cent by volume of O_2.

(a)

Constituents per m^3 of gas (a)	*O_2 required per m^3 of constituents* (b)	*O_2 required per m^3 of gas* (c)=(a)×(b)	*Products after combustion per m^3 of gas*			
			CO_2	H_2O	O_2	N_2
H_2 = 0·52	0·5	0·26		0·52		
CH_4 = 0·20	2	0·40	0·20	0·40		
CO = 0·16	0·5	0·08	0·16			
CO_2 = 0·03	—	—	0·03			
O_2 = 0·02	—	−0·02			0·216	
N_2 = 0·07	—					3·619
Total = 1·00		0·72	0·39	0·[illegible]2	0·216	3·619

Volume of air required $= 0\cdot72 \times \frac{100}{20\cdot9} = 3\cdot45\ m^3/m^3$ of gas. Ans.

(b) Excess O_2 supplied $= 0\cdot72 \times 0\cdot3 = 0\cdot216\ m^3$

Total N_2 supplied $= (0\cdot72 + 0\cdot216) \times \frac{79\cdot1}{20\cdot9} = 3\cdot549\ m^3$

Total volume of dry products of combustion

$$= 0\cdot39 + 0\cdot216 + 3\cdot619 = 4\cdot225\ m^3$$

∴ Volumetric analysis of dry products of combustion

$CO_2 = 9\cdot23\%$; $O_2 = 5\cdot11\%$; $N_2 = 85\cdot66\%$ Ans.

(c) Higher calorific value, H.C.V.

$$= 0\cdot52 \times 6{,}800 + 0\cdot20 \times 21{,}000 + 0\cdot16 \times 6{,}570$$

$$= 8{,}787\ kcal/m^3$$ Ans

Mass of steam generated per m^3 of gas

$$= \frac{0\cdot92}{22\cdot41} = 0\cdot04106\ kg$$

Hence, H.C.V. $-$ L.C.V. $= 585 \times 0\cdot04106 = 24$ kcal

∴ Lower calorific value,

L.C.V. $= 8{,}787 - 24 = 8{,}763\ kcal/m^3$ Ans.

7-7. Volumetric composition before and after combustion of C_7H_{16} with excess air :

Find the percentage volumetric composition of the pre-combustion and the post-combustion mixtures when heptane (C_7H_{16}) is burnt with 20 per cent excess air. Find also the molecular weight, the specific volume and the gas constant at S.T.P. of the post-combustion mixture. Assume that air contains 21 per cent of oxygen and 79 per cent of nitrogen by volume. Molecular weights $CO_2 = 44$; $H_2O = 18$; $O_2 = 32$; and $N_2 = 28$. The volume of the kg mol at S.T.P. is 22·41 m^3 and the universal gas constant is 848 kg m/kg °K.

Pre-combustion volumetric composition :

Constituents of C_7H_{16}	Proportional mass	Proportional volume	Percentage volume
	(a)	(b) = (a)/mol wt	(c) = [illegible] × 100
C	12×7=84	7	46·67
H_2	2×8=16	8	[illegible]
Total		15	[illegible]

Post-combustion volumetric composition :

Vol. per 100 m³ of gas	O_2 reqd. per m³ of constituents	O_2 reqd. per 100 m³ of fuel	Products of combustion per 100 m³ of fuel			
(a)	(b)	(c) = (a) × (b)	CO_2	H_2O	O_2	N_2
C=46·67	1	[illegible]	[illegible]		[illegible]	[illegible]
H=53·33	0·5	[illegible]		[illegible]		
Total		73·33	[illegible]	[illegible]	[illegible]*	[illegible]*

*Excess O_2 supplied = 73·33 × 0·2 = 14·67 m³

*Total N_2 supplied = (73·33 + 14·67) × $\frac{79}{21}$ = [illegible] m³

Total volume after combustion = [illegible]

= [illegible] m³

∴ Post combustion volumetric composition

$\underline{CO_2} = \frac{46.67}{[illegible]} \times 100 = $ [illegible]

$\underline{H_2O} = \frac{[illegible]}{[illegible]} \times 100 = $ [illegible]

$\underline{O_2} = \frac{[illegible]}{[illegible]} \times 100 = $ [illegible]

$\underline{N_2} = $ [illegible]

Constituents	Mol of burnt gases (a)	Molecular weight (b)	Mass of constituents per mol of flue gas (c)=(a)×(b)
CO_2	0·1043	44	4·59
H_2O	0·1195	18	2·15
O_2	0·0342	32	1·09
N_2	0·7420	28	20·77
Total	1·000		28·60

Average molecular weight of the gases = 28·6 Ans.

∴ Specific volume (volume per kg)

$$=\frac{22\cdot41}{\text{Mol wt}}=\frac{22\cdot41}{28\cdot60}=0\cdot7834\ \text{m}^3 \quad \text{Ans.}$$

and Gas constant, $R=\dfrac{R_m}{\text{Mol wt}}=\dfrac{848}{28\cdot60}=29\cdot6$ Ans.

✓ 7·8. C.I. engine : Percentage excess air by C, and O_2-H_2 balance.

A compression ignition engine uses light diesel oil of chemical composition $C_{12}H_{26}$ (Dodecane). The Orsat analysis gave CO_2 7·2 per cent, O_2 10·8 per cent. Assuming remainder to be nitrogen, determine the air-fuel ratio and percentage excess air by (i) carbon balance method and (ii) oxygen-hydrogen balance method. Comment on the accuracy of results obtained by two methods. Assume air contains 76·8 per cent nitrogen by mass.

(a) Carbon balance method



Fuel — $C_{12}H_{26}$

$$\therefore \quad \text{By mass, } C=\frac{12\times12}{12\times12+26}\times100=84\cdot7\%, \text{ and } H_2=15\cdot3\%$$

Dry products of combustion,

$CO_2=7\cdot2\%$, $O_2=10\cdot8\%$, $N_2=82\%$ by volume

Since mol is a unit of volume, the given fractions can be treated as mol fractions.

Mass of N_2 in one mol of dry products $= 0.82 \times 28 = 22.96$ kg

$\therefore$ Mass of air per mol $= 22.96 \times \frac{100}{76.8} = 29.98$ kg

Mass of CO_2 in one mol of products $= 0.072 \times 44$ kg

$\therefore$ Carbon content in one mol of gas

$$= (0.072 \times 44) \times \left(\frac{12}{44}\right) = 0.864 \text{ kg}$$

$\therefore$ Corresponding mass of fuel burnt $= \frac{0.864}{0.847} = 1.02$ kg

Thus one mol of dry products is produced when 1·02 kg of fuel is burnt with 29·98 kg of air, *i.e.*,

Air/fuel ratio $= \frac{29.98}{1.02} = 29.35$ **Ans.**

Minimum air required for 1 kg of fuel for complete combustion

$$= \frac{100}{23.2}\left(0.847 \times \frac{32}{12} + 0.153 \times 8\right) = 15 \text{ kg}$$

$\therefore$ Percentage excess air $= \frac{29.35 - 15}{15} = 95.6\%$ **Ans.**

(*b*) *Oxygen-hydrogen method*

Mass of O_2 in one mol of dry products of combustion

$$= \text{Mass associated with } CO_2 + \text{mass of } O_2$$

$$= (44 \times 0.072) \times \frac{32}{44} + 0.108 \times 32$$

$$= 5.76 \text{ kg}$$

Oxygen associated with nitrogen

$$= 0.82 \times 28 \times \left(\frac{23.2}{76.8}\right) = 6.936 \text{ kg}$$

$\therefore$ O_2 combined with H_2 to form water vapour

$$= 6.936 - 5.76 = 1.176 \text{ kg}$$

$\therefore$ Mass of hydrogen in the fuel $= \frac{1.176}{8} = 0.147$ kg

and corresponding mass of fuel $= \frac{0.147}{0.153} = 0.9607$

Thus one mol of dry products is produced when 0·9607 kg of fuel is burnt with 29·98 kg of air, *i.e.*,

$$\text{Air/fuel ratio} = \frac{29·98}{0·9607} = 31·2 \text{ kg} \qquad \text{Ans.}$$

and
$$\text{Percentage excess air} = \frac{31·2-15}{15} = 108\% \qquad \text{Ans.}$$

Note. The carbon balance method is more accurate as carbon is the predominent element in the fuel. In oxygen-hydrogen balance relevant quantities are relatively small and their measurements are open to error. The carbon balance will give inaccurate results if there is unburnt carbon in the exhaust.

7·9. Heat taken by dry exhaust gas and steam : pp of steam.

The fuel oil used in a test on a compression ignition engine was found to have a hydrogen content of 12½% by mass and there was no incombustible matter. The measured air-fuel ratio was 20 : 1. The temperature and pressure in the exhaust manifold were found to be 350°C and 1·5 kgf per cm². The test room temperature was 20°C. The percentage volume of steam in the exhaust was 8%. Determine the heat carried away by the exhaust gases per minute if the fuel consumption was 0·5 kg per minute. Take the specific heat of dry exhaust gases as 0·24 and specific heat of superheated steam as 0·48.

Steam formed per kg of fuel $= 9 \times 0·125 = 1·125$ kg

Mass of dry flue gases per kg of fuel

$$= (20+1) - 1·125 = 19·875 \text{ kg}$$

Heat to dry flue gases $= 19·875 \times 0·24(250-20)$

$$= 1{,}574 \text{ kcal per kg} \qquad ...(1)$$

As the total pressure of exhaust mixture is given and as percentage volume of steam is given, we can find out the partial pressure of steam. According to the Dalton's law of partial pressure

Partial pressure ∝ Number of mols

$$\frac{\text{pp of steam}}{1·5} = \frac{8}{100}$$

∴ pp of steam $= 0·12$ kgf/cm²

From Steam Tables,

Mass of N_2 in one mol of dry products $=0.82\times28=22.96$ kg

$\therefore$ Mass of air per mol $=22.96\times\frac{100}{76.8}=29.98$ kg

Mass of CO_2 in one mol of products $=0.072\times44$ kg

$\therefore$ Carbon content in one mol of gas

$$=(0.072\times44)\times\left(\frac{12}{44}\right)=0.864\text{ kg}$$

$\therefore$ Corresponding mass of fuel burnt $=\frac{0.864}{0.847}=1.02$ kg

Thus one mol of dry products is produced when 1·02 kg of fuel is burnt with 29.98 kg of air, *i.e.*,

Air/fuel ratio $=\frac{29.98}{1.02}=$ 29.35 Ans.

Minimum air required for 1 kg of fuel for complete combustion

$$=\frac{100}{23.2}\left(0.847\times\frac{32}{12}+0.153\times8\right)=15\text{ kg}$$

$\therefore$ Percentage excess air $=\frac{29.35-15}{15}=$ 95.6% Ans.

(b) Oxygen-hydrogen method

Mass of O_2 in one mol of dry products of combustion

$$=\text{Mass associated with } CO_2+\text{mass of } O_2$$

$$=(44\times0.072)\times\frac{32}{44}+0.108\times32$$

$$=5.76\text{ kg}$$

Oxygen associated with nitrogen

$$=0.82\times28\times\left(\frac{23.2}{76.8}\right)=6.936\text{ kg}$$

$\therefore$ O_2 combined with H_2 to form water vapour

$$=6.936-5.76=1.176\text{ kg}$$

$\therefore$ Mass of hydrogen in the fuel $=\frac{1.176}{8}=0.147$ kg

and corresponding mass of fuel $=\frac{0.147}{0.153}=0.9607$

Constituents	*Mol of burnt gases* (*a*)	*Molecular weight* (*b*)	*Mass of constituents per mol of flue gas* (*c*)=(*a*)×(*b*)
CO_2	0·1043	44	4·59
H_2O	0·1195	18	2·15
O_2	0·0342	32	1·09
N_2	0·7420	28	20·77
Total	1·000		28·60

Average molecular weight of the gases=28·6 Ans.

∴ Specific volume (volume per kg)

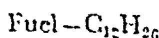
$$=\frac{22{\cdot}41}{\text{Mol wt}}=\frac{22{\cdot}41}{28{\cdot}60}=0{\cdot}7834 \text{ m}^3 \qquad \text{Ans.}$$

and Gas constant, $R=\dfrac{R_m}{\text{Mol wt}}=\dfrac{848}{28{\cdot}60}=29{\cdot}6$ Ans.

7·8. C. I. engine : Percentage excess air by C, and O_2-H_2 balance.

A compression ignition engine uses light diesel oil of chemical composition $C_{12}H_{26}$ (Dodecane). The Orsat analysis gave CO_2 7·2 per cent, O_2 10·8 per cent. Assuming remainder to be nitrogen, determine the air-fuel ratio and percentage excess air by (i) carbon balance method and (ii) oxygen-hydrogen balance method. Comment on the accuracy of results obtained by two methods. Assume air contains 76·8 per cent nitrogen by mass.

(*a*) Carbon balance method


Fuel – $C_{12}H_{26}$

∴ By mass, $C=\dfrac{12\times 12}{12\times 12+26}\times 100=84{\cdot}7\%$, and $H_2=15{\cdot}3\%$

Dry products of combustion,

$CO_2=7{\cdot}2\%$, $O_2=10{\cdot}8\%$, $N_2=82\%$ by volume

Since mol is a unit of volume, the given fractions can be treated as mol fractions.

Mass of N_2 in one mol of dry products $= 0.82 \times 28 = 22.96$ kg

$\therefore$ Mass of air per mol $= 22.96 \times \dfrac{100}{76.8} = 29.98$ kg

Mass of CO_2 in one mol of products $= 0.072 \times 44$ kg

$\therefore$ Carbon content in one mol of gas

$$= (0.072 \times 44) \times \left(\frac{12}{44}\right) = 0.864 \text{ kg}$$

$\therefore$ Corresponding mass of fuel burnt $= \dfrac{0.864}{0.847} = 1{\cdot}02$ kg

Thus one mol of dry products is produced when 1·02 kg of fuel is burnt with 29·98 kg of air, *i e*,

Air/fuel ratio $= \dfrac{29.98}{1.02} = \underline{29{\cdot}35}$ **Ans.**

Minimum air required for 1 kg of fuel for complete combustion

$$= \frac{100}{23.2}\left(0.847 \times \frac{32}{12} + 0.153 \times 8\right) = 15 \text{ kg}$$

$\therefore$ Percentage excess air $= \dfrac{29{\cdot}35 - 15}{15} = \underline{95{\cdot}6\%}$ **Ans.**

(*b*) *Oxygen-hydrogen method*

Mass of O_2 in one mol of dry products of combustion

$$= \text{Mass associated with } CO_2 + \text{mass of } O_2$$

$$= (44 \times 0.072) \times \frac{32}{44} + 0.108 \times 32$$

$$= 5.76 \text{ kg}$$

Oxygen associated with nitrogen

$$= 0.82 \times 28 \times \left(\frac{23.2}{76.8}\right) = 6.936 \text{ kg}$$

$\therefore$ O_2 combined with H_2 to form water vapour

$$= 6.936 - 5{\cdot}76 = 1.176 \text{ kg}$$

$\therefore$ Mass of hydrogen in the fuel $= \dfrac{1.176}{8} = 0.147$ kg

and corresponding mass of fuel $= \dfrac{0.147}{0.153} = 0.9607$

Thus one mol of dry products is produced when 0·9607 kg of fuel is burnt with 29·98 kg of air, *i.e.*,

$$\underline{\text{Air/fuel ratio}} = \frac{29 \cdot 98}{0 \cdot 9607} = \underline{31 \cdot 2 \text{ kg}} \qquad \text{Ans.}$$

and $$\underline{\text{Percentage excess air}} = \frac{31 \cdot 2 - 15}{15} = \underline{108\%} \qquad \text{Ans.}$$

Note. The carbon balance method is more accurate as carbon is the predominent element in the fuel. In oxygen-hydrogen balance relevant quantities are relatively small and their measurements are open to error. The carbon balance will give inaccurate results if there is unburnt carbon in the exhaust.

7·9. Heat taken by dry exhaust gas and steam : pp of steam.

The fuel oil used in a test on a compression ignition engine was found to have a hydrogen content of 12½% by mass and there was no incombustible matter. The measured air-fuel ratio was 20 : 1. The temperature and pressure in the exhaust manifold were found to be 350°C and 1·5 kgf per cm². The test room temperature was 20°C. The percentage volume of steam in the exhaust was 8%. Determine the heat carried away by the exhaust gases per minute if the fuel consumption was 0·5 kg per minute. Take the specific heat of dry exhaust gases as 0·24 and specific heat of superheated steam as 0·48.

Steam formed per kg of fuel $= 9 \times 0 \cdot 125 = 1 \cdot 125$ kg

Mass of dry flue gases per kg of fuel

$$= (20+1) - 1 \cdot 125 = 19 \cdot 875 \text{ kg}$$

Heat to dry flue gases $= 19 \cdot 875 \times 0 \cdot 24(250 - 20)$

$$= 1{,}574 \text{ kcal per kg} \qquad \ldots(1)$$

As the total pressure of exhaust mixture is given and as percentage volume of steam is given, we can find out the partial pressure of steam. According to the Dalton's law of partial pressure

Partial pressure $\propto$ Number of mols

$$\frac{\text{pp of steam}}{1 \cdot 5} = \frac{8}{100}$$

$\therefore$ pp of steam $= 0 \cdot 12$ kgf/cm²

From Steam Tables,

Thus one mol of dry products is produced when 0·9607 kg of fuel is burnt with 29·98 kg of air, *i.e.*,

$$\underline{\text{Air/fuel ratio}}=\frac{29\cdot98}{0\cdot9607}=\underline{31\cdot2\text{ kg}} \quad \text{Ans.}$$

and

$$\underline{\text{Percentage excess air}}=\frac{31\cdot2-15}{15}=\underline{108\%} \quad \text{Ans.}$$

Note. The carbon balance method is more accurate as carbon is the predominent element in the fuel. In oxygen-hydrogen balance relevant quantities are relatively small and their measurements are open to error. The carbon balance will give inaccurate results if there is unburnt carbon in the exhaust.

7·9. Heat taken by dry exhaust gas and steam : pp of steam.

The fuel oil used in a test on a compression ignition engine was found to have a hydrogen content of 12½% by mass and there was no incombustible matter. The measured air-fuel ratio was 20 : 1. The temperature and pressure in the exhaust manifold were found to be 350°C and 1·5 kgf per cm². The test room temperature was 20°C. The percentage volume of steam in the exhaust was 8%. Determine the heat carried away by the exhaust gases per minute if the fuel consumption was 0·5 kg per minute. Take the specific heat of dry exhaust gases as 0·24 and specific heat of superheated steam as 0·48.

Steam formed per kg of fuel $=9\times0\cdot125=1\cdot125$ kg

Mass of dry flue gases per kg of fuel

$$=(20+1)-1\cdot125=19\cdot875\text{ kg}$$

Heat to dry flue gases $=19\cdot875\times0\cdot24(250-20)$

$$=1{,}574\text{ kcal per kg} \quad \ldots(1)$$

As the total pressure of exhaust mixture is given and as percentage volume of steam is given, we can find out the partial pressure of steam. According to the Dalton's law of partial pressure,

Partial pressure $\propto$ Number of mols

$$\frac{\text{pp of steam}}{1\cdot5}=\frac{8}{100}$$

$\therefore$ pp of steam $=0\cdot12$ kgf/cm²

From Steam Tables,

The enthalpy of dry steam at 0·12 k [illegible]

$= 618·6$ kcal/kg

Heat to steam $1·125\ \{618·6 + 0·5$ [illegible]$\}$ [illegible]

$= 836$ kcal/kg of fuel

Heat to exhaust gases per minute

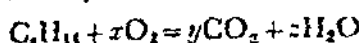
$= 0·5(1{,}574 + 836) = \underline{1{,}205 \text{ kcal}}$ Ans.

7·10. Experimental engine : rich mixture, A/F given ; volume analysis.

An experimental engine is supplied with a rich mixture of hexane (C_6H_{14}) and air. Fuel and air metering shows that 12·67 kg of air is supplied per kg of fuel. Calculate the mixture strength in terms of the chemically-correct value. Determine the volumetric analysis of dry exhaust gases assuming that all of the hydrogen in the fuel is burnt.

Assume oxygen is 21 per cent by volume in air.

Let the combustion equation be



$$C_6H_{14} + xO_2 = yCO_2 + zH_2O$$

(Nitrogen has been left out because it is same on both sides of equation)

Equating atoms of the same element before and after combustion, i.e., on the left hand and right hand side of the equation

$$y = 6,\ z = 7,\ \text{and therefore}\ x = 9·5$$

Now the combustion equation can be written as



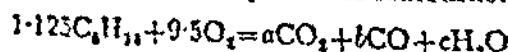
$$C_6H_{14} + 9·5O_2 = 6CO_2 + 7H_2O$$

$\therefore$ Gravimetric air-fuel ratio $= \dfrac{(9·5 \times 32) \times 100/23}{12 \times 6 + 1 \times 14} = 15·37$

The actual mixture strength (fuel-air ratio) expressed in terms of the chemically correct value is

$$\frac{15·37}{13·67} \times 100 = \underline{112·4\%} \qquad \text{Ans.}$$

i.e., the actual mixture is 12·5% rich in fuel. The combustion is therefore incomplete for carbon but hydrogen is completely burnt as given in problem. The actual equation of combustion now becomes



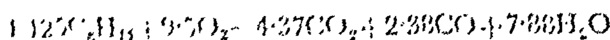
$$1·125C_6H_{14} + 9·5O_2 = aCO_2 + bCO + cH_2O$$

Equating atoms of the same element before and after combustion

$$1{\cdot}125\times 6=a+b,\ 1{\cdot}125\times 14=2c,\ 9{\cdot}5\times 2=2a+b+c$$

$$\therefore \qquad a=4{\cdot}37,\ b=2{\cdot}38 \text{ and } c=7{\cdot}88$$

and the combustion equation is written as



$$1{\cdot}125C_8H_{18}+9{\cdot}5O_2=4{\cdot}37CO_2+2{\cdot}38CO+7{\cdot}88H_2O$$

Total volume of dry flue gases including nitrogen

$$=4{\cdot}37+2{\cdot}38+9{\cdot}5\times\frac{100}{21}$$

$$=42{\cdot}47 \text{ units of volume}$$

$\therefore$ Analysis of dry flue gases by volume

$$\left.\begin{aligned} \underline{CO_2} &= \frac{4{\cdot}37}{42{\cdot}47}\times 100=\underline{10{\cdot}29\%} \\ \underline{CO} &= \frac{2{\cdot}38}{42{\cdot}47}\times 100=\underline{5{\cdot}65\%} \\ \underline{N_2 \text{ (by difference)}} &= \underline{84{\cdot}06\%} \end{aligned}\right\} \quad \text{Ans.}$$

Note. Do not forget N_2 while calculating the total volume of dry gas.

7·11. Volumetric analysis with deficient air supply : heat due to incomplete combustion.

A petrol has a composition by mass C = 86% and H_2 = 14%. When used in an engine the air supply is 95 per cent of that theoretically required for complete combustion. Assuming that all the hydrogen is burnt and that the carbon burns to CO and CO_2 so that there is no free carbon left, calculate (a) the percentage analysis of the dry exhaust gases by volume, and (b) the percentage of the gross calorific value of the fuel lost by incomplete combustion. Gross calorific value in kcal per kg :

$$C \text{ to } CO_2=8{,}300,\quad C \text{ to } CO=2{,}470,\quad H_2=34{,}400$$

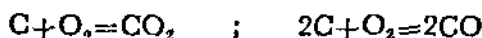
Air contains 23 per cent oxygen by mass.

Minimum air required per kg of fuel for complete combustion

$$=\frac{100}{23}\left[0{\cdot}86\times\frac{32}{12}+0{\cdot}14\times 8\right]=14{\cdot}84 \text{ kg}$$

Deficiency of air supplied per kg of fuel

$$=14{\cdot}84\times0{\cdot}05=0{\cdot}742 \text{ kg}$$



$$C+O_2=CO_2 \quad ; \quad 2C+O_2=2CO$$

$$12+32=44 \quad ; \quad 12+16=28$$

Air saved by burning C to CO instead of CO_2 per kg of carbon

$$\frac{100}{23}\left[\frac{32-16}{12}\right]=5{\cdot}8 \text{ kg}$$

Hence, C burned to CO per kg of fuel $=\frac{0{\cdot}742\times0{\cdot}86}{5{\cdot}8}=0{\cdot}11$ kg

and C burned to CO_2 per kg of fuel $=0{\cdot}86-0{\cdot}11=0{\cdot}75$

Dry products of combustion (a)	*Mass per kg of fuel* (b)	*Proportional volume* $(c)=\frac{(b)}{mol.\ wt.}$	*Percentage volume* $(d)=\frac{(c)}{\Sigma(c)}\times100$	
CO_2	$0{\cdot}75\times\frac{44}{12}=2{\cdot}75$	$\frac{2{\cdot}75}{44}=0{\cdot}0625$	13 60	Ans.
CO	$0{\cdot}11\times\frac{28}{12}=0{\cdot}257$	$\frac{0{\cdot}257}{28}=0{\cdot}00917$	2 00	
N_2	$0{\cdot}77\times14{\cdot}84\times0{\cdot}95=10{\cdot}86$	$\frac{10{\cdot}86}{28}=0{\cdot}3879$	84 40	
Total		0 4596	100 00	

Gross calorific value of coal $=0{\cdot}86\times8{,}300+0{\cdot}14\times34{,}400$

$$=11{,}950 \text{ kcal per kg}$$

Percentage heat loss due to incomplete combustion

$$=\frac{0{\cdot}11(8{,}300-2{,}470)\times100}{11{,}950}=\underline{5{\cdot}368\%} \qquad \text{Ans.}$$

7.12. Excess air supplied given CO_2 analysis

In a boiler test CO_2 recorder read 12·5 per cent. Calculate the percentage excess air supplied per kg of the fuel if the percentage composition of coal by mass is C = 87%, H_2 = 4% and O_2 = 3·5%.

Constituents per kg of fuel (a)	*Proportional vol. composition* (a)/Mol wt	*O_2 required by volume*	*Dry products of combustion by volume* CO_2
C=0·870	$\frac{0·87}{12}$=0·0725	0·07250	0·0725
H_2=0·040	$\frac{0·04}{2}$=0·0200	0·01000	
O_2=0·035	$\frac{0·035}{32}$=0·001094	−0·00109	
	Total	0·08141	

Minimum air required by volume$=\frac{100}{21}\times 0·08141=0·3877$

Volume of N_2 with minimum air $=0·3877-0·08141=0·3063$

Let the volume of excess air (N_2+O_2) be E

Percentage CO_2 (in dry products)$=\frac{CO_2}{CO_2+N_2+E}$

or $12·5=\frac{0·0725\times 100}{0·0725+0·3063+E}$ $\quad\therefore\quad E=0·2013$

$\therefore$ Percentage excess air $=\frac{0·2013}{0·3877}=51·93\%$ Ans.

7-13. Air supplied : loss due to incomplete combustion, no N_2 given.

Account for the simultaneous presence of CO and O_2 in burnt gases.

An analysis of the flue gases in a boiler trial gave 12·5% CO_2, 1% CO. The chemical analysis of dry fuel gave 84% C and 5% H. Determine the mass of air used per kg of fuel consumed. What percentage of fuel is lost due to incomplete combustion, if the calorific value of dry coal is 8,500 kcal per kg.

The calorific value of C=8,050 kcal per kg when burnt to CO_2 and 2,400 kcal when burnt to CO.

The presence of CO in burnt gases indicates incomplete combustion whereas free O_2 indicates excess air. However it is usual to get some quantities of CO with free O_2 in flue or exhaust gas analysis.

This simultaneous presence of CO and O_2 in burnt gases is generally due to imperfect air distribution and insufficient mixing in the case of boilers with large grate areas working on natural draught system. However, it may also be due to subsequent leakage of air in the flue. In engines it is due to lack of turbulence.

$$\text{Proportion of C burnt to CO} = \frac{CO}{CO_2+CO} = \frac{1}{12\cdot5+1} = 0\cdot074$$

$\therefore$ C burnt to CO per kg of fuel $= 0\cdot84 \times 0\cdot074 = 0\cdot0622$

and C „ CO_2 „ $= 0\cdot84 - 0\cdot0622 = 0\cdot7778$

$$\text{Mass of } CO_2 \text{ per kg of fuel} = \frac{0\cdot7778 \times 44}{12}$$

$$\text{Volume of 1 kg of } CO_2 = \frac{22\cdot41}{44} \text{ m}^3$$

Volume of CO_2 per kg of fuel burnt

$$= \frac{0\cdot7778 \times 44}{12} \times \frac{22\cdot41}{44} = 1\cdot452 \text{ m}^3$$

Volume of CO per kg of fuel burnt

$$= \frac{22\cdot41}{12} \times 0\cdot0622 = 0\cdot1161 \text{ m}^3$$

Volume of $(CO+CO_2)$ per kg of fuel burnt

$$= 1\cdot452 + 0\cdot1161 = 1\cdot5681 \text{ m}^3$$

The dry flue gases consist of CO_2, CO, N_2 and unused air (E).

$$\text{Percentage by volume of } CO_2 = \frac{1\cdot452 \times 100}{1\cdot5681 + N_2 + E} = 12\cdot5$$

$$\therefore \qquad N_2 + E = 10\cdot048$$

Total volume of dry gases per kg of fuel

$$= 10\cdot048 + 1\cdot5681 = 11\cdot616 \text{ m}^3$$

Minimum O_2 required per kg of fuel

$$= 0\cdot0622 \times \frac{32}{24} + 0\cdot7778 \times \frac{32}{12} + 0\cdot05 \times 8 = 2\cdot557 \text{ kg}$$

$$\therefore N_2 \text{ in min air} = 2\cdot557 \times \frac{77}{23} \times \frac{22\cdot41}{28} = 6\cdot85 \text{ m}^3$$

and unused air $= 10\cdot048 - 6\cdot85 = 3\cdot198$ m³

N_2 in unused air $= 3\cdot198 \times 0\cdot79 = 2\cdot526$ m³

Constituents per kg of fuel (a)	Proportional vol. composition (a)/Mol wt	O_2 required by volume	Dry products of combustion by volume CO_2
C=0.870	$\frac{0.87}{12}=0.0725$	0.07250	0.0725
H_2=0.040	$\frac{0.04}{2}=0.0200$	0.01000	
O_2=0.035	$\frac{0.035}{32}=0.001094$	−0.00109	
	Total	0.08141	

$$\text{Minimum air required by volume}=\frac{100}{21}\times 0{\cdot}08141=0{\cdot}3877$$

$$\text{Volume of } N_2 \text{ with minimum air } =0{\cdot}3877-0{\cdot}08141=0{\cdot}3063$$

Let the volume of excess air (N_2+O_2) be E

$$\text{Percentage } CO_2 \text{ (in dry products)}=\frac{CO_2}{CO_2+N_2+E}$$

or $$12{\cdot}5=\frac{0{\cdot}0725\times 100}{0{\cdot}0725+0{\cdot}3063+E} \qquad \therefore \quad E=0{\cdot}2013$$

$\therefore$ Percentage excess air $=\frac{0{\cdot}2013}{0{\cdot}3877}=51{\cdot}93\%$ Ans.

7.13. Air supplied : loss due to incomplete combustion, no N_2 given.

Account for the simultaneous presence of CO and O_2 in burnt gases.

An analysis of the flue gases in a boiler trial gave 12·5% CO_2, 1% CO. The chemical analysis of dry fuel gave 84% C and 5% H. Determine the mass of air used per kg of fuel consumed. What percentage of fuel is lost due to incomplete combustion, if the calorific value of dry coal is 8,500 kcal per kg.

The calorific value of C=8,050 kcal per kg when burnt to CO_2 and 2,400 kcal when burnt to CO.

The presence of CO in burnt gases indicates incomplete combustion whereas free O_2 indicates excess air. However it is usual to get some quantities of CO with free O_2 in flue or exhaust gas analysis.

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$$\text{Proportion of C burnt to CO} = \frac{CO}{CO_2+CO} = \frac{1}{12\cdot5+1} = 0\cdot074$$

$\therefore$ C burnt to CO per kg of fuel $= 0\cdot84 \times 0\cdot074 = 0\cdot0622$

and C ,, CO_2 ,, $= 0\cdot84 - 0\cdot0622 = 0\cdot7778$

$$\text{Mass of } CO_2 \text{ per kg of fuel} = \frac{0\cdot7778 \times 44}{12}$$

$$\textit{Volume of 1 kg of } CO_2 = \frac{22\cdot41}{44}\,\text{m}^3$$

Volume of CO_2 per kg of fuel burnt

$$= \frac{0\cdot7778 \times 44}{12} \times \frac{22\cdot41}{44} = 1\cdot452\ \text{m}^3$$

Volume of CO per kg of fuel burnt

$$= \frac{22\cdot41}{12} \times 0\cdot0622 = 0\cdot1161\ \text{m}^3$$

Volume of $(CO+CO_2)$ per kg of fuel burnt

$$= 1\cdot452 + 0\cdot1161 = 1\cdot5681\ \text{m}^3$$

The dry flue gases consist of CO_2, CO, N_2 and unused air (E).

$$\text{Percentage by volume of } CO_2 = \frac{1\cdot452 \times 100}{1\cdot5681 + N_2 + E} = 12\cdot5$$

$$\therefore \qquad N_2 + E = 10\cdot048$$

Total volume of dry gases per kg of fuel

$$= 10\cdot048 + 1\cdot5681 = 11\cdot616\ \text{m}^3$$

Minimum O_2 required per kg of fuel

$$= 0\cdot0622 \times \frac{32}{24} + 0\cdot7778 \times \frac{32}{12} + 0\cdot05 \times 8 = 2\cdot557\ \text{kg}$$

$$\therefore\ N_2 \text{ in min. air } = 2\cdot557 \times \frac{77}{23} \times \frac{22\cdot41}{28} = 6\cdot85\ \text{m}^3$$

and unused air $= 10\cdot048 - 6\cdot85 = 3\cdot198\ \text{m}^3$

N_2 in unused air $= 3\cdot198 \times 0\cdot79 = 2\cdot526\ \text{m}^3$

$\therefore$ Total $N_2 = 6{\cdot}85 + 2{\cdot}526 = 9{\cdot}376 \text{ m}^3$

$$\text{Percentage } N_2 = \frac{9{\cdot}376}{11{\cdot}616} \times 100 = 80{\cdot}7\%$$

As all N_2 has come from air,

$$\text{Mass of air} = \frac{9{\cdot}376}{22{\cdot}41} \times 28 \times \frac{100}{77} = 15{\cdot}22 \text{ kg}$$

[Check : Mass of air per kg of fuel

$$= \frac{N \times C}{33(CO_2 + CO)} = \frac{80{\cdot}6 \times 84}{33(12{\cdot}5 + 1)} = 15{\cdot}22 \text{ kg}]$$

Heat lost in incomplete combustion

$$= 0{\cdot}0621(8{,}050 - 2{,}400) = 351{\cdot}4 \text{ kcal per kg of fuel}$$

$\therefore$ Percentage loss due to incomplete combustion

$$= \frac{351{\cdot}4}{8{,}300} = 4{\cdot}23\% \qquad \text{Ans.}$$

7·14. Gaseous fuel : volumetric composition of wet gas ; percentage contraction in volume ; temperature after combustion.

The composition of a gas by volume is CH_4, 36 ; H_2, 46 ; CO, 8 ; N_2, 10. It is burnt with 6 times its volume of air. Treating the H_2O produced as a perfect gas, find, (a) the percentage composition by volume of the resulting hot gas, and (b) the percentage volumetric contraction on combustion.

A volume of 0·56 m³ of this mixture is at 101°C and 1 kgf/cm² before combustion. Estimate the temperature after combustion when the pressure is 5 kgf/cm² and the volume 0·51 m³, allowing for contraction. Air contains 20·9 per cent of oxygen by volume.

Total air supplied per m³ of gas $= 1 \times 6 = 6 \text{ m}^3$

$$\text{Volume of } O_2 \text{ in air supplied} = \frac{20{\cdot}9}{100} \times 6 = 1{\cdot}254$$

Volume of N_2 in air supplied $= 6 - 1{\cdot}254 = 4{\cdot}746 \text{ m}^3$

Constituents by vol. per mol of gas (a)	Oxygen required per mol of constituent (b)	Oxygen reqd. per mol of gas (c=a×b)	Products after combustion			
			CO_2	H_2O	O_2	N_2
CH_4=0 36	2	0·72	0 36	0 72		
H_2=0 46	½	0 23	—	0 43		
CO=0 08	½	0 04	0 08	—		
N_2=0 10	—	—	—	—	—	0·10
Air supplied=6 00	—	—	—	—	1·254 −0 990	4 746 (used)
Total=7 00		0·99	0 44	1 18	0 264	4 864

From the above table

Total volume of burnt gases$=0{\cdot}44+1\ 18+0\ 264+4\ 864=6{\cdot}73$

Percentage composition by volume

$$CO_2=\frac{0{\cdot}44}{6\ 73}=6\ 54\%,\quad H_2O=\frac{1\ 18}{6\ 73}=17\ 54\%$$
$$O_2=\frac{0\ 264}{6\ 73}=3\ 92\%\quad\text{and}\quad N_2=\frac{4\ 864}{6\ 73}=72\ 0\%$$
Ans.

Initial volume=gas+air $=1+6=7\ m^3$

$\therefore$ Contraction$=7\ 00-6\ 73=0{\cdot}27\ m^3$

Percentage volumetric contraction $=\frac{0\ 27}{7}=3\ 86\%$ Ans.

Considering mixture as a perfect gas

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$\frac{1\times0\ 56}{(273+101)}=\frac{5\times0\ 51}{T_2}\qquad T_2=1{,}703°K\ \text{or}\ 1{,}430°C\ \text{Ans.}$$

7·15. Gas engine : air gas ratio on N_2, O_2 and CO_2 balance.

Explain to what extent the percentage of CO_2 and O_2 in the flue gases may be takn as a guide to the effiriency of combustion.

The percentage analysis by volume of the gas supplied to a gas engine was $CH_4=19{\cdot}5$, $CO=18\ 0$, $H_2=44{\cdot}4$, $C_3H_8=1{\cdot}6$, O_2

$N_2=13{\cdot}1$, $CO_2=3{\cdot}0$ and dry exhaust gas analysis was $CO_2=9{\cdot}4$, $O_2=6{\cdot}0$ and $N_2=84{\cdot}5$.

Estimate the air gas ratio by volume and percentage of excess air on the basis of (a) N_2 balance, (b) O_2 balance, and (c) CO_2 balance.

High percentage of CO_2 usually indicates that the excess air is being kept to the minimum, but it does not necessarily indicate the best attainable combustion as there may be considerable CO present. High percentage of CO_2 with little or no CO shows the best combustion.

Percentage of O_2 is a better indicator of excess air than CO_2, because it is same for any coal. CO_2 percentage with excess air is different for different coals.

Constituent per 100 mol of gas	*Combustion equation*	*Oxygen required per 100 mol of gas*	*Dry products after combustion*	
			CO_2	N_2
$CH_4=19{\cdot}5$	$CH_4+2O_2=CO_2+2H_2O$	$19{\cdot}5\times2=39$	19·5	
$CO=18{\cdot}0$	$2CO+O_2=2CO_2$	$18\times\frac{1}{2}=9$	18·0	
$H_2=44{\cdot}4$	$2H_2+O_2=2H_2O$	$44{\cdot}4\times\frac{1}{2}=22{\cdot}2$	—	
$C_3H_6=1{\cdot}6$	$2C_3H_6+9O_2=6CO_2+6H_2O$	$1{\cdot}6\times\frac{9}{2}=7{\cdot}2$	4·8	
$O_2=0{\cdot}4$		$-0{\cdot}4$	—	
$N_2=13{\cdot}1$			—	13·1
$CO_2=3{\cdot}0$	—	—	3·0	with air $=\frac{77\times79}{21}=289{\cdot}7$
Total 100·0		77		302·8

$$\text{Minimum air required}=\frac{77}{21}\times100=366{\cdot}7 \text{ mol}$$

Let excess air supplied be E mol

(*a*) This excess air will also contain N_2

$$\therefore \quad \text{Total } N_2=302{\cdot}8+\frac{79}{100}\times E=302{\cdot}8+0{\cdot}79E$$

$$\text{Percentage } N_2 = \frac{N_2}{\text{Total flue gas}}$$

$$81.5 = \frac{[illegible] + 0.79E}{45.3 + 302.8 + E} \times 100 \qquad \therefore\ E = 156.7 \text{ mol}$$

Air supplied = Minimum air supply + Excess air

$$= 366.7 + 156.7 = 523.4$$

$$\underline{\text{Air : gas} = 5.234 : 1}$$ Ans.

$$\underline{\text{Percentage excess air} = \frac{156.7}{366.7} = 42.7\%}$$ Ans.

(b) O_2 in excess air, $= \frac{21}{100} \times E = 0.21E$

$$\text{Percentage } O_2 = \frac{O_2}{\text{Total dry flue gas}} \times 100$$

$$6 = \frac{0.21E \times 100}{45.3 + 302.8 + E} \qquad \therefore\ E = 139.2 \text{ mol}$$

Air supplied = 366.7 + 139.2 = 505.9

$$\underline{\text{Air : gas} = 5.059 : 1}$$ Ans.

$$\underline{\text{Percentage excess air} = \frac{139.2}{366.7} = 38\%}$$ Ans.

(c) $$\text{Percentage } CO_2 = \frac{CO_2 \times 100}{\text{Total flue gas}}$$

$$9.4 = \frac{45.3 \times 100}{45.3 + 302.8 + E} \qquad \therefore\ E = 133.7 \text{ mol}$$

Air supplied = 366.7 + 133.7 = 500.4 mol

$$\text{Air : gas} = 500.4 : 100 = \underline{5.004 : 1}$$ Ans.

$$\underline{\text{Percentage excess air} = \frac{133.7}{366.7} = 36.4\%}$$ Ans.

7.16 Gas producer : mass of air and steam required

What are the advantages of drawing steam [illegible] producer plant ? Sketch a gas producer.

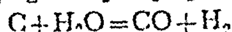
In a gas producer air and steam are drawn [illegible] anthracite coal and the gas consists of hydrogen [illegible] monoxide. The coal analysis gave 91% of carbon [illegible] bustible.

Heat of formation for steam is 35,400 kcal/kg of hydrogen and for CO 2,470 kcal/kg of carbon. Assuming that no heat interchanges other than those in the chemical reactions take place, show that 0·51 kg of steam and 3·48 kg of air will be required per kg of coal. Temperature of water is 18°C. Also find the volumetric analysis of the gas.



$$2C + O_2 = 2CO$$

$$1 \text{ kg of } C + 1\tfrac{1}{3} \text{ kg of } O_2 = 2\tfrac{1}{3} \text{ kg of } CO + 2{,}470 \text{ kcal} \qquad (1)$$



$$C + H_2O = CO + H_2$$

$$1 \text{ kg of } C + 1\tfrac{1}{2} \text{ kg of steam} = 2\tfrac{1}{3} \text{ kg of } CO + \tfrac{1}{6} \text{ kg of } H_2$$
$$+ 2{,}470 \text{ kcal} - \tfrac{1}{6} \times 35{,}400 \text{ kcal} - 1{\cdot}5[(100-18)+539]$$
$$= 2\tfrac{1}{3} \text{ kg of } CO + \tfrac{1}{6} \text{ kg of } H_2 - 4{,}361 \text{ kcal} \qquad (2)$$

To balance the heat given in equation (1) to heat taken in equation (2), multiply (2) by $\frac{2{,}470}{4{,}361} = 0{\cdot}566$.

$$0{\cdot}566 \text{ kg of } C + 0{\cdot}85 \text{ kg of steam}$$
$$= 1{\cdot}32 \text{ kg of } CO + 0{\cdot}0945 \text{ kg of } H_2 - 2{,}470 \text{ kcal} \qquad (3)$$

Total carbon burnt in equation (1) and (3) = 1·566 kg

Steam required per kg of carbon $= \frac{0{\cdot}85}{1{\cdot}566}$

Steam required per kg of coal $= \frac{0{\cdot}85}{1{\cdot}566} \times 0{\cdot}94 = 0{\cdot}51$ kg **Ans.**

Total air required per kg of coal $= \frac{4 \times 0{\cdot}94}{3 \times 1{\cdot}566} \times \frac{100}{23}$

$= 3{\cdot}48$ kg **Ans.**

The two operations yield, by mass

$CO = 2{\cdot}33 + 1{\cdot}32 = 3{\cdot}65$ kg, $H_2 = 0{\cdot}0945$ kg

and $N_2 = \frac{1{\cdot}33 \times 77}{23} = 4{\cdot}453$ kg

Constituents (a)	*Mol wt* (b)	*Proportional volume* (c) = (a)/(b)	*Percentage volume*
CO—3·65	28	0·1304	38·8
H_2—0·0945	2	0·0472	14·0
N_2—4·453	28	0·1591	47·2
Total		0·3367	100·0

Ans.

EXAMPLES 7

7·1 Percentage volumetric composition of dry flue gases, with excess air.

The percentage composition of a sample of anthracite coal was found to be carbon=90%, hydrogen=3·3%, oxygen=3%, nitrogen=0·8%, sulphur=0·9% and ash=2%. If 50% of excess air is supplied, find the percentage composition of dry flue gases by volume

[minimum O_2=2·643 kg ; in dfg CO_2=3 3 kg ; O_2=1 3215 kg ; N_2=12·938 kg ; % by volume, CO_2=13% ; O_2=7·1% and N_2=80%]

7 2. Determination of the calorific value of a gaseous fuel.

Distinguish between the higher and lower calorific values of a fuel, and explain why it is usual to calculate the thermal efficiency of an engine from the lower calorific value.

The following observations were made during a test on coal gas :

Volume of gas used, litres	...	*60*
Pressure of gas above atmosphere, cm of water	...	*4·5*
Temperature of gas, °C	...	*14*
Barometer, cm of mercury	...	*75*
Room temperature, °C	...	*16*
Cooling water used, kg	...	*9 8*
Cooling water rise of temperature °C	...	*6 3*
Condensed steam collected, gm	...	*9*

Determine from these observations the higher and lower calorific values of the gas.

[gas pressure=75 33 cm ; vol at N.T P =56 58 litres heat to water=61·74 Kcal ; H.C V =1091 Kcal/m³ , L C V =998 Kcal m³]

7 3 Heat lost in flue gases ; mean specific heat

The analysis by mass of the fuel used in a boiler trial was C=88%, H_2=3 6%, O_2=4 8%, ash=3 6% and the volumetric analysis of dry flue gases was CO_2=10 9%, CO=1 0%, O_2=7 1%, N_2=81%. Estimate (a) the mean specific heat of the dry flue gases, and (b) the quantity of total heat carried away by the dry flue gases and

vapour of combustion per kg of fuel burnt, if the temperature of the gases is 288°C and that of the air in boiler house, 20°C. Given specific heats of CO_2, CO, O_2 and N_2 as 0·216, 0·245, 0·218, and 0·244 respectively.

[mean sp. heat = 0·2376 ; dfg per kg of fuel = 18·5 kg ; heat in dfg per kg of fuel = 1,178 kcal, in H_2O (assuming p.p. 0·07 kgf/cm²) = 231 kcal, total = 1409 kcal]

7·4. Percentage excess air : heat lost due to incomplete combustion and excess air.

Find from the following data which refer to a boiler trial, (a) the percentage excess air supplied, (b) the heat loss expressed as a percentage of the heat in the fuel, due to incomplete combustion, and (c) the heat carried away by the excess air also expressed as a percentage of the heat in the fuel (C_p=0·24). Percentage coal analysis by mass C=81·2% ; H=5·8% ; S=1·0%, and ash=4·0% ; percentage volumetric analysis of the flue gases, CO_2=9·7% ; CO=1·3% ; N_2=80·0% ; and O_2=9·0% ; boiler room temperature 19°C ; flue gas temperature =276°C ; C.V. of carbon burnt to CO_2=8,080 kcal per kg and burnt to CO=2,440 kcal per kg ; composition of air by mass O_2=23·2% ; N_2=76·8% ; C.V. of coal =7,160 kcal/kg. Any formula used must be established.

[minimum air = 11·04 kg ; dfg = 18·41 kg ; excess air = 7 088 kg ; % excess air = 64·2% ; C burnt to CO = 0·09603 kg ; % loss due to incomplete combuiton = 7·55% ; % heat in excess air = 6·11%]

7·5. Calorific value of C_6H_{14} and air mixture at given temperature and pressure.

Determine the net calorific value of 1 m³ of mixture, Hexane (C_6H_{14}) and air at 12·7 cm of water pressure and 20°C, when 20 per cent excess air is supplied. Assume atmospheric pressure as 1·02 kgf/cm² and net calorific value of Hexane vapour, 10,700 kcal/kg.

[$2C_6H_{14} + 19O_2 = 12CO_2 + 14H_2O$; 2 mols of C_6H_{14} in 110·5 mols of mixture ; mass of C_6H_{14} in 1 m³ mixture = 0·06944 kg ; calorific value at N.T.P. = 743/m³ ; calorific value at given condition = 691 kcal/m³]

76. Air/fuel ratio, given volumetric analysis of products of combustion.

A petrol has the following ultimate analysis : carbon 85 5 per cent, hydrogen 14·5 per cent. The volumetric analysis of the dry products of combustion was CO_2 11·6 per cent, O_2 3 9 per cent and remainder N_2. Determine the air/fuel ratio used by (i) carbon balance method, and (ii) oxygen-hydrogen balance method. Comment on the accuracy of results obtained by two methods. Assume air contains 76·9 per cent nitrogen by mass.

[(i) mass of air/mol = 30 77 kg ; mass of petrol burnt = 1 699 kg ; air/fuel ratio = 18 9. (ii) mass of O_2 = 4 96/mol of products of combustion ; O_2 to form water vapour = 2 048 ; mass of petrol = 1·765 ; air/fuel ratio = 17·44].

77. Petrol engine (rich mixture) . mass analysis ; air/petrol ratio

The dry products of combustion of an air-petrol mixture had the volumetric analysis CO_2, 12 4 , CO, 3 0 , N_2, 84 6 per cent. Assuming that fuel consisted only of carbon and hydrogen and that none was left unburnt, determine

(a) the percentage mass analysis of the petrol.

(b) the air/petrol ratio by mass.

Air contains 23·2 per cent by mass of oxygen

[Write combustion equation and equate coefficients ; C = 84·7% H_2 = 15·3% ; air supplied = (714 kg O_2 + 2360 kg N_2) = 3,074 kg/mol of combustion products , petrol = 218·6 kg mol of products : ∴ Air/fuel = 14 1]

78. Incomplete combustion : mass of products of combustion.

Petrol having the composition by mass C, 85 and H, 15 per cent was burned with 11·5 times its mass of air, this amount of air being too little for combustion. Assuming that all the hydrogen was burned, find the mass of each of resulting gases per kg of petrol.

Air contains 23 2 per cent of oxygen by mass.

[defficiency of air per kg of fuel=3·45 kg ; air saved by combustion of C to CO instead of CO_2=5·75 kg ; C burnt to CO=0·6 ; CO_2=0·917 kg : CO=1·4 kg : H_2O=1·35 kg : N_2=8·833 kg. Check ; total mass=11·5+1]

7·9. Air supplied per kg ; speed of flue gases, no N_2 given.

A boiler is fired with coal having a percentage composition ; Carbon=85·1%, Hydrogen=4·2%, Oxygen, ash etc.=10·7%. The analysis of dry flue gases shows 10·2% CO_2. Estimate the mass of air supplied to the furnace per kg of fuel fired.

If the measured temperature of the flue gases at the chimney base is 410°C, when consumption of boiler is 746 kg of coal per hour, find the mean speed of the flue gases entering the chimney if its cross sectional area is 2 m² ; R=29·27 and atmospheric pressure 1·033 kgf/cm².

[air per kg of fuel=20·24 kg ; volume of air (approx. equal to gases) per second=8·34 m³ ∴ mean velocity=4·17 m/sec]

7·10. Gaseous fuel : minimum air for combustion ; volumetric analysis of exhaust ; volume of excess air, given $\%O_2$.

Gas having the following analysis by volume is used in an engine : CH_4=39%, H_2=46%, N_2=0·5%, CO=8%, H_2O=2%, CO_2=4·5%. Estimate the volume of air required for the combustion of a m³ of this gas. What will be the volumetric analysis of the products of combustion at 100°C ?

If the exhaust from the engine using this gas shows 12% oxygen, what is the volume of excess air supplied per m³ of gas burned and also the total volume of air ?

[min air=5m³ ; % vol, CO_2=8·99, H_2O=21·99, N_2=69 02 ; excess air=7·64 m³ ; total air=12·64 m³]

7·11. Gas producer : volume of gas produced ; quantity of air and steam required.

The gas from a producer, obtained by drawing air and steam through incandescent anthracite, consists of hydrogen, carbon monoxide

and nitrogen only. The analysis of the anthracite is 94% carbon, 6% ash by mass.

Determine the mass of steam and air required per kg of anthracite burnt and the volumetric analysis of producer gas. Neglect all heat interchanges except those in the chemical reactions. L.C.V. of $H_2=28,300$ *kcal/kg ; C.V. of* $CO=2,440$ *kcal/kg ; C.V. of* $C=8,100$ *kcal/kg ; heat given to steam = 622 kcal/kg*

Air contains 23·1% O_2 *by mass and 21%* O_2 *by volume.*

[Balancing the heat, H_2O per kg of anthracite = 0·6037 kg ; air per kg of anthracite = 3·082 kg ; % by volume CO = 39·9%, H_2 = 17·3% and N_2 = 42·8%]

7.12. Gas producer plant : efficiency of producer, engine and combined plant.

What is the effect of using steam along with air (a) on the working of producer gas plant , (b) on its efficiency.

In a gas producer feed with bituminous coal of lower calorific value 7,450 kcal/kg the yield of gas per kg of coal is 13·2 m³ at N.T.P. of lower calorific value 450 kcal/m³. Percentage analysis by volume of the gas is $H=18·3$, $CH_4=3·4$, $CO=25·4$, $CO_2=6·89$, $N_2=46$ *and a gas engine takes 4·9 m³ of this gas per brake horse-power-hour. Calculate (a) the m³ of air necessary for the combustion of 1 m³ of gas, (b) the thermal efficiency of producer from the lower calorific value of gas, (c) the brake thermal efficiency of gas engine, and (d) the overall efficiency of power plant.*

[air per m³ of gas = 1·3643 m³ , thermal η of producer = 79·8% ; brake thermal η = 30% ; overall η = 23·8%]

8

Boilers—Combustion and Performance

8·1. Boiler Draught. The draught may be defined as the small pressure difference which causes flow of gases to take place inside the boiler. This pressure difference is normally very small and is usually measured in mm of water rather than in kgf/cm^2. Draught produced by a chimney is known as natural draught, whereas draught produced by forced draught (F.D.) and induced draught (I.D.) fans, steam jet, etc., is known as artificial draught.

Losses in draught are encountered in passing through the fuel bed and grate, boiler tubes, dampers and passage to chimney.

8·2. Natural Draught. Natural draught is obtained by the use of a chimney. It is produced by the difference in mass between the column of hot gases inside the chimney and an equal column of air of the same area outside the chimney. Chimney also carries the products of combustion to such a height before discharging them that it will not be objectionable to the surroundings. A chimney may be built either of masonary, steel or concrete. Strictly speaking the term chimney relates to a masonary structure while stack refers to a metallic chimney.

Comparison of various chimneys. (1) The life of brick or concrete chimney is about 50 years and that of steel chimney is about 15 years. The life of steel chimney depends more on its care *i.e.*, painting. The painting is required to avoid rust and corrosion.

(2) Because of cracks and imperfect binding of bricks, the brick chimney may have the considerable leakage which reduces the draught. Concrete and steel chimneys do not have such leakage.

(3) Concrete chimneys are lighter than brick chimneys but steel chimneys are the lightest.

(4) Steel chimneys are least costly and most rapid to install.

(5) Steel chimneys require least space and has less weight.

8·3. Chimney Calculations. Assumptions : From the equation $C+O_2=CO_2$, we see that the volume of CO_2 produced is equal to the volume of O_2 required. In air, nitrogen is present which does not take part in combustion. When we add nitrogen associated with this oxygen and also the excess air, it is seen that the volume of products of combustion is equal to the volume of air supplied, measured at the same temperature.

The major constituent of boiler fuels is carbon. In addition there is some hydrogen which on combustion would form steam. However, the volume of steam is negligible and hence it may be assumed that the volume of flue gases in the chimney is equal to the volume of air supplied, both volumes being measured at the same temperature.

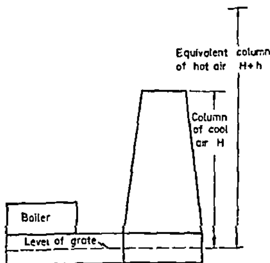


Fig. 8·1. Chimney height

Let, m=mass of air per kg of fuel.

T_0=absolute temperature at freezing point. (273°K)

ρ = density of air at temperature T_0 ($=1{\cdot}293$ kg/m³)

T_1 = atmospheric temperature.

T = temperature of flue gases inside the chimney.

P = difference of pressure in kgf/m².

H = height of chimney in metre.

h = height of column of hot gases in metre which would produce pressure, P kgf/m²

h' = height of column of water in mm equivalent to pressure, P kgf/m²

Density of outside air at temperature $T_1 = \dfrac{\rho \times T_0}{T_1}$ kg/m³

Mass of chimney gases $=(m+1)$ kg

Volume of chimney gases at temperature T

$$= \text{Volume of air supplied at temperature } T$$

$$= \frac{m}{\rho} \times \frac{T}{T_0} \text{ m}^3$$

$\therefore$ Density of chimney gases $= \dfrac{(m+1)}{\dfrac{m}{\rho} \times \dfrac{T}{T_0}} = \dfrac{\rho(m+1)T_0}{mT}$ kg/m³

Pressure of chimney gases at the grate level

$$= \frac{H\rho(m+1)T_0}{mT} \text{ kgf/m}^2$$

Pressure at the grate due to H metre column of outside air

$$= \frac{H\rho T_0}{T_1}$$

(*a*) **Draught produced.**

Pressure causing draught, in kgf/m²

$$P = \frac{H\rho T_0}{T_1} - \frac{H\rho(m+1)T_0}{mT} = H\rho T_0 \left[\frac{1}{T_1} - \frac{m+1}{m} \times \frac{1}{T} \right] \quad [8{\cdot}1\ (a)]$$

Substituting the values of ρ and T_0,

$$P = 353H \left[\frac{1}{T_1} - \frac{m+1}{m} \times \frac{1}{T} \right] \text{ kgf/m}^2 \quad [8{\cdot}1(b)]$$

Heght of column of hot gases, $h = \dfrac{\text{Pressure in kgf/m}^2}{\text{Density of hot gases in kg/m}^3}$

$$= \frac{H\rho T_0\left[\frac{1}{T_1} - \frac{m+1}{m} \times \frac{1}{T}\right]}{\frac{\rho(m+1)T_0}{mT}}$$

$$= H\left(\frac{m}{m+1} \times \frac{T}{T_1} - 1\right) \text{ in metre} \qquad (8{\cdot}2)$$

The pressure of 1 kgf/m² is equivalent to 1 mm of water column.

∴ Height of column of water

$$h' = 353H\left[\frac{1}{T_1} - \frac{m+1}{m} \cdot \frac{1}{T}\right] \qquad (8{\cdot}3)$$

(*b*) **Condition for maximum discharge**

Mass of hot gases discharged ∝ velocity × density

But velocity $\propto \sqrt{h}$ and density $\propto \frac{1}{T}$

∴ Mass of hot gases discharged $\propto \sqrt{h} \times \frac{1}{T}$

$$\propto \sqrt{\left(\frac{m}{m+1} \times \frac{T}{T_1} - 1\right)} \times \frac{1}{T}$$

For maximum discharge, differentiating and equating to zero,

we get, $T = \frac{2(m+1)}{m} \times T_1$ and $h = H$ Result (8·4)

(c) **Efficiency of a chimney** In the case of natural draught system the temperature of the flue gases leaving a chimney is higher than in artificial draught system because certain minimum temperature is required to produce a given draught with the given height of a chimney. This causes loss of heat and lowers the overall efficiency of a plant.

Let T_2 be the temperature of the flue gases in the case of artificial draught.

∴ Extra heat carried away by one kg of flue gases

$$= C_p(T - T_2) \text{ heat units}$$

The draught pressure produced by the natural draught system in metre of hot gases, $h = H\left(\frac{m}{m+1} \times \frac{T}{T_1} - 1\right)$

Maximum energy this head would give to one kg of air, at the cost of extra heat $= 1 \times H \left(\frac{m}{m+1} \times \frac{T}{T_1} - 1 \right)$ kgfm

$$\therefore \quad \text{Efficiency of chimney} = \frac{H\left(\frac{m}{m+1} \times \frac{T}{T_1} - 1 \right)}{JC_p(T - T_2)} \qquad (8\cdot5)$$

8·4. Artificial Draught. When draught required is more than about 40 mm artificial draught is more economical provided heat of flue gases is utilised in economiser, air preheater, etc. The other advantages of artificial draught are higher rate of combustion and evaporation, higher furnace temperatures, complete draught control according to demand, prevention of smoke, less excess air, use of lower quality of fuel and independence from climatic conditions.

Disadvantages of the artificial draught are increased capital cost and power and maintenance required for fans.

With artificial draught fuel burnt per m^2 of grate area is 200 to 300 kg/hr whereas in natural draught system it is 50 to 120 kg/hr. Also it has been found that the reduction in consumption of fuel per hp due to artificial draught is about 15 per cent.

Artificial draught may be a mechanical or steam jet draught. Draught produced by means of fans is known as *mechanical draught*.

Mechanical draught. It may be classified as *forced draught* or *induced draught* depending upon the system used to produce it.

(*i*) *Forced draught*. In a forced draught system the air under pressure is forced through fuel on stoker or grate either direct or through an air preheater. Forced draught is obtained in two ways :

(*a*) *Closed stokehold system*. This is quite popular in marine boilers. In this system the stokehold or room containing boilers is made air tight. Thus the stokers are under greater pressure than outside atmospheric air. Communication between the stokehold and outside is by means of double doors with a space between them forming an air lock.

(*b*) *Closed ashpit system*. In this system the front of the ashpit is closed and the delivery pipe from the fan leads into ducts which

conduct the air to the ashpits of the various furnaces. Part of the air is delivered above the grate but the greater portion passes from the ashpit through the grate.

(*ii*) *Induced draught.* In an induced draught system the air pressure at the fuel bed is reduced below that of atmosphere by means of a fan placed at or near to the bottom of the chimney. The action of this is analogous to the action of a chimney. Induced draught is generally used with economiser and air preheater.

(*iii*) *Balanced draught.* In many cases both forced and induced draught are used in such a way that atmospheric or slightly negative pressure is maintained in the combustion chamber. This is the standard practice now-a-days. If the pressure is higher than atmospheric, furnace flames shoot out when fire door is opened and gases may leak from the furnace into the boiler room. If combustion chamber pressure is much below atmospheric, air may leak in the combustion chamber and flues, increasing the excess air and thereby increasing the chimney gas losses. A slightly negative pressure (of about 10 mm of water) induces a small current of air through the settings and prevents overheating. This combination of draught is termed as *balanced draught.*

Comparison of F.D. and I D fans.

(1) In forced draught fan size and power required are one-third to one-half of that in an induced fan installation, because forced draught fan handles cold (atmospheric) air.

(2) Induced draught fan must be specially designed to handle hot gases and carbon particles. (Temperature from 250°C to 450°C)

(3) Induced draught fan, due to higher temperature may require water cooled bearings.

Choice of Fan *If a boiler has underfeed stoker or if the loss* of draught at stoker is more as compared to rest of the boiler, the logical choice of fan would be forced draught fan Conversely, if the major part of the loss were through the boiler and secondary heating surface, an induced draught fan would be better.

Steam Jet Draught. In steam jet draught the jet of steam is directed into the chimney or the jet is installed in the ashpit. In the

former case it is induced draught whereas in the latter case it is forced draught. The steam jet draught is simple, cheap to install and requires very little attention. In addition to this, when used as forced draught system, many low grade fuels can be burnt because steam avoids the formation of clinker. The main disadvantage of steam jet draught is that it cannot start until steam pressure is available. Also, in case of forced draught system the steam passing into the furnace will carry away heat in the same manner as moisture in the fuel. If the nozzles are periodically inspected and renewed when eroded, the steam required to create the draught should not exceed 5 per cent of the output of the boiler.

Steam jet draught of induced type is very much popular in locomotive boilers. The exhaust from non-condensing steam engine is directed into the smoke box. With this arrangement the draught is automatically adjusted to suit the requirements of the boiler.

8-5. Primary Air. The air supply to the fire bed passes mainly through the fire bars. The air which follows this course is called primary air. During its passage through the ashpit and fire-box it becomes heated and by the time it has passed through the fire bed it reaches the correct temperature.

Secondary Air. In order to ensure that sufficient air has reached the combustion chamber, some air is usually supplied over the fire bed. This enters the combustion zone through specially arranged inlets in the firing door of the boiler. This air is called secondary air.

To ensure complete combustion quantity of air supplied is more than required theoretically. This is normally expressed as percentage excess air. The excess air supplied is generally as follows :

Hand fired boilers	50 to 100 per cent
Mechanical stoker fired boilers	20 to 50 per cent
Gas or pulverised fuel fired boilers	10 to 30 per cent

8-6. Equivalent Evaporation. To compare the capacity of boilers evaporating steam at different pressures and temperatures and from feed water at different temperatures, a term "equivalent evaporation" is used. Equivalent evaporation from and at 100°*C* means the evaporation of feed water at 100°*C* to dry and saturated

steam at 100°C. For equivalent evaporation of 1 kg of steam, latent heat at atmospheric pressure *i e* 539 kcal is required.

Let m be the actual mass of water evaporated per kg of fuel. Then equivalent evaporation in kg per kg of fuel

$$=\frac{m \times \text{heat supplied per kg of steam}}{\text{Latent heat corresponding to saturation temperature of } 100^\circ C}$$

$$=\frac{m \times (\text{enthalpy of steam} - \text{enthalpy of feed water})}{539 \text{ kcal}}$$

Boiler Efficiency. Boiler efficiency is a measure of the heat usefully employed in generation of steam to the heat supplied in the fuel in the same period

$$\text{Boiler efficiency} = \frac{m\,(h_1 - h_2)}{CV}$$

where CV = calorific value of fuel per kg

h_1 = enthalpy of steam from boiler

h_2 = enthalpy of feed water

Heat Balance. The boiler calculations are generally based upon the higher calorific value of 1 kg of fuel considered as 100 per cent. The terms for heat balance may be as follows

1. *Heat utilised for generation of steam*

Useful heat absorbed, $H_1 = m(h_1 - h_2)$

= Equivalent evaporation 539 kcal

2. *Loss due to moisture in fuel* The moisture in the fuel is evaporated and superheated and thus the heat is lost

Loss due to moisture in fuel, $H_2 = m_1(h_1' - h_2')$

where m_1 = mass of moisture per kg of fuel as fired

h_1' = enthalpy of steam formed

h_2' = enthalpy of liquid at temperature of boiler furnace.

3. *Loss due to H_2O vapour, from the combustion of hydrogen.* This is found similar to loss due to moisture in fuel

4. *Loss due to moisture in air.* This is also found in the similar way as above and is negligible.

5. *Loss due to dry flue gases.* This is the largest loss.

$$H_3 = m_2\, C_p(t_g - t_a)$$

where m_2 = mass of dry flue gases per kg of fuel

C_p = specific heat of dry flue gases

t_g = temperature of flue gases

t_a = temperature of atmospheric air

6. *Loss due to incomplete combustion of carbon.* This loss is caused by incomplete combustion of carbon to carbon mono-oxide instead of carbon dioxide

$$H_4 = m_3 \times CV \text{ of } CO$$

$$= \frac{CO \times C}{CO_2 + CO} \times CV \text{ of } CO$$

where C = mass of carbon actually burned per kg of fuel

CO and CO_2 = percentages by volume

CV = No. of heat units generated by burning 1 kg of carbon contained in CO to CO_2 = 5,690 kcal

7. *Loss due to unconsumed combustibles in refuse.* This loss is due to some unburnt carbon falling into the ashpit

$$H_5 = m_4 \times CV$$

where m_4 = unburnt mass of carbon in refuse per kg of fuel

CV = calorific value of carbon.

8. *Loss unaccounted for.* These losses due to unconsumed hydrogen and hydrocarbons, radiation, etc., and are found by difference.

Boiler trial. Fig. 8·2 shows the schematic diagram of a boiler plant. In a trial, boiler is run on 'test conditions' for a few hours to obtain steady conditions. Observations are started after the boiler has run on steady conditions for about three hours. Readings are taken at an interval of 10 to 15 minutes. Light signals may be arranged for simultaneous taking of readings. The duration of trial may be about 6 hours in coal-fired boilers and 4 hours in oil-fired boilers. It should be aimed that conditions of fuel, draft, temperature, pressure, water level, rate of feeding and evaporation, etc., are same throughout the test.

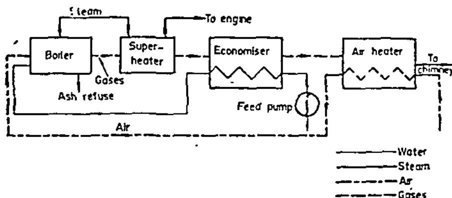


Fig. 8 2. Schematic diagram of a boiler plant.

ILLUSTRATIVE EXAMPLES

8 1. Equivalent evaporation per kg of coal and per m² of heating surface ; boiler efficiency

What is the procedure of a boiler trial ? Give schematic sketch of a boiler plant.

The following particulars relate to a trial of two marine boilers of the Scotch type : Duration of trial 16 hrs Total heating surface, 2,940 m². Total grate area 3 9 m². Mean absolute pressure of steam, 12·5 kgf/cm² Total coal burnt, 6,848 kg. Total mass of water evaporated, 62,400 kg. Temperature of feed water, 40°C. Condition of steam 0 95 dry.

Calculate, (a) the mass of coal burnt per m² of grate area per hour, (b) the mass of water evaporated per kg of coal under actual conditions, (c) the equivalent evaporation from and at 100°C per kg of coal, (d) the equivalent evaporation from and at 100°C per m² of total heating surface per hour, and (e) the efficiency of the boiler Take calorific value of coal 6,930 kcal/kg.

(*a*) Steam formed per hour $= \frac{62,400}{16} = 3,900$ kg

Coal burnt per hour $= \frac{6,848}{16} = 428$ kg

Coal burned per m² of grate per hour $= \frac{428}{3 9} = 10 98$ kg

(b) Mass of water evaporated per kg of coal

$$=\frac{3{,}900}{428}=9{\cdot}114 \text{ kg} \qquad \text{Ans.}$$

(c) Equivalent evaporation from and at 100°C per kg of coal

$$=\frac{\text{Heat given to steam per kg of coal burnt}}{\text{Latent heat from and at } 100°C}$$

$$=\frac{9{\cdot}114[(191{\cdot}7+0{\cdot}95\times473{\cdot}6)-40]}{539}=10{\cdot}17 \text{ kg} \qquad \text{Ans.}$$

(d) Equivalent evaporation per m² of heating surface per hr.

$$=\frac{10{\cdot}17\times428}{2{,}940}=1{\cdot}481 \text{ kg} \qquad \text{Ans.}$$

(e) Efficiency of the boiler $=\frac{10{\cdot}17\times539}{6{,}600}=83{\cdot}1\%$ Ans.

8·2. Boiler η ; heat added in economiser, boiler and super heater.

Why excess air is supplied in boilers ? What do you understand by secondary air ?

A steam generator is supplied with coal of calorific value 6,800 kcal/kg and produces 7 kg of steam per kg of coal burnt. The feed water delivered from the feed pump is at a pressure of 30 kgf/cm² and at a temperature of 43·7°C. The temperature of the water leaving the economiser is 125·3°C. The steam passes from the generator to the superheater with a dryness of 0·95 and leaves the superheater at 400°C.

The pressure may be taken to be 30 kgf/cm² throughout. Find the efficiency of the boiler and the proportion of heat given to the steam in each section. The volume of 1 kg of water may be taken to be zero.

For theory—see text.

Heat supplied in 1 kg of steam $=(771{\cdot}4-43{\cdot}7)=727{\cdot}7$ kcal

∴ Efficiency of the boiler plant $=\frac{727{\cdot}7\times7}{6{,}800}=74{\cdot}9\%$ Ans.

Heat added in economiser per kg of steam

$$=1\times1(125{\cdot}3-43{\cdot}7)=81{\cdot}6 \text{ kcal or } 11{\cdot}2\% \qquad \text{Ans.}$$

Heat added in boiler per kg of steam

$$=(239{\cdot}6+0{\cdot}95\times430)-125{\cdot}3=522{\cdot}8 \text{ kcal or } \underline{71{\cdot}9\%} \qquad \text{Ans.}$$

Heat added in super heater per kg of steam

$$=[727{\cdot}7-(81{\cdot}6+522{\cdot}8)]=123{\cdot}3 \text{ kcal or } \underline{16{\cdot}9\%} \qquad \text{Ans.}$$

←*Heat supplied*=100→
Superheater=16·9
Economiser=11·2
Boiler =71·9

8·3 Leakage in economiser ; heat lost.

In a trial of a boiler fitted with an economiser the following volumetric analysis of the gas entering and leaving the economiser were made :

Constituents	*Entering*	*Leaving*
CO_2	*8·3%*	*7·9%*
O_2	*11·4%*	*11·5%*
N_2	*80·3*	*80·6*

The temperature of the flue gases on entering and leaving the economiser were 350°C and 180°C respectively. The temperature of the water entering and leaving the economiser were 15°C and 115 °C. Mass of feed water per hour 10,000 kg, mass of coal used per hour 1,060 kg, carbon in 1 kg of coal 0·8 kg. Assuming the average specific heat of the gases to be 0·25, estimate per kg of coal burnt, (a) *the air leakage into the economiser,* (b) *the heat lost by the gases in passing through the economiser, and* (c) *the heat gained by the feed water. Assume air temperature 15°C.*

(a) Flue gases entering the economiser

Constituents (a)	*Vol. per m³* (b)	*Mol wt* (c)	*Proportional mass* (d)=(b)×(c)	*C per kg of flue gas* $(e)=\frac{(d)}{\Sigma(d)}\times\frac{12}{44}$
CO_2	0·083	44	3·65	$\frac{3\cdot65}{29\cdot75}\times\frac{12}{44}=0\cdot03355$
O_2	0·114	32	3·65	
N_2	0·803	28	22·45	
Total	1·000		29·75	

$\therefore$ Dry flue gases per kg of coal$=\dfrac{0\cdot8}{0\cdot03355}=23\cdot8$ kg

Flue gases leaving the economiser :

Constituents (a)	*Vol. per m³* (b)	*Mol wt* (c)	*Proportional mass* (d)=(b)×(c)	*C per kg of flue gas* $(e)=\frac{d)}{\Sigma(a)}\times\frac{12}{44}$
CO_2	0·079	44	3·48	$\frac{3\cdot48}{29\cdot76}\times\frac{12}{44}=0\cdot0319$
O_2	0·115	32	3·68	
N_2	0·806	28	22·60	
Total	1·000		29·76	

$\therefore$ Dry flue gas per kg of coal$=\dfrac{0\cdot8}{0\cdot0319}=25$ kg

The increase in flue gases after leaving the economiser is due to the air leakage.

$\therefore$ Air leakage in economiser per kg of coal

$=25-23\cdot8=1\cdot2$ kg Ans.

(b) Heat lost by gases in passing through the economiser

$=$Heat in entering flue gases$+$heat in leakage air

$-$heat in leaving flue gases

$=23\cdot8\times0\cdot25\times350+1\cdot2\times0\cdot25\times15-25\times0\cdot25\times180$

$=964\cdot5$ kcal Ans.

(c) Heat gained by the feed water $= \frac{10,000}{1,060} \times (115-15)$

$= 943$ kcal Ans.

8.4. Height of chimney.

Find the height of a chimney to get net draught of 13 mm. Assume total draught losses 5mm ; temperature of chimney gases 320°C ; temperature of air 30°C and the mass of air used per kg of fuel 20 kg. One kg of air occupies a volume of 0 7734 m³ at NTP.

Chimney produces draught due to the difference between the mass of the column of hot gases inside the chimney and the mass of an equal column of cool air outside.

Density of external air $= \frac{1}{0.7734} \times \frac{273}{(273+30)}$

$= 1.165 \text{ kg/m}^3$

Density of chimney gases at $0°C = \frac{(20+1)}{20 \times 0.7734}$

$\therefore$ Density of chimney gases at 320°C

$= \frac{273}{(273+320)} \times \frac{21}{20 \times 0.7734}$

$= 0.625 \text{ kg/m}^3$

Let H = height of chimney in metre

A = cross-sectional area of chimney in m²

Mass of chimney gases $= AH \times 0.625$ kg

Mass of equal column of external air $= AH \times 1.165$ kg

Difference in the masses $= AH(1.165 - 0.625) = 0.54\,AH$ kg

As per definition this should be equal to the draught force.

$\therefore$ Draught in kgf/m², $P = 0.54H$ kg/m²

$= 0.54H$ mm of water

[1 kgf/m² = 1 mm of water]

Draught required $= 13+5 = 18$ mm of water

$0.54H = 18$

$\therefore$ Height of chimney, $H = 33.3$ metre

8-5. Draught ; efficiency and extra heat carried

Apart from the consideration of draught the minimum temperature of the waste gases leaving a certain boiler installation is 130°C. A natural draught is produced by means of a chimney 50 metre high, the gases within the chimney having a temperature of 310°C. The air supplied to the furnaces is 19 kg per kg of fuel burned, the temperature of the air is 7°C and the mean specific heat of the waste gases is 0·24. Calculate the draught produced in mm of water and efficiency of the chimney as an instrument for moving the air and the products of combustion through the furnaces and flues.

If the calorific value of the fuel is 7,800 kcal/kg express the extra expenditure of heat on the production of the chimney draught as a percentage of the heat of the fuel.

From Eq. (8-3c)

$$h' = 353H\left[\frac{1}{T_1} - \frac{m+1}{m} \times \frac{1}{T}\right]$$

$$= 353 \times 50\left[\frac{1}{280} - \frac{19+1}{19} \times \frac{1}{583}\right]$$

$$= 31{\cdot}2 \text{ mm}$$ Ans.

Draught in height of column of hot gases

$$h = H\left(\frac{m}{m+1} \times \frac{T}{T_1} - 1\right)$$

$$= 50\left(\frac{19}{20} \times \frac{583}{280} - 1\right) = 48{\cdot}9 \text{ metre}$$

Maximum energy total to static heat of 48·9 m will impart to 1 kg of flue gases $= 48{\cdot}9 \times 1 = 48{\cdot}9$ kgfm

Extra heat carried away by the flue gases per kg due to temperature being higher than in artificial draught system

$$= 1 \times 0{\cdot}24 \times (310 - 130) = 43{\cdot}2 \text{ kcal}$$

∴ Efficiency of the chimney

$$= \frac{48 \times 100}{427 \times 43{\cdot}2} = 0{\cdot}26\%$$ Ans.

Extra heat carried away by the flue gases per kg of fuel burned

$$= \frac{20 \times 43{\cdot}2}{7{,}800} \doteq 11{\cdot}05\%$$ Ans.

8.6. Chimney gas temp. ; h' and η for maximum discharge.

(a) Derive a condition for the maximum discharge of flue gases from chimney.

(b) A chimney has a height of 100 metres. For the maximum discharge condition, calculate the temperature of the chimney gases and the draught produced, if the air supplied per kg of fuel is 18 kg.

Also determine the efficiency of this chimney as an instrument for creating the draught, if the minimum temperature of chimney gases in artificial drught system is limited to 120°C. Take the boiler house temperature as 40°C and specific heat of flue gases 0 24.

For theory see-text.

Temperature of chimney gases, $T=\frac{2(m+1)}{m}\times T_1$

$$=\frac{2\times(18+1)}{18}\times 313=661°C \quad \text{Ans.}$$

Draught, $h'=353H\left[\frac{1}{T_1}-\frac{m+1}{m}\times\frac{1}{T}\right]$

$$=353\times 100\left[\frac{1}{313}-\frac{18+1}{18}\times\frac{1}{661}\right]=56{\cdot}4 \text{ mm of water. Ans.}$$

Extra heat carried away by the flue gases per kg due to temperature being higher than in artificial draught system.

$$=1\times 0\,24(661-393)=67 \text{ kcal or } 28{,}610 \text{ kgfm}$$

For maximum discharge, $h=H=100$ m

Static head of 100 metres will impart 100 kgfm of energy to 1 kg of gases

$\therefore$ Efficiency of the chimney $=\frac{100}{28{,}610}\times 100=0\,3496\%$ Ans.

8.7. Boiler trial : heat balance , η eq evaporation.

The following observations were made during a boiler trial :

Mass of feed water per hour 635 kg ; Temperature of feed water 65°C ; Steam pressure 10 5 kgf/cm² ; Oil fired per hour 52 kg , Higher calorific value 10,700 kcal/kg

Percentage composition of oil by mass, C=84·75 ; H_2=14·00 ; S=1·25.

Analysis of dry flue gases by volume, $CO_2=12{\cdot}4$; $O_2=4{\cdot}3$; $N_2=83{\cdot}3$.

Temperature of gases leaving boiler 362°C ; Specific heat of dry flue gases 0·24 ; Boiler room temperature 21°C ; Throttling calorimeter temperature at outlet 125°C ; Pressure of steam after throttling 10·1 cm of mercury ; Barometer reading 76 cm of mercury ; Heating surface of boiler 20 m² ; Specific heat of superheated steam 0·5. Partial pressure of steam in flue gases 0·07 kgf/cm².

Draw up a complete heat balance sheet and calculate the boiler efficiency and equivalent evaporation per kg of fuel and per m² of heating surface/hour.

Let x be the dryness fraction of steam coming from boiler.

Absolute pressure after throttling $=\dfrac{76+10{\cdot}1}{73{\cdot}55}=1{\cdot}17$ kgf/cm²

From steam tables,

Total heat before throttling = Total heat after throttling

$$183{\cdot}5+x\times480{\cdot}2=640+0{\cdot}5(125-103{\cdot}6)$$

[By interpolation

∴ Dryness fraction of steam, $x=0{\cdot}973$

Steam produced per kg of oil $=\dfrac{635}{52}=12{\cdot}21$ kg

Heat to steam per kg oil burnt

$$=12{\cdot}21[183{\cdot}5+0{\cdot}973\times480{\cdot}2-65]=7{,}153 \text{ kcal}$$

Heat supplied by 1 kg of oil = 10,700 kcal

Moisture formed per kg of oil burnt $=0{\cdot}13\times9=1{\cdot}17$ kg

Heat carried away by H_2O vapour (p.p. of steam 0·07 kgf/cm²)

$$=1{\cdot}17[614{\cdot}1+0{\cdot}5(362-38{\cdot}7)-21]=884 \text{ kcal}$$

Constituents of dfg/m³ (a)	*Mol Wt.* (b)	*Proportional mass* (c) = (a) × (b)	*C Per kg of fuel* (d) $=\dfrac{(c)}{\Sigma(c)}\times\dfrac{12}{44}$
$CO_2=0{\cdot}124$	44	5·456	$\dfrac{5{\cdot}456}{30{\cdot}156}\times\dfrac{12}{44}$
$O_2=0{\cdot}043$	32	1·376	=0·04934
$N_2=0{\cdot}833$	28	23·324	
1·000		30·156	

Mass of dry flue gases per kg of fuel$=\frac{0.8575}{0.04931}=17.78$ kg

∴ Heat carried away by dfg/kg of fuel burnt

$$=17.8\times0.24(362-21)=1,422 \text{ kcal}$$

Heat balance sheet on the basis of one kg of fuel

Heat supplied	*kcal*	%	*Heat consumed*	*kcal*	%
Heat supplied (based on HCV)	10,700	100	(1) Heat to steam	7,153	66.85
			(2) Heat carried by dfg	1,422	13.29
			(3) Heat carried away by moisture in the flue gases	884	8.26
			(4) Heat lost in rediation, etc. (by difference)	1,241	11.60
Total	10,700	100		10,700	100.00

Boiler efficiency$=\frac{7,153}{10,700}=$ 66.85% **Ans.**

Equivalent evaporation per kg of fuel$=\frac{7,153}{539}=$ 13.27 kg **Ans.**

Equivalent evaporation per m²/hour

$$=\frac{13.27\times15}{20}=9.953 \text{ kg}$$ **Ans.**

8.8 Coal/hr.; volumetric and gravimetric analysis of flue gases.

A boiler is supplied with coal which contains 15 per cent of moisture. The dried coal analysed is found to have 87% mass carbon, 4% hydrogen, 7% incombustibles and 2% presumably oxygen. The boiler is rated at 85% thermal efficiency and has to supply 5,500 kg of steam per hour at a temperature of 200°C and a pressure of 15 kgf/cm². Find (a) the mass of coal used per hour, (b) the volumetric and gravimetric analysis of the products of combustion. The temperature of the feed water is 60°C. CO_2 content in the flue gases is 14% by volume and CO content is nil.

(Given 1 kg of carbon burning to CO_2 gives 8,000 kcal and 1 kg of H_2 to steam gives 28,000 kcal).

Assuming that the calorific value of H_2 given is lower calorific value (as *HCV* of H_2 is 34,400 kcal).

$$\text{LCV of dry coal} = 8{,}000C + \left(H - \frac{0}{8}\right) \times 28{,}000$$

$$= 8{,}000 \times 0{\cdot}87 + \left(0{\cdot}04 - \frac{0{\cdot}02}{8}\right) \times 28{,}000 = 8{,}010 \text{ kcal}$$

Coal contains 15 per cent of moisture

$\therefore$ LCV of coal as fired $= 0{\cdot}85 \times 8{,}010 = 6{,}810$ kcal/kg

From steam tables. Total heat of 1 kg of steam at 15 kgf/cm² and 200°C = 668·4 kcal

$$\text{Heat required per kg of steam} = \frac{668{\cdot}4 - 60}{0{\cdot}75} = 811{\cdot}2 \text{ kcal}$$

$\therefore$ Coal used $= \dfrac{5{,}500 \times 811{\cdot}2}{6{,}810} = 655{\cdot}2$ kg/hr   Ans.

Minimum O_2 required per kg of dry coal

$$= 0{\cdot}87 \times \frac{32}{12} + 0{\cdot}04 \times 8 - 0{\cdot}02 = 2{\cdot}62 \text{ kg}$$

Let m kg of air be supplied per kg of dry coal.

Analysis of dry flue gas per kg of dry coal

Constituent (a)	*Part by mass* (b)	*Mol. wt.* (c)	*Parts by vol.* (d)=(b)/(c)
CO_2	$\frac{44}{12} \times 0{\cdot}87 = 3{\cdot}19$	44	0·0725
O_2	$0{\cdot}23m - 2{\cdot}62$	32	$(0{\cdot}00719m - 0{\cdot}0819)$
N_2	$0{\cdot}77m$	28	$(0{\cdot}0275m)$
		Total	$(0{\cdot}03469m - 0{\cdot}0094)$

$$\% \text{ of } CO_2 \text{ by volume, } 10 = \frac{0{\cdot}0725 \times 100}{(0{\cdot}03469m - 0{\cdot}0094)}$$

$$m = 21{\cdot}16 \text{ kg}$$

Composition of wet flue gas per kg of wet coal

Constituents (a)	By mass (b)	% by mass (c)	Mol wt (d)	By volume (e)=(b)/(d)	% by volume (f)
CO_2	3 19×0 85 =2 712	14 32	44	0 0616	9 56
O_2	(0·23×21·16−2 62)0 85=1·910	10 08	32	0 0599	9 31
H_2O	0·15+0 36×0 85 =0 456	2 41	18	0 0254	3 96
N_2	0 77×21 16×0 85 =13 86	73 20	28	0 4960	77·17
	18 938	100 01		0 6429	100 00

8 9. Mass of air; pp of steam; heat carried by moist flue gas.

The percentage analysis by mass of the coal used in a boiler was C 83, H_2 6, O_2 5, ash, etc, 6 The dry flue gas contained CO_2 10 50, CO 1·30, O_2 7 67, N_2 80 53 per cent by volume.

The temperatures of the air and flue gas were 15°C and 215°C respectively. Find

(a) the total mass of air supplied per kg of coal;

(b) the partial pressure of the steam in the hot flue gas, if the pressure in the flue is 1·03 kgf/cm².

(c) the heat carried away by the moist flue gas per kg of coal fired, including that due to the CO present.

Specific heat of dry flue gas 0 23 and of superheated steam 0 49. Air contains 23 1 per cent of O_2 by mass.

Calorific value of 1 kg CO to CO_2, 2410 kcal/kg

(a)

Constituent per mol of dfg (a)	*Mol wt* (b)	*Proportional mass of constituents* (c)=(a)×(b)	*Mass of carbon*
$CO_2=0{\cdot}1050$	44	4·620	$4{\cdot}620\times\frac{12}{44}=1{\cdot}26$
$CO=0{\cdot}0130$	28	0·364	$0{\cdot}364\times\frac{12}{28}=0{\cdot}156$
$O_2=0{\cdot}0767$	32	2·460	
$N_2=0{\cdot}8053$	28	22·550	
Total=1·0000		29·994	1·416

$$\text{Mass of dfg/kg of coal}=\frac{29{\cdot}994}{1\ 416}\times 0{\cdot}83=17\ 6\text{ kg}$$

H_2O produced from combustion$=0{\cdot}06\times 9\ =0{\cdot}54$ kg/kg of coal

Since 0·06 kg is ash, mass of air$=[17{\cdot}6+0{\cdot}54-(1-0{\cdot}06)]$

$=17\ 2$ kg/kg of fuel Ans.

(b) Partial pressure of steam$=\dfrac{H_2O\text{ mol}}{\text{Total mol}}\times$total pressure

$$=\frac{\frac{0\ 54}{18}}{\frac{17\ 6}{29{\cdot}994}+\frac{0{\cdot}54}{18}}\times 1{\cdot}03=0{\cdot}05\text{ kgf/cm}^2$$ Ans.

(c) From steam tables at 0 05 kgf/cm², $h_g=611{\cdot}5$; $t_s=32{\cdot}55°C$

Heat in vapour$=0{\cdot}54\ [(611{\cdot}5-15)+0{\cdot}49(2\times 15-32{\cdot}55)]$

$=370{\cdot}4$ kcal/kg of coal

Heat in dfg$=0{\cdot}23\times 17{\cdot}6(215-15)\ =809{\cdot}6$ kcal/kg of coal

Heat in $CO=\dfrac{0{\cdot}364}{29{\cdot}994}\times 17{\cdot}6\times 2{,}410=515$ kcal/kg of coal

∴ Total heat carried away by moist gases

$=370{\cdot}4+809{\cdot}6+515$

$=1{,}695$ kcal/kg of coal

8-10. Heat balance sheet ; air heater efficiency.

The following information was obtained during the trial of a boiler which included an economiser and an air heater.

Evaporation per kg of coal fired 9 18 kg, steam pressure and temperature 40 kgf/cm² and 350°C. Moisture in coal fired 1 5 per cent, carbon in ashes negligible. Analysis of dry coal, carbon 84, hydrogen 4, oxygen 7 and ash 5 per cent Calorific value of dry coal 7,800 kcal/kg.

Feed water temperatures, entering economizer 27°C leaving economizer 150°C

Air temperatures, entering air heater 280°C, leaving air heater and entering chimney 160°C.

Analysis of dry flue gas, CO_2 12 3, O_2 7·7, N_2 80 0 per cent by volume

Draw up a heat balance for the complete boiler plant in kcal per kg of dry coal, using a datum of 15°C and determine the boiler efficiency. Estimate also the efficiency of heat transmission in the air heater. Take C_p for air and dry flue gas = 0 24 kcal/kg °C

Assume partial pressure of steam vapour 0 07 kgf/cm² and its C_p = 0 48 kcal/kg °C

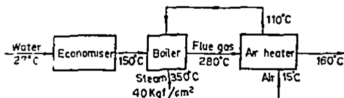


Fig 8 3

The problem is solved on the basis of 1 kg of dry fuel

Constituents per mol of dfg (a)	*Mol wt* (b)	*Proportional mass of constituents* (c) = (a) × (b)	*Mass per kg of dfg* $(d) = \frac{(c)}{\Sigma(c)}$	*Mass of carbon per kg dfg*
N_2 = 80 0	28	2,240	0·7398	$\frac{0\ 1788 \times 12}{44}$ = 0 04876
CO_2 = 12 3	44	541 2	0 1788	
O_2 = 7·7	32	246 4	0 0814	
Total = 100 0		3,027 6	1 0000	

(a)

Constituent per mol of dfg (a)	*Mol wt* (b)	*Proportional mass of constituents* (c)=(a)×(b)	*Mass of carbon*
CO_2=0·1050	44	4·620	$4·620 \times \frac{12}{44} = 1·26$
CO=0·0130	28	0·364	$0·364 \times \frac{12}{28} = 0·156$
O_2=0·0767	32	2·460	
N_2=0·8053	28	22·550	
Total=1·0000		29·994	1·416

$$\text{Mass of dfg/kg of coal} = \frac{29·994}{1·416} \times 0·83 = 17·6 \text{ kg}$$

H_2O produced from combustion $= 0·06 \times 9 = 0·54$ kg/kg of coal

Since 0·06 kg is ash, mass of air $= [17·6 + 0·54 - (1 - 0·06)]$

$= 17·2$ kg/kg of fuel Ans.

(b) Partial pressure of steam $= \frac{H_2O \text{ mol}}{\text{Total mol}} \times$ total pressure

$$= \frac{\frac{0·54}{18}}{\frac{17·6}{29·994} + \frac{0·54}{18}} \times 1·03 = 0·05 \text{ kgf/cm}^2 \quad \text{Ans.}$$

(c) From steam tables at 0·05 kgf/cm², $h_g = 611·5$; $t_s = 32·55°C$

Heat in vapour $= 0·54\,[(611·5 - 15) + 0·49(2 \times 15 - 32·55)]$

$= 370·4$ kcal/kg of coal

Heat in dfg $= 0·23 \times 17·6(215 - 15) = 809·6$ kcal/kg of coal

Heat in $CO = \frac{0·364}{29·994} \times 17·6 \times 2{,}410 = 515$ kcal/kg of coal

∴ Total heat carried away by moist gases

$= 370·4 + 809·6 + 515$

$= 1{,}695$ kcal/kg of coal Ans.

8-10. Heat balance sheet ; air heater efficiency.

The following information was obtained during the trial of a boiler which included an economiser and an air heater.

Evaporation per kg of coal fired 9 18 kg, steam pressure and temperature 40 kgf/cm² and 350°C. Moisture in coal fired 1·5 per cent, carbon in ashes negligible. Analysis of dry coal, carbon 84, hydrogen 4, oxygen 7 and ash 5 per cent. Calorific value of dry coal 7,800 kcal/kg.

Feed water temperatures, entering economizer 27°C leaving economizer 150°C.

Air temperatures, entering air heater 280°C, leaving air heater and entering chimney 160°C.

Analysis of dry flue gas, CO_2 12·3, O_2 7·7, N_2 80·0 per cent by volume

Draw up a heat balance for the complete boiler plant in kcal per kg of dry coal, using a datum of 15°C and determine the boiler efficiency. Estimate also the efficiency of heat transmission in the air heater. Take C_p for air and dry flue gas = 0 24 kcal/kg °C.

Assume partial pressure of steam vapour 0 07 kgf/cm² and its C_p = 0 48 kcal/kg °C.

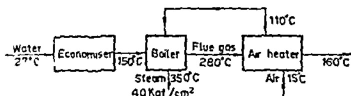


Fig. 8 3.

The problem is solved on the basis of 1 kg of dry fuel

Constituents per mol of dfg (a)	Mol wt (b)	Proportional mass of constituents (c) = (a) × (b)	Mass per kg of dfg $(d) = \frac{(c)}{\Sigma(c)}$	Mass of carbon per kg dfg
N_2 = 80 0	28	2,240	0 7398	$\frac{0\ 1788 \times 12}{44}$
CO_2 = 12 3	44	541 2	0 1788	= 0·04876
O_2 = 7·7	32	246 4	0 0814	
Total = 100 0		3,027 6	1 0000	

$$\text{Mass of dfg/kg of dry fuel} = \frac{\text{mass of carbon/kg of fuel}}{\text{mass of carbon/kg of dfg}}$$

$$= \frac{0.84}{0.04876} = 17.23 \text{ kg}$$

H_2O produced from combustion $= 0.04 \times 9 = 0.36$

Air supplied/kg of dry fuel $= 17.23 - (1 - \text{ash} - H_2O \text{ formed})$

$= 17.23 - (1 - 0.005 - 0.36)$

$= 16.64$ kg

Mass of dry fuel/kg of fuel fired

$= 1.000 - 0.015 = 0.985$ kg

$\therefore$ Moisture/kg of dry fuel $= \dfrac{0.015}{0.985} = 0.0152$ kg

Total H_2O/kg of dry fuel $= 0.36 + 0.0152 = 0.3752$ kg

Evaporation per kg of dry fuel

$= \dfrac{9.18}{0.985} = 9.32$ kg of steam

From steam tables, specific enthalpy at 40 kgf/cm² and 350°C = 737·9 kcal/kg ; at 0·07 kgf/cm², $t_s = 38.66$°C, $h_g = 614.1$ kcal/kg

Heat in steam produced above 15°C

$= 9.32(737.9 - 15) = 6,740$ kcal

Heat in dry flue gas $= 17.23 \times 0.24(160 - 15) = 600$ kcal

Heat in H_2O in flue gas $= 0.3752[(614.1 - 15) + 0.48(160 - 38.66)]$

$= 246$ kcal

Heat in fuel $= 7,800$ kcal

Heat in feed water $= 9.32(27 - 15) = 112$ kcal

Heat balance sheet per kg of dry fuel

Credit	kcal	%	Debit	kcal	%
1 kg of dry coal	7,800	98·58	Steam	6,740	85·17
Feed water	112	1·42	Dry flue gas	600	7·60
			H_2O in flue gas	246	3·11
			Heat to radiation, etc., by diff.	326	4·12
Total	7,912	100·00	Total	7 912	100·00

Boiler efficiency $= \frac{\text{heat produced in the steam}}{\text{heat supplied by the fuel}}$

$$= \frac{9.32(737.9 - 27)}{1 \times 7,800} = 85\% \qquad \text{Ans.}$$

Air heater :

Heat gained by the air $= 16.61 \times 0.24 \times (110 - 15)$
$= 379.4$ kcal/kg of dry fuel

Heat given up by the dfg $= 17.23 \times 0.24 \times (280 - 160)$
$= 496.2$ kcal/kg of dry fuel

Heat given up by the H_2O $= 0.3752 \times 0.48 \times (280 - 160)$
$= 21.6$ kcal/kg of dry fuel

The efficiency of heat transmission

$$= \frac{\text{heat to the air}}{\text{heat given up by the flue gases}}$$

$$= \frac{379.4}{496.2 + 21.6} = 73.4\% \qquad \text{Ans.}$$

EXAMPLES 8

8-1. Equivalent evaporation : fuel consumption

Define (a) equivalent evaporation from and at 100°C and (b) boiler efficiency.

The equivalent evaporation of a boiler from and at 100°C is 1,300 kg/hour. Calculate the actual evaporation if the feed water is supplied at 110°C and the steam is generated at a pressure of 15 kgf/cm² and temperature 200°C. If the efficiency of this boiler is 72 per cent, find (a) the fuel consumption per hour taking C.V. of coal as 6,100 kcal per kg, and (b) the grate area, if the rate of evaporation is 100 kg/m² per hour.

[heat added per kg of steam = 558.4 kcal, actual evaporation = 1,255 kg/hr ; fuel consumption = 159.5 kg/hr, grate area = 12.55 m²]

8.2. Rise of temperature of feed water in economiser and of air in preheater.

The coal fed to a boiler had the percentage analysis by mass : C—84, H_2—4, O_2—4, and non-combustible 8, —100 plus 5 per cent of hygroscopic moisture. The volumetric analysis of the dry flue gas gave CO_2—13.2, O_2—6.07, CO—0.3 per cent. The ashes falling into the pit were 0.07 of the coal as fired and contained 9 per cent of carbon.

The flue gases left the boiler proper at 400°C entering an economiser which they left at 250°C, then entering an air per-heater which they left at 30°C.

Mass of steam generated 9.3 kg/kg of fuel as fired ; specific heat of air and dry products of combustion 0.24 ; specific heat of steam in flue gas 0.48 ; efficiency of heat transfer in economiser and air-heater 72 and 82 per cent respectively. Air contains 23.1 per cent by mass of oxygen. Determine the rise of temperature (a) of the feed water in economiser, and (b) of the air in the per-heater.

[Carbon in ash per kg of coal $= \frac{0.07 \times 0.09}{0.95}$; dfg/kg of dry coal—15.62 kg ; dfg per kg of coal as fired—14.85 kg ; H_2O of combustion—0.36 ; total vapour per kg of coal as fired—0.392 kg ; air per kg of coal as fired—14.3 kg ; ΔT in economiser—43.56°C ; ΔT in pre heater—107.6°C]

8.3. Draught r q t extra heat ; temp. for maximum discharge.

A chimney 30 metre high is discharging hot gases at 300°C, when the outside air temperature is 20°C. The quantity of air supplied per kg of fuel is 20 kg. Determine

(a) the draught produced in mm of water column ;

(b) the efficiency of the chimney, if the minimum temperature of artificial draught is 150°C. The mean specific heat of the gases is 0.24 ;

(c) the percentage of heat spent in natural draught system for creating draught, if the net calorific value of fuel supplied be 7,320 kcal/kg ;

(d) the temperature of the chimney gases for maximum discharge in a given time, and what would be the draught produced correspondingly.

[h'=16·7 mm ; equivalent height of column of hot gas=25·9 metre ; η=0·1686% ; extra heat=10·32% ; temperature of hot gas =615·3°C ; h=18·1 mm]

8-4. Air used for given draught and chimney height.

What is the function of a boiler chimney ? Why no chimney is provided in a locomotive boiler ?

How much air is used per kg of coal burnt in a boiler having a chimney of 35 m height to create a draught of 20 mm of water, when the temperature of the flue gases in the chimney is 370 C and the temperature of the boiler house is 31°C. Take volume of 1 kg of air at N.T.P. as 0·7734 m³.

Does this chimney satisfy the condition for maximum discharge ?

[equating draught pressure per unit area equal to difference in the mass of hot and cold air, m=18·81 kg.

Note. As denominator is very small, the result may vary for very small difference in calculation.

For maximum discharge, $T=2\left(\frac{m+1}{m}\right)\times T_1=646\cdot6°K$ whereas actual temperature is 643°K ∴ condition satisfied]

8·5 h.p. for I.D. and F.D. fans.

What are the advantages of artificial draught system over natural draught system ? Explain the term "balanced draught".

Estimate the bhp of a motor required to drive a fan which maintains a draught of 5 cm of water under the following conditions for (a) induced draught fan, (b) forced draught fan. Temperature of flue gases leaving the boiler in each case is 175°C and of air in the boiler house is 30°C ; air supplied per kg of fuel in each case is 18·0 kg, mass of coal burnt per hour is 25·40 kg and assume the efficiency of fans to be 78 per cent. Take volume of 1 kg of air at N.T.P. = 0·7734 m³.

[volume of flue gases for I.D. fan=871·4 m³, hp=7·54, hp for F.D. fan=$\frac{7\cdot54\times T_1}{T}$=5·05]

8·6. Heat carried in flue gases from a boiler.

A boiler is fired with a fuel having a composition by mass : C, 86% ; H, 3·9% ; O, 1·4% ; ash, 8·7%

The volumetric analysis of the dry flue gas was : CO_2, 12·7% ; CO, 1·4% ; O, 4·1% ; N, 81·8%.

The temperature of the air supply to the furnace was 20°C and the temperature of the flue gases leaving the boiler 220°C. Taking the dew-point of the wet flue gases as 50°C, the specific heat of steam (superheated) as 0·36 per unit mass, the specific heat of dry flue gas as 0·24 per unit mass, and the calorific value of CO as 2,430 kcal/kg, calculate per kg of fuel burned :

(a) *the heat carried away by the dry flue gas ;*

(b) *the heat carried away by the moisture from combustion ;*

(c) *the heat loss due to incomplete combustion of the carbon.*

To what points would you pay particular attention in order to increase overall efficiency of the installation and what is the advantage of a low dew point ?

[dfg/kg of fuel=15·35 kg ; H_2O formed=0·351 kg ; heat carried by dfg=737 kcal ; heat in H_2O=232 kcal ; heat lost due to CO formation=485 kcal]

8·7. Heat balance sheet of a boiler.

Draw out a heat balance sheet for a boiler with the following test results—General, duration of test=8 hours, ambient temperature of air 35°C, coal fired in 8 hrs.=6,400 kg, ash collected in 8 hrs=480 kg, combustibles in ash 18 kg, moisture in coal=2·16 per cent.

Analysis of coal, C=83·39%, H_2=4·56%, O_2=5·05%, S=0·64% N_2=1·03%, ash=5·33% *Calorific value of coal as fired=8,000 kcal/kg.*

Steam evaporation in 8 hrs.=66,000 kg ; Steam temperature 198·9°C.

Boiler feed temperature 150°C.

Flue gas temperature 315°C, specific heat 0·24. Flue gas analysis, CO_2=12·8%, O_2=6·4%, CO=0·2% and N=80 6%.

Partial pressure of steam = 0 07 kgf/cm², specific heat = 0 48.

[per kg of coal as fired, dfg = 15 79 kg ; moisture = 0·423 kg ; heat supplied in raising steam 5,330 kcal (66 62%) ; heat carried by dfg 1,070 kcal (13 38%) ; moisture 301 kcal (3·76%) ; ashpit 22 (0 28%) ; difference 1,277 (15 96%) ; assumption steam dry and saturated.

Note. The amount of ash collected is wrong as ash is only 5·33% in dry coal. In solution this value is not required]

8.8. Heat balance sheet of a boiler : excess of air.

The following particulars refer to a trial on a coal-fired water tube boiler ; steam pressure 15 kgf/cm², dryness fraction 0 95, feed water per hour 2,040 kg, dry coal fired per hour 232 kg, mean feed temperature 65°C, mean boiler house temperature 25 3°C, mean flue gas temperature 440°C. Analysis of dry flue gas by volume CO_2 10·50%, CO 1 3%, O_2 7 67%, and N_2 = 80·53% Specific heat of dry flue gas 0·238. Analysis of dry coal by mass C = 83%, H_2 = 6%, O_2 = 5%, ash = 6%. Higher calorific value of dry coal 8,200 kcal.

Find (a) the total mass of the flue gases per kg of fuel fired (b) the percentage of excess air supplied over the minimum quantity required for the complete combustion of the fuel Also make out a complete heat balance sheet for the boiler per kg of dry coal.

[dfg/kg of fuel = 17·58 kg ; minimum air = 11 49 kg , % excess air = 49 51% ; heat supplied in raising steam = 5,086 kcal (62 02%) ; heat taken in dfg = 1,730 kcal (21 1%) ; in H_2O of combustion = 436 kcal (5 32%), difference = 948 (11 56%) ; assumption pp of H_2O = 0 07 kgf/cm²]

8 9. Heat balance sheet of a boiler.

The following data refer to a boiler trial : coal used per hour, 6,100 kg having a moisture content of 2 per cent and gross calorific value (dry) of 8,450 kcal/kg, the ultimate analysis of dry coal being C 84 ; O_2 4 ; H_2 4 ; ash 8 per cent by mass

The dry flue gas analysis was, by volume, CO_2 9 5 , O_2 10 48 and N_2 80 02 per cent.

Temperature of flue gas was 300°C ; stokehold temperature 30°C, mean specific heat of dry flue gas 0·24 and of steam in flue gas 0·49 ; mass of steam generated 55,000 kg/hour, dry saturated at 20 kgf/cm² from feed water at 50°C.

Draw up a heat balance per kg of dry coal, above stokehold temperature, given that air contains 23·1 per cent by mass of O_2. Assume partial pressure of water vapour in flue gases, 0·07 kgf/cm².

[Mass of dfg=22 kg ; total H_2O=0·3804 kg ; mass of air =21·44 kg ; air for correct combustion=10·9 kg ; heat supplied in raising steam=5,870 kcal (68%) ; heat in dfg, correct=742 kcal (8·6%) ; dfg, excess=685 kcal (7·94%) ; in vapour=271 kcal (3·14%), radiation, etc., 1,066 (12·32%)]

9

I.C. Engines—Combustion and Performance

9.1. Introduction. In an I C. engine fuel is burnt inside a cylinder and work is done against a piston The power produced inside the cylinder is known as indicated horse-power It is called "indicated" because it is measured by taking indicator diagram with an instrument known as indicator.

In I.C. engines the calorific value in fuel is spent in,

(*i*) producing power in the engine,

(*ii*) heat taken away by circulating water for cooling the engine cylinder,

(*iii*) heat taken away by exhaust gases, and

(*iv*) radiation and other losses.

The total power produced or gross ihp of the engine is given by the area of the +ve loop in the indicator diagram A part of this power, known as pumping power is spent in suction and exhaust strokes. This is indicated by the −ve loop of the indicator diagram. The difference of the gross ihp and pumping power (area of +ve loop *minus* area of −ve loop) is known as net ihp. The power available at the shaft is net ihp *minus* friction losses and is known as brake horse-power (bhp), as this power is measured by a brake. Fig. 9.1 shows the +ve and −ve area in P-V diagram for an I.C. engine.

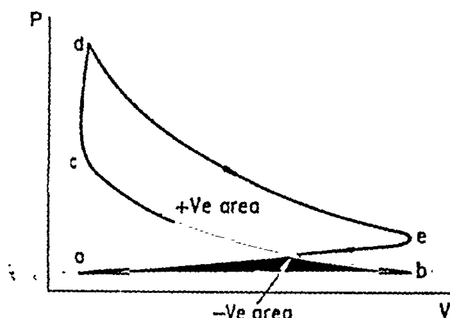
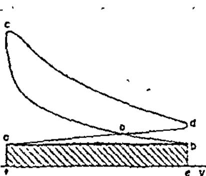


Fig. 9.1. Positive and negative loops of indicator diagram.

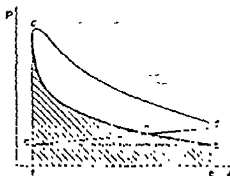
9·2. Four-Stroke Cycle Engine. In four-stroke cycle operation each of the four operations, suction, compression, expansion and exhaust requires one stroke. In petrol engine suction consists of a mixture of air and fuel and ignition takes place at the end of the compression stroke by means of a spark. The combustion is nearly at constant volume. In Diesel engines the suction consists of air only and fuel is injected at the end of compression stroke whence ignition takes place automatically due to high temperature of compressed air. The combustion in slow speed engines is nearly at constant pressure and in high speed engines partly at constant volume and partly at constant pressure.

The work done in the 4-strokes and net work done is shown in Fig. 9·2.

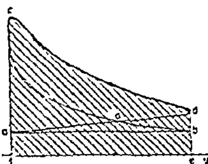
9·3. Two-Stroke Cycle Engine. In a two-stroke engine the cycle is completed in two strokes of piston, namely compression stroke and power-stroke. At the end of the power-stroke, exhaust ports are uncovered by piston (*see* Fig. 9·3 and Fig. 9·4). Immediately afterwards suction ports are uncovered and the fresh charge, compressed in the crank case or a separate compressor rushes in the cylinder helping in the driving out or the scavenging of burnt gases. The top shape of piston is usually made as shown in Fig. 9·3 and 9·4 to help the fresh charge in scavenging ; or the ports are made inclined at 45° and top of the piston is also bevelled at 45° to achieve the same end (*see* Fig. 9·5). During the return stroke the admission ports are first closed, immediately followed by the exhaust ports. Unidirectional.



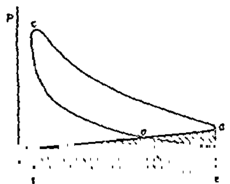
(a)
Suc ion stroke, W.D. = +area $abef$.



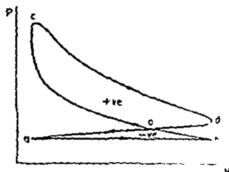
(b)
Compression stroke, W.D. = −area $befe$



(c)
Expansion stroke, W.D. = +$cdef$.



(d)
Exhaust stroke, W D. = −area $dafe$.



(e)

∴ Net work done = +area $abef$ − area $befe$ + area $cdef$ − area $dafe$
= area of +ve loop $docd$ − area of −ve loop $aoba$

Fig 9 2. Four-stroke cycle.

scavenging has been achieved in opposed piston two-stroke diesel engines, where admission ports are at top of cylinder and are uncovered

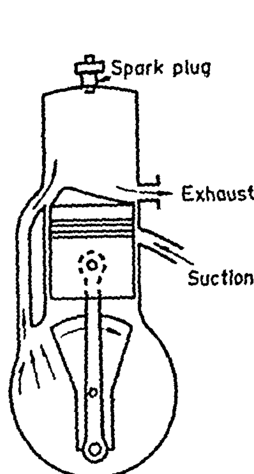


Fig. 9·3. 2-stroke petrol engine.

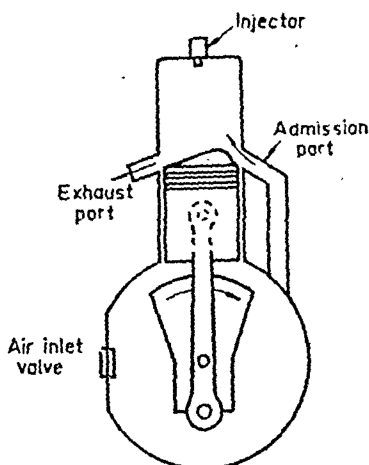


Fig. 9·4. 2-stroke diesel engine.

by the top piston and exhaust ports are at bottom, and are uncovered by the bottom piston. The fresh charge enters from top ports and drives the burnt gases out from bottom ports.

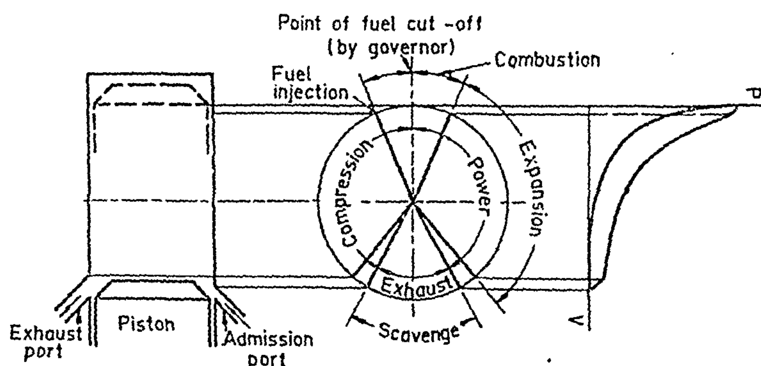


Fig. 9·5. 2-stroke engine : valve diagram and indicator diagram.

9·4. Comparison of Two Stroke and Four Stroke Cycle Engine.

(*a*) *Merits of two stroke cycle.*

(*i*) A two-stroke cycle engine has twice as many power strokes as a four-stroke cycle engine, at the same engine speed. Theoreti-

cally, therefore, a two-stroke engine should develop twice the power of a four stroke engine of the same dimensions. However, the extra power developed is only 70 to 90% (greater value applicable in slow speed engines) due to power absorbed in compressing the charge, reduction in the effective stroke and the compression ratio due to valve ports and (in high speed engines) due to short time available for exhaust of gases

(*ii*) For the same power a two-stroke cycle engine is lighter and occupies less floor area. This makes it more suitable for use in marine engines.

(*iii*) As the number of working strokes are double than in four-stroke engine turning moment is more uniform and hence a lighter flywheel is required.

(*iv*) The more uniform turning moment results in lighter foundation of the engine.

(*v*) The mechanism is very simple as there are no valves. In some cases mechanically operated valve may be provided.

(*vi*) In the absence of valves, a simple arrangement can be used for reversing the engine.

Demerits of two stroke cycle.

(*i*) In two stroke engines, particularly high speed ones, scavenging (driving out of exhaust gases) is not complete due to short time available for exhaust and hence the fresh charge is polluted. This pollution of charge has been reduced in opposed piton two-stroke diesel engines by unidirectional scavenging.

(*ii*) As inlet and exhaust ports open simultaneously some fresh charge, containing fuel in the case of petrol and gas engines and compressed air in the case of diesel engines, is lost

The thermal efficiency of two-stroke engines is likely to be lower than four-stroke engines due to the above reasons and due to lower effective compression ratio.

(*iii*) As the number of power strokes are double, cooling system presents difficulty.

(*iv*) Consumption of lubricating oil is greater.

(*v*) As number of power strokes are twice there is more wear and tear.

(*vi*) The exhaust is noisy due to short time available.

There are more four-stroke engines than two-stroke engines in use, particularly those working on petrol, due to proved economy. Two-stroke cycle is, however, very suitable for low speed diesel engines. For marine engines it is ideally suited due to low weight/hp ratio and less head room, which are very important consideration in ships.

9·5. Efficiencies.

(*a*) *Mechanical efficiency.* It is the ratio of power obtained at shaft (bhp) to the indicated power (ihp). Thus it accounts for the loss in friction.

$$\text{Mechanical efficiency} = \frac{\text{bhp}}{\text{ihp}}$$

(*b*) *Indicated thermal efficiency.* It is the ratio of the indicated work done to the energy supplied by the fuel

$$\text{Indicated thermal efficiency} = \frac{\text{ihp} \times C}{m_f \times CV}$$

where m_f = Mass of fuel, kg per hour

CV = Calorific value, heat units per kg

C = One horse-power-hour equivalent, 632·5

(*c*) *Brake thermal efficiency.* It is the ratio of brake or shaft work obtained to the energy supplied by the fuel.

$$\text{Brake thermal efficiency} = \frac{\text{bhp} \times C}{m_f \times CV}$$

In I. C. engines this term is also known as *overall efficiency.*

(*d*) *Efficiency ratio or relative efficiency.* This is the ratio of indicated thermal efficiency to the corresponding air standard cycle efficiency.

$$\text{Efficiency ratio} = \frac{\text{Indicated thermal efficiency}}{\text{Air standard efficiency}}$$

(e) *Volumetric efficiency.* Volumetric efficiency is the ratio of actual volume inhaled during suction stroke measured at intake conditions to the swept volume of the piston. This is also defined as the ratio of mass of air which enters or is forced into the cylinder in suction stroke to the mass of free air equivalent to the piston displacement at intake temperature and pressure conditions.

$$\eta_v = \frac{\text{Charge aspirated per stroke reduced to intake condition}}{\text{Swept volume}}$$

or [illegible]

Volumetric efficiency may also be defined with reference to N.T.P. conditions. This definition is particularly used in case of petrol engines as there is artificial depression of inlet temperature due to vaporisation of petrol.

Power output of an engine is proportional to volumetric efficiency, provided the combustion is complete.

9·6. Measurement of Indicated Horse-Power Indicated power is the power developed in the cylinder

(*i*) It is found by taking an indicator diagram from an instrument called indicator.

Indicated horse-power, ihp = area of P-v diagram

$$= \frac{p_m LAN}{75 \times 60} \qquad (9{\cdot}1)$$

where p_m = mean effective pressure in kgf/cm²

$$= \frac{\text{area of diagram} \times \text{strength of indicator spring}}{\text{length of diagram}}$$

L = stroke in m

A = area of piston in cm²

N = number of working cycles/min

(*ii*) *Morse Test.* Indicated horse-power for multi-cylinder engines can be found from Morse test.

In 'Morse Test' engine is run at a constant setting of throttle, angle of advance, etc., and at constant speed. Firstly dynamometer reading is taken with all cylinders working. Then each cylinder is cut-out in turn (by shorting each plug in petrol engines and cutting individual fuel supply in diesel engines) and the load is reduced to keep the speed same as before and dynamometer reading is again taken.

[*Note.* Fuel consumption and heat taken away by water may also be noted with all cylinders working, if thermal efficiency and heat balance sheet is required.]

The assumptions in this method of finding the ihp are that the friction and pumping horse power of the shorted cylinder remains the same after shorting as they were when the cylinder was fully operative. This assumption is a reasonable assumption because the test can be carried out in a very short time. As one cylinder is cut, load is immediately adjusted and reading is taken. Before cutting out the next cylinder, engine should be run on all cylinders for a short time.

Taking the case of a 4-cylinder engine, let

B = bhp with all cylinders operating
B_a = bhp with cylinder no. 1 cut-off
B_b = ,, ,, 2 ,,
B_c = ,, ,, 3 ,,
B_d = ,, ,, 4 ,,
P = Total pumping horse-power of all cylinders working
F = Total frictional horse-power of all cylinder working
G = Total gross ihp

G_1, G_2, etc. = Gross ihp of individual cylinders.

Then B = Net ihp − Frictional hp $= (G-P)-F$ (1)

When one cylinder is cut-off gross hp of the cylinder is zero, but its pumping and friction horse-power remain same, as the piston is moving in the cylinder and sucking and exhausting the air.

$$\therefore \quad B_a = G_2 + G_3 + G_4 - P - F$$

Similarly, $B_b = G_3 + G_4 + G_1 - P - F$

$B_c = G_4 + G_1 + G_2 - P - F$

$B_d = G_1 + G_2 + G_3 - P - F$

Adding the above equations, we get,

$$B_a + B_b + B_c + B_d = 3(G_1 + G_2 + G_3 + G_4) - 4P - 4F = 3G - 4P - 4F \quad (2)$$

Multiplying equation (1) by 4 and subtracting from equation (2), we get,

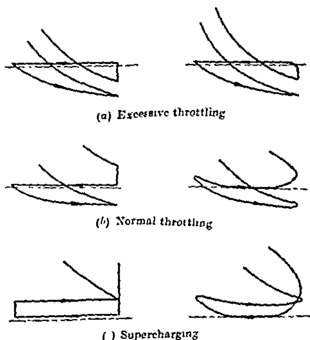
$$4B - (B_a + B_b + B_c + B_d) = G = \text{Net ihp of cylinders} \quad (3)$$

if pumping hp is negligible in comparison with gross ihp.

According to Morse this method of finding ihp gives a slightly higher value than the actual, since the pumping and friction hp of an idle cylinder may be greater than the hp of a working cylinder.

However, this is a very easy method of finding ihp without the use of a costly indicator which has to be of high speed type for petrol and automotive diesel engines.

9·7. Light Spring Diagrams. For study of suction and exhaust strokes and measurement of pumping horse-power light spring



(a) Excessive throttling

(b) Normal throttling

( ) Supercharging

Ideal diagrams Actual diagrams

Fig. 9·6 Light spring diagrams

diagrams are taken with spring strength of 0·4 to 1 kgf/cm²/cm. The light spring diagrams for excessive throttling, normal throttling and for supercharged engines are shown in Fig. 9·6.

Supercharging. In supercharging intake pressure is artificially raised by a blower or compressor above the atmospheric pressure. Supercharging is restored to increase the power output on land installations or to *maintain* the power output as altitude is increased in aircrafts.

Besides increasing power output supercharging has the following effects. Due to increased turbulence it provides better mixing of air-fuel mixture. It may encourage more even distribution of the charge to the cylinders. The temperature of the charge is raised as it is compressed producing better vaporization of the fuel. Due to higher temperature supercharging increases the possibility of detonation in petrol engines and lessens the possibility of knocking in Diesel engines.

9-8. Measurement of Brake Horse-power. The net power obtained at the shaft is termed as brake horse-power (bhp) and is measured by a rope brake arrangement as shown in Fig. 9·7.

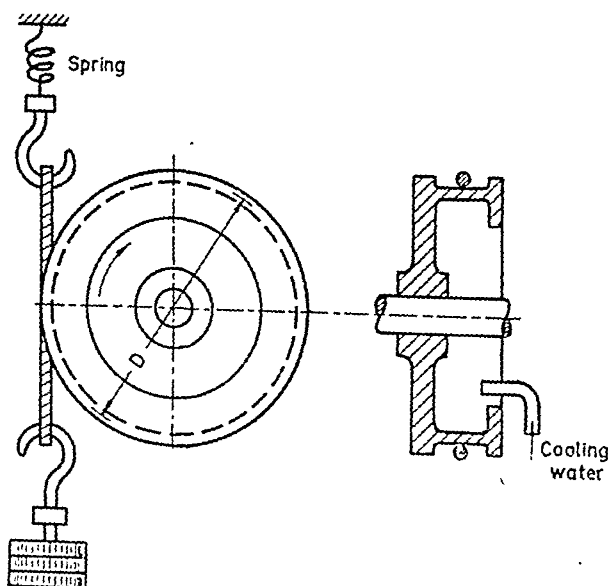


Fig. 9·7. Rope brake arrangement.

$$\text{bhp} = \frac{\pi ND(W-S)}{75\times 60} = \frac{2\pi NT}{75\times 60}$$

where $T = \text{Torque} = (W-S)\dfrac{D}{2}$ in, kgf m units

N = rpm of brake wheel

The brake horse-power is also measured with the help of dynamometers.

9-9. Measurement of Flow of Air In I.C. engines consumption of air may be measured by box meter which consists of an

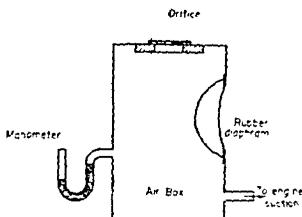


Fig 9 8. Orifice meter.

air-tight box fitted with a sharp edged orifice at a point well removed from engine suction connection (Fig 9 8) Due to the suction of the engine there is a depression of pressure in the orifice which may be assumed constant if the box has a large capacity, say 500-600 times the swept volume in the case of slow speed single cylinder engines. The capacity may be less with multi cylinder or high speed engines.

Internal baffles or diaphragm on one face may be provided to reduce pressure pulsations The pressure difference across the orifice is a measure of air flow. It is measured by means of a water manometer. The pressure difference should be limited to about 100 mm to make the compressibility effect of air negligible.

Let d = diameter of orifice, in mm

A = arc of orifice, in mm²

C_d = coefficient of discharge for given orifice plate

h = pressure difference in mm of water measured from U-tube manometer.

H = head causing flow of air, in mm of air

V = velocity of air, in m/s

ρ_a = mass of air/m³ under atmospheric conditions, in kg

ρ_w = mass of water/m³, in kg

Head causing flow of air, $H = h \times \frac{\rho_w}{\rho_a}$ mm $= \frac{h \times 10^{-3}}{p_a}$

Quantity (or volume) of air, Q

$$= C_d \times \text{area} \times \text{velocity}$$
$$= C_d \times \frac{A}{10^6} \times \sqrt{2g\,\frac{H}{10^3}}\ \text{m}^3$$

$\therefore$ Mass of air per sec $= Q\rho_a = \left[C_d \times \frac{A}{10^6} \times \sqrt{2g\frac{H}{10^3}}\right]\rho_a$ kg

Substituting the value of H, m (= 10³ kg/m³ for water), and g in m/s², we get

$$\text{Mass of air/s} = C_d \times \frac{A}{10^6}\sqrt{2 \times 9.81.\frac{h}{10^3}} \times \rho_a$$
$$= 4{\cdot}43 \times 10^{-6}\, C_d\, A\sqrt{h\,\rho_a}$$
$$= 3{\cdot}48 \times 10^{-6}.C_d.d^2\sqrt{h\,\rho_a}$$

The expression can also be derived in terms of pressure at orifice in kgf/cm² and temperature of air at orifice inlet in °K.

$$PV = \rho_a RT, \text{ if } V = 1\ \text{m}^3$$

$\therefore$ Mass per unit volume, $\rho_a = \frac{p \times 10^4}{29{\cdot}27\,T}$

Substituting in the equation above

$$\text{Mass of air/s} = 3{\cdot}48 \times 10^{-6}\, C_d \times d^2\sqrt{\frac{h \times p \times 10^4}{29{\cdot}27\,T}}$$
$$= 0{\cdot}0000644\, C_d.d^2\sqrt{\frac{ph}{T}}$$

9.10. Performance of I.C. Engines The set-up for conducting engine trial is shown in Fig. 9.9. The trial may be carried

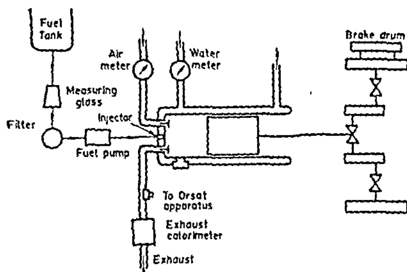


Fig. 9.9. Set-up for I.C. engine trial.

at variable speed and full throttle or variable load and constant speed. The spark ignition engines are generally used for variable speed operation, and hence the performance curves are drawn for the speed range of the engine The typical spark-ignition engine performance curves for a multi cylinder automobile engine are shown in Fig. 9.10.

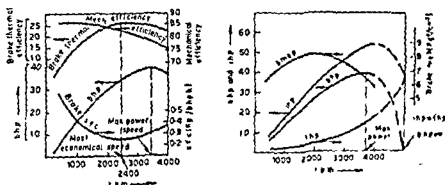


Fig. 9.10. Performance curves of petrol engines.

The typical performance curves of a compression ignition (Diesel) engine at constant speed are shown in Fig. 9·11.

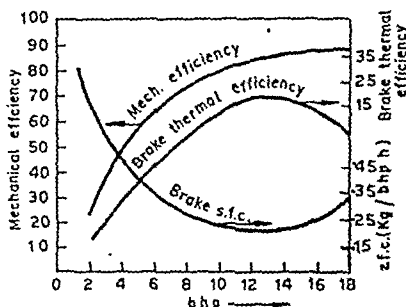


Fig. 9·11. Performancecurves of Diesel engines.

IMPORTANT POINTS

1. In the equation ihp$=\frac{P_m l A N}{4,500}$, N stands for working strokes per minute which are $\frac{1}{2}\times$rpm in four-stroke engines, and equal to rpm in two-stroke engines. In gas engines, governed by hit and miss method, N is equal to actual number of explosions.

2. In calculating bhp by rope-brake, N in the equation $\frac{2\pi NT}{4,500}$ stands for the rpm of brake drum, irrespective of whether it is a four-stroke or a two-stroke engine or a gas engine with hit and miss governing. If diameter of rope is given it should be taken into account for effective brake drum diameter.

3. Whenever volumetric efficiency is determined, it should be carefully noted whether it is on ambient condition or N.T.P. condition. In petrol and gas engines volumetric efficiency may be calculated taking air only into account or taking air and gas into account as required in the problem.

4. Net ihp of an engine is the (Gross ihp—Pumping horsepower). The former is represented by the +ve loop of indicator diagram and the latter by the —ve loop. However, pumping horsepower, being very small, is generally neglected.

(5) In the heat balance sheet bhp should be included and not ihp. Fhp should not form a separate item of heat balance because part of its value reappears in circulating or cooling water and exhaust gases.

ihp — bhp=fhp, *may be shown separately outside the heat* balance.

(6) It is usual to give an indicator spring rating in the form $\frac{1}{x}$, where x represents the pressure for one cm vertical rise of the stylus pencil point.

(7) The results of I C. engine trials may be verified from the following approximate values :

Fuel consumption per bhp-hr in diesel engines=about 0·2 kg

Fuel consumption per bhp-hr in petrol engines=about 0·25 kg

Brake thermal efficiency of diesel engines=about 30%

Brake thermal efficiency of petrol engines=about 24%

Heat in fuel approximately distributed as follows :

$\frac{1}{3}$ in hp produced, $\frac{1}{3}$ in cooling water and $\frac{1}{3}$ in exhaust gases. Indicated thermal efficiency is about 55 to 60 per cent of the ideal efficiency.

ILLUSTRATIVE EXAMPLES

9 1. Petrol engine : efficiencies ; RAC rating.

Explain what is meant by RAC rating.

A four cylinder, four-stroke petrol engine was subjected to a laboratory test and the following data were obtained :

Cylinder diameter=64 mm ; stroke length=90 mm , clearance volume=50 c c , fuel consumption=7 5 litres per hr , rpm=2,400 ; calorific value of fuel=11,400 kcal/kg , specific gravity of fuel=0 717 ; brake drum diameter=73 5 cm , rope diameter=2 5 cm , load on brake drum running at $\frac{1}{3}$ engine speed by belts, spring balances read 60 kg and 8 kg ; mechanical efficiency=80 per cent

Determine, (a) *the air standard efficiency,* (b) *the brake thermal efficiency* (c) *the indicated thermal efficiency, and* (d) *the relative efficiency.* $\gamma=1{\cdot}4$.

Also compare the output of this engine with the RAC rating which assumes mep 90 lbf/in² (6·327 kgf/cm²), mechanical efficiency 75 per cent and piston speed 1,000 ft/min (305 metres per min).

Royal Automobile Club (R.A.C.) rating was adopted in past taking the average values of mep, mechanical efficiency and piston speed prevailing in those days. Now a days these values have greatly increased.

(*a*) Compression ratio $= \dfrac{V_s + V_c}{V_c} = \dfrac{\frac{\pi}{4} \times (6{\cdot}4)^2 \times 9 + 50}{50} = 6{\cdot}79$

Air standard efficiency $= 1 - \dfrac{1}{r^{\gamma - 1}}$

$$= 1 - \frac{1}{6{\cdot}79^{1.4-1}} = 53{\cdot}5\% \quad \text{Ans.}$$

(*b*) Fuel consumption per hour $= 7{\cdot}5 \times 1 \times 0{\cdot}717 = 5{\cdot}378$ kg

Heat supplied per minute $= \dfrac{5{\cdot}378 \times 11.400}{60} = 1{,}024$ kcal

$$\text{bhp} = \frac{\pi DN \times F}{75 \times 60} = \frac{\pi \times \frac{76}{100} \times \left(\frac{2{,}400}{3}\right) \times (60 - 8)}{75 \times 60} = 22{\cdot}1$$

$\therefore$ Brake thermal efficiency $= \dfrac{22{\cdot}1 \times 632{\cdot}5}{60 \times 1{,}024} = 22{\cdot}7\%$ Ans.

(One bhp-hr. $= 632{\cdot}5$ kcal)

(*c*) Indicated thermal efficiency $= \dfrac{\text{Brake thermal efficiency}}{\text{Mechanical efficiency}}$

$$= \frac{22{\cdot}7}{0{\cdot}8} = 28{\cdot}38\% \quad \text{Ans.}$$

(*d*) Relative efficiency $= \dfrac{\text{Indicated thermal efficiency}}{\text{Air standard efficiency}}$

$$= \frac{28{\cdot}38}{53{\cdot}5} = 53{\cdot}04\% \quad \text{Ans.}$$

bhp as per R.A.C. rating

$$= \frac{P_m \times \text{piston speed per min}}{75 \times 60} \times \text{mech. efficiency}$$

$$= \frac{6{\cdot}327 \times \left(\frac{\pi}{4} \times 6{\cdot}4^2\right) \times 305}{75 \times 60} \times 0{\cdot}75 = 10{\cdot}35$$

Actual bhp : R.A.C. bhp $= 22{\cdot}1 : 10{\cdot}35 = 2{\cdot}136 : 1$ Ans.

9·2. Two-stroke diesel engine : mep ; specific fuel consumption.

Sketch a rope-brake arrangement for determining bhp of an engine.

A two-stroke diesel engine has a bore of 110 mm and stroke of 150 mm. Running at a mean speed of 300 meters per minute it develops a torque of 5·74 kgfm. The mechanical efficiency of the engine is 80 per cent and the indicated thermal efficiency 40 per cent. Assuming calorific value of fuel used 10,700 kcal/kg, determine, (a) the ihp, (b) the indicated mep, and (c) the fuel consumption per bhp-hr.

(a) Piston speed $=300$ m/min $=2\times l\times N$

$$\therefore \quad N=\frac{300}{2\times 0{\cdot}15}=1{,}000 \text{ rpm}$$

$$\text{bhp}=\frac{2\pi NT}{75\times 60}=\frac{2\pi\times 1{,}000\times 5{\cdot}74}{75\times 60}=8$$

$$\text{ihp}=\frac{\text{bhp}}{\eta_{mech}}=\frac{8}{\cdot 8}=\underline{10} \qquad \textbf{Ans.}$$

(b) $$\text{ihp}=\frac{P_m lAN}{75\times 60} \quad \therefore \quad P_m=\frac{10\times 75\times 60}{0{\cdot}15\times\frac{\pi}{4}(0{\cdot}11)^2\times 1{,}000\times 10^4}$$

$$=\underline{3{\cdot}156 \text{ kgf/cm}^2} \qquad \textbf{Ans.}$$

(c) $$\underline{\text{Fuel consumption per bhp-hr}}=\frac{\text{Heat equivalent to hp-hr}}{\eta_{indicated}\times\eta_{mech}\times CV}$$

$$=\frac{75\times 60\times 60}{427\times 0{\cdot}4\times 0{\cdot}8\times 10{,}700}=\underline{0{\cdot}185 \text{ kg}} \qquad \textbf{Ans.}$$

9·3. Two stroke engine : apparent and actual compression ratio.

(a) State the relative merits and demerits of two-stroke cycle engines over four-stroke cycle engines

(b) An engine working on two stroke cycle has diameter 110 mm, stroke 150 mm and clearance volume 75 c c.

In a test the following readings were taken :

Air port opens ... 32 mm before BDC
Air port closes ... 32 mm after BDC

Exhaust port opens	... *40 mm before BDC*
Exhaust port closes	... *40 mm after BDC*
Fuel injection starts	... *4 mm before TDC*
Fuel injection stops	... *2 mm before TDC*

Sketch a probable valve timing diagram and find the apparent and actual compression ratios of the engine.

$$\text{Swept volume} = \frac{\pi}{4} \times 11^2 \times 15 = 1{,}425 \text{ c c}$$

$$\therefore \ \underline{\text{Apparent compression ratio}} = \frac{1{,}425 + 75}{75} = \underline{20} \qquad \text{Ans.}$$

$$\text{Effective swept volume} = \frac{\pi}{4} \times 11^2 \times (15 - 4) = 1{,}045 \text{ c c}$$

$$\therefore \ \underline{\text{Actual compression ratio}} = \frac{1{,}045 + 75}{75} = \underline{14{\cdot}94} \qquad \text{Ans.}$$

9·4. Gas engine : air and gas consumption given vol. η.

A single-acting gas engine working on the four-stroke cycle has an overall volumetric efficiency of 71 per cent when the brake horse-power is 30. The barometer pressure is 1·036 kgf/cm² and the temperature is 20°C. The piston diameter is 32 cm and the stroke 38 cm. The speed is 280 rpm and the gas used has a calorific value 4,450 kcal/m³ measured at 15°C and 1 033 kgf/cm². The air/fuel ratio by volume is 7·5.

If the brake thermal efficiency is 28·5 per cent, estimate (a) the number of missed cycles per minute, (b) the air consumption in kg/min assuming that during each missed cycle a volume of air equal to the usual volume of gas is admitted, and (c) the gas consumption as metered, if the gas pressure is 12 cm of water. R=29·27.

(a) $\text{Stroke volume} = \frac{\pi}{4} (32)^2 \times 38 = 30{,}560 \text{ c c}$

Volume inhaled at 1·036 kgf/cm² and 293°K

$$= 30{,}560 \times 0{\cdot}71 = 21{,}700 \text{ c c}$$

Equivalent volume inhaled at 1·033 kgf/cm² and 288°K

$$=\frac{1\ 036\times 21,700\times 288}{293\times 1\ 033}=21,400 \text{ c c}$$

Since air/fuel ratio by volume is 7 5,

∴ Volume of gas inhaled per cycle $=\frac{21,400}{(7\ 5+1)}=2,518$ c c

Equivalent heat input per cycle $=2,518\times 10^{-6}\times 4,450$

$=11·21$ kcal

$$\text{bhp}=\frac{\text{Work done per cycle}\times\text{no. of cycles per min}\times\text{brake thermal }\eta}{75\times 60}$$

$$30=\frac{11·21\times 427\times x\times 0\ 285}{75\times 60}$$

∴ No. of cycles per minute, $x=99$

Missed cycles per minute $=\frac{280}{2}-99=41$ Ans.

(*b*) Air consumption per minute

$$=\frac{21,400\times 7\ 5}{8\ 5}\times 99+21,400\times 41$$

$=2746000$ c c or 2 746 m³

∴ Mass of air consumed per minute

$$=\frac{PV}{RT}=\frac{1\ 033\times 10^4\times 2\ 746}{29\ 27\times 288}=3\ 365 \text{ kg}$$ Ans.

(*c*) Gas consumption per minute at 1 033 kgf/cm² and 288°K

$=2,518\times 99=2,49,300$ c c

Gas pressure $=1\ 036+\frac{12}{1,000}=1·048$ kgf/cm²

∴ Gas consumption metered at 12 cm of water and 20°C

$$=\frac{1·033\times 2,49,300\times 293}{288\times 1\ 048}=2,50,000 \text{ c c or } 0\ 25 \text{ m}^3$$ Ans.

9·5. Oil engine : stroke volume given air/fuel ratio.

What is the limit of compression ratio in petrol engines and why ?

Determine the stroke volume of a four-stroke single-cylinder oil engine designed to the following particulars :

25 bhp at 250 rpm when running on oil having the composition by mass C 85 ; H 15 per cent and a calorific value of 10,000 kcal/kg ; the oil is burned with 24 per cent of excess air ; volumetric efficiency reckoned on atmospheric conditions of 1 033 kgf/cm² and 10°C is 0·8, mechanical efficiency 0 88 and thermal efficiency on ihp basis 0·35 R=29·27.

The efficiency of a petrol engine increases with compression ratio, but it cannot be increased beyond a certain limit as it causes detonation. The usual limit is 6 to 8, though in rare cases compression ratio as high as 10 has been used with special quality of petrol.

$$\text{ihp}=\frac{\text{bhp}}{\text{Mech }\eta}=\frac{25}{0{\cdot}88}=28{\cdot}41$$

∴ Heat input to engine per stroke

$$=\frac{28{\cdot}41\times75\times60}{427\times0{\cdot}35\times125}=6{\cdot}845 \text{ kcal}$$

∴ Fuel used per stroke$=\dfrac{6{\cdot}845}{10{,}000}=0{\cdot}0006845$ kg

Actual air per kg of fuel$=\left(0{\cdot}85\times\dfrac{8}{3}+0{\cdot}15\times8\right)\dfrac{100}{23}\times1{\cdot}24$

$$=18{\cdot}69 \text{ kg}$$

∴ Air required per stroke$=0{\cdot}0006845\times18{\cdot}69=0{\cdot}01279$ kg

Volume of air required per stroke,

$$V=\frac{mRT'}{P}=\frac{0{\cdot}01279\times29{\cdot}27\times283}{1{\cdot}033\times10^4}=0{\cdot}01026 \text{ m}^3$$

Volume of cylinder$=\dfrac{0{\cdot}01026}{0{\cdot}8}=0{\cdot}01282$ m³ or 12·82 litre Ans.

9.6 Two-stroke C.I. engine : mep

The clearance volume of a two-stroke cycle compression ignition engine is $\frac{1}{12}$ of the swept volume and compression begins when the piston has covered 10 per cent of the swept volume. At the beginning of compression the pressure is 1.12 kgf/cm² and the temperature is [illegible]. The analysis by volume of the trapped charge is CO_2, [illegible]; H_2O, [illegible]; O_2, 19.3%; N_2, 78.6%. Fuel which is completely burned is injected in amount equal to $\frac{1}{20}$ of the air charge in the cylinder (by mass) and the indicated thermal efficiency is 0.36.

If the calorific value of the fuel is 19,500 kcal/kg calculate the indicated mean effective pressure. Air contains 21 per cent of oxygen by volume.

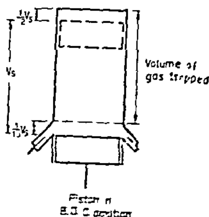


Fig. 9-12

$\therefore$ Mean molecular weight$=\dfrac{2,882\cdot2}{100}=28\cdot822$

Let the stroke volume be $V_s m^3$

$\therefore$ Clearance volume$=\dfrac{V_s}{12}=0\cdot0833 V_s m^3$

$\therefore$ Volume of gases trapped$=0\cdot9V_s+0\cdot0833V_s=0\cdot9833V_s m^3$

and volume of gases trapped at N.T.P. conditions

(*i.e.* $P=1\cdot033$ kgf/cm^2 and $T=288°K$)

$$=0\cdot9833\times\frac{1\cdot12}{1\cdot033}\times\frac{273}{368}\times V_s=0\cdot7912\ V_s m^3$$

$$=\frac{0\cdot7912V_s}{22\cdot41}=0\cdot0353V_s \text{ mol}$$

$\therefore$ Mass of trapped gases$=$Molar quantity$\times$mean mol weight

$$=0\cdot0353\ V\times28\cdot822=1\cdot017\ V_s \text{ kg}$$

O_2 in air, 21% by volume is equivalent to O_2 23% by mass.

Referring to table,

If gas trapped is 100 by mass, O_2 is 21·4 or air associated

$$=\frac{21\cdot4\times100}{23}$$

$\therefore$ Air associated with $1\cdot017\ V_s$ kg of trapped gas

$$=\frac{1\cdot017\ V_s}{100}\times\frac{21\cdot4\times100}{23}=0\cdot946\ V_s \text{ kg}$$

Mass of fuel supplied$=\dfrac{0\cdot946\ V_s}{20}=0\cdot0473\ V_s$ kg

$\therefore$ Heat supplied per cycle$=0\cdot0473\ V_s\times10,500=497\ V_s$ kcal

Work done per cycle$=0\cdot36\times497\ V_s=179\ V_s$ kcal

$$\underline{\text{mep}}=\frac{\text{Work done per cycle}}{\text{Swept volume}}=\frac{179\ V_s\times427}{V_s\times10^4}$$

$$=\underline{7\cdot64\ \text{kgf/cm}^2}\qquad \text{Ans.}$$

9·7. Oil engine : vol. composition of dfg ; bore and stroke.

A six-cylinder four-stroke direct injection oil engine is to develop 150 bhp at 1600 rev/min. The fuel to be used has a calorific value of 10,100 kcal/kg, and its percentage composition by mass is 86 2, hydrogen 13·5, non-combustibles 0 3.

The absolute volumetric efficiency is assumed to be 78 per cent, the indicated thermal efficiency 38 per cent, and the mechanical efficiency 80 per cent. The air consumption is to be 110 per cent in excess of that required for theoretically correct combustion.

(a) Estimate the volumetric composition of the dry exhaust gas.

(b) Determine the bore and the stroke of the engine taking a stroke-bore ratio of 1 5 to 1 The volume of 1 kg of air at 0°C and 1 033 kgf/ cm² is 0 7734 m³

Oxygen in air is 23 1 per cent by mass and 20 8 per cent by volume.

(a)

Constituents per kg of fuel (a)	Mass of O_2 required per kg of fuel ( )	Products of Combustion (c)			
		CO_2	H_2O	O_2	N_2
C=0·862	$0{\cdot}862\times\frac{32}{12}=2\ 3$	3 162			
H=0 135	$0\ 135\times8=1\ 08$		1·215		$7{\cdot}098\times\frac{76\ 9}{23{\cdot}1}$
Ash=0 003	Excess O_2=1·1(2 3+1 08)				
	=3 718			3 718	=23 62
Total 1·000	=7 098				

Air supplied=7 098+23 62=30 72 kg/kg of fuel

Constituents of dry exhaust gas by mass (*a*)	*Mol wt of constituents* (*b*)	*Proportional volume* (*c*)=(*a*)/(*b*)	*Percentage volumetric composition* (*d*)= $\frac{(c)}{\Sigma(c)}$
CO_2= 3·162	44	0·0719	6·97
O_2= 3·718	32	0·1162	11·26
N_2=23·62	28	0·8436	81·77
Total		=1·0317	100·00

Ans. (for the percentage volumetric composition values)

(*b*) $$\text{ihp}=\frac{\text{bhp}}{\text{mechanical }\eta}=\frac{150}{0{\cdot}80}=187{\cdot}5$$

$$\text{Indicated thermal efficiency}=\frac{\text{heat equivalent to ihp}}{\text{heat supplied by the fuel}}$$

$$0{\cdot}38=\frac{\dfrac{(187{\cdot}5\times 4{,}500)}{427}}{m\times 10{,}100}$$

∴ Mass of fuel, $m=0{\cdot}5148$ kg/min

Actual air supplied$=0{\cdot}5148\times 30{\cdot}72=15{\cdot}81$ kg/min

∴ Volume of air at 0°C and 1·003 kgf/cm²

$$=15{\cdot}81\times 0{\cdot}7734=12{\cdot}23\ m^3$$

Now, absolute volumetric efficiency

$$=\frac{\text{volume induced/min at }0°C\text{ and }1{\cdot}033\text{ kgf/cm}^2}{\text{swept volume/min}}$$

∴ Swept volume per min$=\dfrac{12{\cdot}23}{0{\cdot}78}=15{\cdot}68\ m^3$

Total swept volume per min

$$=\left(1{\cdot}5D\times\frac{\pi D^2}{4}\right)\times\frac{N}{2}\times\text{no. of cylinders}$$

$$15{\cdot}68=\left(1{\cdot}5D\times\frac{\pi D^2}{4}\right)\times\frac{1600}{2}\times 6$$

∴ Bore of cylinder, $D=0{\cdot}013$ m or 13 cm Ans.

and stroke$=1{\cdot}5\times 13=19{\cdot}5$ cm Ans.

9-8. Fall in output at high altitude ; power developed.

What is the working principle of a supercharger? How does it effect the efficiency and output of (a) petrol engine, and (b) diesel engine.

A six-cylinder, petrol engine 100 mm diameter by 100 mm stroke running at 1,500 rpm uses a mixture of air to petrol by mass of 13 5 to 1. Assuming that the air drawn into the cylinder per stroke and measured at pressure 1·033 kgf/cm² and temperature 80°C is equal to seven-eighths of swept volume and that thermal efficiency of engine is 22 per cent, find the power developed at ground level where barometer reads 760 mm of mercury. The calorific value of petrol is 9,000 kcal per kg.

What would be the power developed by this engine at an altitude of 1,500 meters where the temperature is 25°C. A drop of 10 mm barometer reading may be assumed for each 100 meter of rise in altitude.

Take $R=29\ 27$

Stroke volume $=\frac{\pi}{4}\times 10^2\times 10=785\ 4$ cc

Air drawn in cylinder $=\frac{7}{8}\times 785\ 4=687{\cdot}2$ cc

Mass of air drawn in cylinder per stroke

$$m=\frac{PV}{RT}=\frac{1\ 033\times 10^4\times 687\ 2\times 10^{-6}}{29\ 27\times 353}=0\ 0006871\ \text{kg}$$

Mass of petrol drawn per stroke $=\frac{0\ 0006871}{13\ 5}=0\ 0000509$ kg

Heat supplied by six cylinders per minute

$$=0\ 0000509\times 6\times\frac{1\ 500}{2}\times 9{,}000=2\ [illegible]\ \text{kcal}$$

$\therefore$ Horse power $=\frac{2\ 061\times 60\times 0\ 22}{632\ 5}=$ [illegible] Ans.

At an altitude of 1,500 meters,

$$\text{Pressure}=760-\frac{10}{100}\times 1\ 500=6[illegible]\ \text{mm of}\ Hg$$

$\therefore$ New pressure $=\frac{1\ 033}{760}\times 610=0\ 829$ kgf/cm²

At 1,500 metres atmospheric temperature is given as 25°C which is not required in the solution. What is required is the temperature of air inside the cylinder. Evidently in the previous case 80°C cannot be the atmospheric temperature. Assuming the same temperature of air in the absence of any other data, hp is directly proportional to pressure,

$$\therefore \text{ hp at 1,500 metres altitude} = \frac{42{\cdot}1 \times 0{\cdot}829}{1{\cdot}033}$$

$$= \underline{33{\cdot}8} \qquad \text{Ans.}$$

9·9. Supercharging ; increase in mep.

In what circumstances it is justifiable to assume the indicated horse-power of a petrol engine to be proportional to its air consumption.

An unsupercharged engine develops a gross imep of 10 kgf/cm² when the pumping imep is 0·35 kgf/cm². The charge pressure and temperature at the beginning of compression are estimated to be 0·96 kgf/cm² and 100°C respectively, and the mean pressure during the induction stroke is 0·91 kgf/cm².

When supercharged by a blower of adiabatic efficiency 70 per cent, the charge after delivery by the blower has its temperature raised 50°C during its entry to the cylinders, and suffers a pressure drop of 0·07 kgf cm², the charge pressure in the cylinders being maintained at 1·45 kgf/cm² during induction stroke.

Estimate the percentage increase in the net imep due to supercharging. Neglect the effects of residuals, assume atmospheric conditions of 1 033 kgf/cm² and 15°C and take $\gamma = 1{\cdot}4$.

When the mixture strength is rich the ihp of a petrol engine will be nearly proportional to its air consumption.

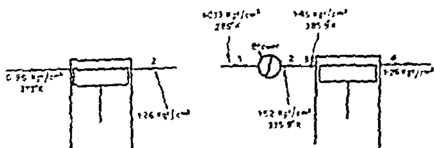
For the supercharger,

$$\text{Adiabatic efficiency} = \frac{T_2' - T_1}{T_2 - T_1}$$

where T_2' = temperature, if the compression is isentropic

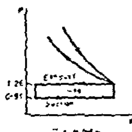
T_2 = temperature, actual

T_1 = temperature, at entry to blower

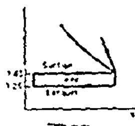


Unsupercharged engine

Supercharged engine



Unsupercharged cycle



Supercharged cycle

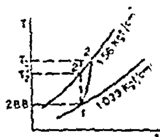


Fig. 9.12.

$$T_2' = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 288\left(\frac{1.45+0.07}{1.033}\right)^{\frac{0.4}{1.4}} = 321.5\ K$$

$$\therefore \qquad 0.7 = \frac{321.5-288}{T_2-288} \qquad \therefore \quad T_2 = 335.9^\circ K$$

Gross imep is proportional to the air consumption. Let suffix *a* refer to unsupercharged engine and *b* to supercharged engine. Volume of air is same in two cases.

$$\therefore \quad \frac{(\text{imep})_b}{(\text{imep})_a} = \frac{(m_{air})_b}{(m_{air})_a} = \frac{P_b T_a}{P_a T_b} = \frac{1{\cdot}45 \times 373}{0{\cdot}96 \times 385{\cdot}9}$$

$$\therefore \quad \text{Gross (imep) supercharged} = 10 \times \frac{1{\cdot}45 \times 373}{0{\cdot}96 \times 385{\cdot}9}$$

$$= 14{\cdot}61 \text{ kgf/cm}^2$$

Original exhaust pressure $= 0{\cdot}91 + 0{\cdot}35 = 1{\cdot}26$ kgf/cm²

Gross imep is given by the area of positive loop of indicator diagram. Pumping imep is the area of negative loop. Net imep is the algebric sum of the two.

$\therefore$ Net imep = gross imep − pumping imep

Assuming exhaust pressure to be unaltered, pumping mep when supercharged $= 1{\cdot}45 - 1{\cdot}26 = 0{\cdot}19$ kgf/cm²

Hence, net mep supercharged $= 14{\cdot}61 + 0{\cdot}19 = 14{\cdot}80$ kgf/cm²

$$\therefore \quad \underline{\text{Increase in net mep}} = \frac{14{\cdot}80 - 9{\cdot}65}{9{\cdot}65} = \underline{53{\cdot}4\%} \qquad \text{Ans.}$$

9·10. Measurement of air by orifice plate.

Show that when the specific volume of air remains sensibly constant the amount flowing per sec through a sharp edged circular orifice is given closely by

$$mass/sec = 0{\cdot}0000644\ C_d\ d^2 \sqrt{\frac{ph}{T}}$$

where c_d is the coefficient of discharge, d is the orifice diameter in mm, p is the pressure at inlet in kgf/cm², h is the pressure head in mm of water and T is the air inlet temperature in °K.

The air supply to an internal combustion engine measured by an orifice of this type fitted to an air box is equivalent to 75 mm of water and the atmospheric pressure and temperature are 1·02 kgf/cm² and 27°C. The diameter of the orifice is 40 mm and the coefficient of discharge is 0·6. Find the mass of air supplied per minute.

For theory—see text.

From text, Eq. (9)

$$\text{Mass of air/second} = 0.0000644\, c_d\, d^2 \sqrt{\frac{ph}{T}}$$

$$= 0.0000644 \times 0.6 \times 40^2 \sqrt{\frac{102 \times 75}{(273+27)}}$$

$$= 0.0311 \text{ kg/s or } \underline{1.866 \text{ kg/min}}$$ Ans.

9·11. Morse test : various efficiencies.

Describe the "Morse Test" for determining the ihp of a multi-cylinder engine. State the assumptions involved and comment on the accuracy of the results.

A four-stroke petrol engine with four cylinders, coupled to a hydraulic dynamometer, was tested at full throttle at constant speed. The cylinders have diameters of 89 mm and stroke 100 mm. Fuel was supplied at the rate of 5.44 kg per hour and the plugs of four cylinders were successively short-circuited without the change of speed. The power measurements were as follows :—

With all cylinders working			*20 hp*
,,	*cylinder 1*	*cut-off*	*13.8 hp*
,,	,, *2*	,,	*14.0 hp*
,,	,, *3*	,,	*14.1 hp*
,,	,, *4*	,,	*13.9 hp*

Calorific value of the petrol used was 10,000 kcal per kg. The clearance volume of each cylinder is 100 cc. Determine (a) the mechanical efficiency (b) the indicated thermal efficiency (c) the air standard efficiency, and (d) the relative efficiency γ = 1.4

For theory—see text

(a) From text, ihp $= 4 \times 20 - (13.8 + 14.0 + 14.1 + 13.9)$
$= 24.2$

$$\underline{\text{Mechanical efficiency}} = \frac{\text{bhp}}{\text{ihp}} = \frac{20}{24.2} = \underline{82.64\%}$$ Ans.

(b) Heat input per hour $= 5.44 \times 10{,}000 = 54{,}400$ kcal

Heat equivalent to ihp $= 24.2 \times 632.3 = 15{,}300$ kcal

$$\underline{\text{Indicated thermal efficiency}} = \frac{15{,}300}{54{,}400} = \underline{28.13\%}$$ Ans.

(c) Stroke volume, $V_s = \frac{\pi}{4}(8^2) \times 10 = 502 \cdot 7$ cc

$$\text{Compression ratio} = \frac{V_s + V_c}{V_c} = \frac{502 \cdot 7 + 100}{100} = 6 \cdot 027$$

$$\text{Air standard efficiency} = 1 - \frac{1}{r^{\gamma - 1}} = 1 - \frac{1}{(6 \cdot 027)^{1 \cdot 4 - 1}}$$

$$= 51 \cdot 25\%$$ Ans.

(d) Relative efficiency $= \frac{28 \cdot 13}{51 \cdot 25} = 54 \cdot 9\%$ Ans.

Note.—In multi-cylinder petrol engines outer cylinders develop less power than inner cylinders due to bad mixture distribution in the inlet manifold and less efficient scavenging.

9·12. Heat balance sheet : exhaust calorimeter.

A 100 bhp diesel engine consumes 16·5 kg of fuel oil per hour containing 86 per cent of carbon and 14 per cent of hydrogen. Calorific value of oil is 10,800 kcal/kg. The water supplied to the jackets also passes subsequently through the exhaust calorimeter. The following observations were made :

Quantity of water supplied per hour	*= 1,220 kg*
Temperature of water entering the jackets	*= 18°C*
,, ,, ,, leaving ,, ,,	*= 57°C*
,, ,, ,, ,, exhaust calorimeter	*= 82°C*
,, ,, exhaust ,, ,, ,,	*= 160°C*
,, ,, ,, ,, engine	*= 410°C*
,, ,, engine room	*= 18°C*

Determine the excess air used as a percentage of that required for complete combustion. Also draw up a heat balance for the engine in kcal/min. Specific heat of exhaust gases 0·25.

$$\text{Fuel supplied per minute} = \frac{16 \cdot 5}{60} = 0 \cdot 275 \text{ kg}$$

Heat supplied by fuel per minute = 0·275 × 10,800

In exhaust calorimeter

Heat taken by cooling water = Heat lost by exhaust gases

$$\frac{1,220}{60}(82-57)=m\times0.25(410-160)$$

$\therefore$ Mass of exhaust gases per minute, $m=8.133$ kg

Actual air supplied per minute $=8.133-0.275=7.858$ kg

Theoretical air required per kg of fuel

$$=\left(0.86\times\frac{8}{3}+0.14\times8\right)\times\frac{100}{23}=14.84 \text{ kg}$$

Theoretical air required per minute $=14.84\times0.275=4.081$ kg

$\therefore$ Excess air $=\dfrac{7.858-4.081}{4.081}=92.5\%$ Ans.

Heat equivalent to 100 bhp $=\dfrac{100\times632.4}{60}=1,054$ kcal/min

Heat carried away by exhaust gases $=mC_p(t_2-t_1)$

$$=8.133\times0.25(410-18)=797 \text{ kcal/min}$$

Heat carried away by cooling water

$$=\frac{1,220}{60}(57-18)=793 \text{ kcal/min}$$

Heat balance sheet on one minute basis :

C_r	kcal	%	D_b	kcal	%
Heat supplied by fuel	2,970	100	1. Heat equivalent to bhp	1,054	35.5
			2. Heat in exhaust	797	26.8
			3. Heat in cooling water	793	26.7
			4. Heat unaccounted for (by subtraction)	326	11.0
Total	2,970	100%		2,970	100%

Note.—The overall efficiency on an I.C. engine plant may be increased by utilising the heat of the exhaust gases in an exhaust gas-boiler.

9·13. Heat balance sheet ; indicator diagram ; hydraulic dynamometer ; heat taken away by dry exhaust and vapour.

During a test on a single-cylinder oil engine 250 mm bore, 600 mm stroke, working on four-stroke cycle, the following observations were made :

Duration of test $=1$ *hr*

Area of indicator diagram $=4{\cdot}51$ cm²

Length of indicator diagram $=7{\cdot}1$ *cm*

Strength of indicator spring $=8{\cdot}31$ *kgf/cm²/cm of compression*

Speed $=350$ *rpm*

Load on hydraulic dynamometer $=100$ *kg*

Dynamometer constant $=700$

Fuel consumption $=11{\cdot}2$ *kg*

Calorific value of fuel used $=10{,}000$ *kcal/kg*

Mass of cooling water $=1{,}020$ *kg*

Temperature rise of cooling water $=25°C$

Analysis of oil by mass $C=85\%$, $H_2=13{\cdot}5\%$, *Incombustible* $=1{\cdot}5\%$

Analysis of exhaust gases by volume $CO_2=8\%$, $O_2=11\%$

$N_2=81\%$ *(remainder)*

Temperature of exhaust gases $=400°C$

Room temperature $=25°C$

Partial pressure of steam in exhaust gases $=0{\cdot}035$ *kgf/cm²*

Specific heat of superheated steam $=0{\cdot}48$

Specific heat of dry exhaust gases $=0{\cdot}24$

Draw up a heat balance sheet in kcal/min and in percentages.

Fuel per minute $= \dfrac{11{\cdot}2}{60} = 0{\cdot}1867$ kg

Heat supplied by oil per minute $=0{\cdot}1867\times10{,}000=1{,}867$ kcal

$$\text{mep, } P_m=\frac{\text{area of indicator diagram}\times\text{spring strength}}{\text{length of card}}$$

$$=\frac{4{\cdot}51\times8{\cdot}31}{7{\cdot}1}=5{\cdot}278 \text{ kgf/cm}^2$$

$$\text{ihp}=\frac{P_m LAN}{75\times60}=\frac{5{\cdot}278\times\frac{600}{1{,}000}\times\frac{\pi}{4}(25)^2\times\frac{350}{2}}{75\times60}=60{\cdot}46$$

$$\text{Heat equivalent to ihp}=\frac{60{\cdot}46\times632{\cdot}4}{60}=637 \text{ kcal}$$

$$\text{bhp}=\frac{WN}{K}=\frac{100\times350}{700}=50$$

$$\text{Heat equivalent to bhp}=\frac{50\times632{\cdot}4}{60}=527 \text{ kcal/min}$$

Heat lost in friction = ihp − bhp = 637 − 527 = 110 kcal/min

$$\text{Heat lost in cooling water}=\frac{1{,}020\times25}{60}=425 \text{ kcal/min}$$

Constituents of dfg by vol (a)	*Mol wt* (b)	*Proportional mass* (c)=(a)×(b)	*Mass of carbon per kg of dfg* (d)
$CO_2=8$	44	44×8=352	$\frac{352}{2{,}972}\times\frac{12}{44}$
$O_2=11$	32	32×11=352	=0·03229
$N_2=81$	28	28×81=2,268	
Total=100		=2,972	

Dry exhaust gases per kg of fuel

$$=\frac{\text{Carbon content per kg of fuel}}{\text{Carbon in 1 kg of dfg}}=\frac{0{\cdot}85}{0{\cdot}03229}=26{\cdot}31 \text{ kg}$$

$\therefore$ Dry exhaust gases per minute $=26{\cdot}31\times0{\cdot}1867=4{\cdot}912$

Heat carried by dry exhaust gases $=4{\cdot}912\times0{\cdot}24\,(400-25)$

$=442$ kcal

H_2O produced by combustion $=0{\cdot}135\times9\times0{\cdot}1867$

$=0{\cdot}2268$ kg/min

From Steam Tables, heat lost per kg of steam at given conditions

$=(608{\cdot}9-25)+0{\cdot}48(400-26{\cdot}4)=763{\cdot}2$ kcal

$\therefore$ Heat lost in steam per minute $= 0{\cdot}2268 \times 763{\cdot}2 = 173$ kcal

Heat balance on one minute basis :

C_r	*kcal*	%	D_b	*kcal*	%
Heat supplied by fuel	1,867	100	Heat equivalent to ihp	657	34·12
			Heat lost in friction	110	5·89
			1. Heat equivalent to bhp	527	28·23
			2. Heat in cooling water	425	22·77
			3. Heat in dry exhaust gases	442	23·67
			4. Heat in steam in exhaust gas	173	9·27
			5. Other losses (by difference)	300	16·06
Total	1,867	100		1,867	100·00

Note.—(*i*) It is not mentioned whether the area of the indicator diagram is net area *i.e.* (area of the +ve loop minus area of −ve loop) ; if it is the area of +ve loop the value of ihp is gross ihp and mechanical efficiency calculated on this bases would give a lower value than actual.

(*ii*) Analysis of gas by Orsat apparatus always gives for dry gas.

Other losses (by difference) = 16·06
Heat in steam in exhaust = 9·27
Heat in dry exhaust gases = 23·67
Heat in cooling water = 22·77
Heat in bhp = 28·23
Heat supplied = 100

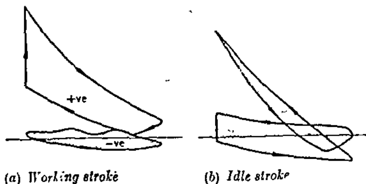
Fig. 9.14

9·11. Gas engine : heat balance ; negative loop.

A single-cylinder 4-stroke gas engine of 200 mm bore and 400 mm stroke with a hit and miss governing was tested with the following results :

Barometer 759 mm of mercury, atmospheric and gas temperature 17°C. Gas consumption 153 litre/min at 8·75 cm of water above atmospheric pressure. Gross calorific value 4,350 kcal/m³ and density 0·592 kg/m³ both at N.T.P., hydrogen present in the gas 12 per cent by mass ; air used 1·45 kg/min ; C_p for dry exhaust gas 0·25 ; mean effective pressure of positive loop 5·72 and of negative loop 0·274 kgf/cm² in firing and 0·38 kgf/cm² in missing strokes ; speed 285 rpm ; explosions per minute 114 ; brake torque 34 kgf metre. Cooling water 9·2 kg per minute, temperature raised 20°C. Exhaust temperature 400°C. The total heat of steam at atmospheric pressure is 639·1 kcal per kg and C_p for superheated steam 0·48.

Calculate the percentage of the indicated power used for pumping and for mechanical friction and draw up a heat balance for the engine in kcal per minute.



(a) *Working stroke* (b) *Idle stroke*

Fig. 9·15.

$$\text{Gas pressure} = \left[759 + \frac{87\cdot5}{13\cdot6}\right] \times \frac{1}{735\cdot5} = 1\cdot041 \text{ kgf/cm}^2$$

$$\therefore \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \quad \frac{1\cdot041 \times 153}{290} = \frac{1\cdot033 \times V_2}{273}$$

$\therefore$ Volume of gas per min at N.T.P., $V_2 = 145\cdot1$ litre

Heat supplied per minute $=145 \cdot 1 \times 10^{-3} \times 4,350 = 631$ kcal

$$\text{bhp} = \frac{2\pi NT}{75 \times 60} = \frac{2 \times \pi \times 285 \times 34}{75 \times 60} = 13 \cdot 52$$

$$\text{Heat equivalent to bhp} = \frac{13 \cdot 52 \times 632 \cdot 4}{60} = 142 \cdot 6 \text{ kcal}$$

$$\text{hp equivalent to } +ve \text{ loop, (Gross ihp)} = \frac{p_m lAN}{75 \times 60}$$

$$= \frac{5 \cdot 72 \times \left(\frac{400}{1,000} \right) \times \left(\frac{\pi}{4} \times 20^2 \right) \times 114}{75 \times 60} = 18 \cdot 2$$

$$\text{Heat equivalent to } +ve \text{ loop} = \frac{18 \cdot 2 \times 632 \cdot 4}{60} = 192 \text{ kcal}$$

hp equivalent to $-ve$ loop of firing stroke

$$= \frac{18 \cdot 2 \times 0 \cdot 274}{5 \cdot 72} = 0 \cdot 872$$

$$\text{No. of missed cycles} = \frac{285}{2} - 114 = 28 \cdot 5 \text{ per min}$$

hp equivalent to $-ve$ loop of missed cycles

$$= \frac{0 \cdot 38 \times \left(\frac{400}{1,000} \right) \times \left(\frac{\pi}{4} \times 20^2 \right) \times 28 \cdot 5}{75 \times 60} = 0 \cdot 3024$$

Net pumping horse-power $= 0 \cdot 872 + 0 \cdot 3024 = 1 \cdot 1744$

$$\text{Heat equivalent to pumping hp} = \frac{1 \cdot 1744 \times 632 \cdot 4}{60} = 12 \cdot 4 \text{ kcal}$$

Net indicated horse-power $= 18 \cdot 2 - 1 \cdot 1744 = 17 \cdot 026$

Frictional horse-power $= 17 \cdot 026 - 13 \cdot 52 = 3 \cdot 506$

Percentage of gross indicated power in pumping

$$= \frac{1 \cdot 1744}{18 \cdot 2} = \underline{6 \cdot 448\%}$$ Ans.

Percentage of gross indicated power in mechanical friction

$$= \frac{3 \cdot 506}{18 \cdot 2} = \underline{19 \cdot 26\%}$$ Ans.

Heat equivalent to mechanical friction

$$=\frac{3\,506\times632\cdot4}{60}=37 \text{ kcal/min}$$

Heat in cooling water $=9\ 2\times20=184$ kcal/min

Mass of gas used $=145\cdot1\times10^{-3}\times0\ 592=0\ 0859$ kg

Mass of steam formed $=0\ 12\times0\ 0859\times9=0\ 09276$ kg per min

$\therefore$ Heat carried away by steam

$$=0\ 09276[(639\ 1-17)+0\ 48(400-100)]$$
$$=71\ 1 \text{ kcal per min}$$

Mass of dry exhaust gas $=0\ 0859+1\ 45-0\ 09276=1\ 4431$ kg

Heat carried by dfg $=1\ 4431\times0\ 25(400-17)=138\ 1$ kcal

Heat balance sheet on one minute basis :

Credit	*kcal*	%	*Debit*	*kcal*	%
Heat supplied in fuel	631	100	ihp of +ve loop (gross ihp)	192 0	30 43
			ihp of −ve loop	12 4	1·97
			Net ihp	179 6	28 46
			Friction	37 0	5 86
			1. Heat equivalent to bhp	142 6	22 60
			2. Heat taken by water	184 0	29·16
			3. Heat to dry exhaust gas	138·1	21·89
			4. Heat to steam	71·1	11·27
			5. Losses (by diff.)	95·2	15·08
Total	631	100		631·0	100 00

Note.—Percentage of power used in pumping and friction has been calculated on the basis of gross indicated power *i.e.* total power produced in engine in power stroke.

EXAMPLES 9

9-1. Petrol engine : cooling water ; volume of air through radiator.

A petrol engine uses 0·226 kg of petrol per bhp/hour. The calorific value of petrol used is 10,400 kcal/kg. Assuming that 30 per cent of heat supplied is carried away by the jacket water and that the rise in temperature of the water as it passes through the jacket is 25°C, what mass of water must pass through the jacket per bhp per minute ? The heated water is brought to its original temperature by passing it through a radiator. Determine the minimum volume of air which must pass through the radiator per bhp/minute, assuming that its specific heat is 0·24 and rise of temperature 25°C. One kg of air has a volume of 775 litres.

If the bhp of this engine is measured by a rope brake, calculate the quantity of water required per bhp/min for brake if the temperature rise of water is limited to 30°C.

[Cooling water/bhp/min = 0·4704 kg ; vol. of air/bhp/min = 1·523 m³ ; full bhp dissipated in brake, water quantity/bhp/min = 0·3512 kg]

9-2. Gas engine test : bhp ; ihp ; thermal efficiency.

During a test on a single-cylinder gas engine of 23 cm bore and 41 cm stroke working on the four-stroke cycle, the following readings were taken :

Speed = 240 rpm ; misses per minute = 10 ; area of indicator diagram = 5·87 cm² ; spring no. = 8·3 ; length of indicator diagram = 7·6 cm ; effective brake load = 78 kg ; brake wheel diameter = 152 cm ; rope diameter = 3 cm ; gas used as measured at the meter = 14·44 m³ per hour at a temperature 18°C and pressure 8·9 cm of water when barometer reads = 73·6 cm of mercury. Take calorific value of gas = 4,000 kcal/m³ at 15°C and 1·033 kgf/cm². Calculate the bhp, ihp and thermal efficiency of the engine.

[bhp = 20·24 ; P_m = 6·41 kgf/cm² ; no. of firing stroke = 110 ; ihp = 26·69 ; volume of gas at N.T.P. = 13·96 m³/hr ; indicated thermal η = 30·2%]

9·3. Petrol engine : cylinder dimensions, given air-fuel ratio.

It is assumed that an automobile engine can operate at a thermal efficiency of 22 per cent when operating conditions are as follows :—

Volumetric efficiency of 80 per cent ; mechanical efficiency of 82 per cent ; heat value of petrol 11,000 kcal per kg ; theoretical air required per kg of petrol 14·5 kg ; excess air of 25 per cent ; petrol vapour has density twice the density of air and the pressure at the end of suction stroke is at a pressure of 0·85 kgf/cm² and a temperature of 60°C. Gas constant for air is 29·2.

Find the cylinder dimensions of a six-cylinder engine of the above conditions when the engine develops its rated horse-power of 90 at a speed of 4,200 rpm. Assume that stroke is 25 per cent greater than the diameter.

[Fuel/stroke/cylinder=$3{\cdot}793\times10^{-5}$ kg ; air=$6\ 876\times10^{-4}$ kg , ρ_{air}=0 8742 [kg/m³ ; volume of air/stroke=787 c c ; volume of petrol/stroke=21·7 c c , $d=10\ 1$ cm , $l=12\ 6$ cm]

9·4. Oil engine : air-fuel ratio ; volumetric efficiency.

The mass analysis of the fuel supplied to a single cylinder four-stroke oil engine is $C=85$; $H=12\ 5$, $O=2$ and residue$=0\ 5$ per cent. The dry exhaust gas has the volumetric analysis $CO_2=7{\cdot}5$; $O_2=10\ 86$ and $N_2=81\ 64$ per cent

The engine has a cylinder 18 cm bore by 38 cm stroke and runs at 280 rpm. Fuel is injected at the rate of 2 56 kg per hour Find :

(a) the air to ful ratio by mass, given that air contains 23 2 per cent of O_2 by mass or 20 9 per cent by volume.

(b) the volumetric efficiency based on surrounding air conditions of 1·033 kgf/cm² and 21°C $R=29\ 25$ kgm/kg deg C.

[dfg=28 kg ; air/fuel ratio=28 13 1 , volumetric η=73·8%]

9·5. Petrol engine : indicated thermal efficiency with rich mixture.

Define volumetric efficiency as applied to petrol engines. How does the volumetric efficiency of an engine limits the power output ?

A six-cylinder petrol engine of bore 76 mm and stroke 100 mm develops 12 bhp at 500 rpm when running on mixture 15 per cent rich. The fuel has a calorific value of 10,500 kcal per kg and contains 85 per cent carbon and 15 per cent hydrogen by mass. Assuming a volumetric efficiency of 75 per cent at 27°C and a mechanical efficiency of 83 per cent, find the indicated thermal efficiency of the engine. Specific volume of air at N.T.P. is 775 litres.

[Air/cylinder/stroke = 0·0004 kg ; minimum air/kg of fuel = 15·07 kg ; actual air-fuel ratio = 15·07 : 1·05 ; indicated thermal η = 31·7%]

9·6. Measurement of air flow ; air-fuel ratio ; volumetric efficiency.

How the air consumption is related to ihp in a petrol engine and in a Diesel engine ?

A single-cylinder four-stroke Diesel engine running at 400 rpm with cylinder displacement of 0·006 m^3 was arranged to draw air through a calibrated orifice in an air box, the pulsations being sufficiently damped by this procedure. The readings obtained were ; barometer 736 mm H_g ; air temperature 17°C ; depression in the air box 125 mm of water ; diameter of orifice 25 mm ; coefficient of discharge of orifice, 0·62 ; fuel used 2·1 kg/hr. Calculate the air fuel ratio and the volumetric efficiency of the engine referred to inlet conditions. Neglect the difference in the density of air on the two sides of the orifice. Assume R = 29·27 kgf m/kg/°C.

[air flow = 0·0139 m^3/s ; vol η = 69·6% ; air flow = 0·01636 kg/s ; air/fuel ratio = 28]

9·7. Morse test : mech η ; fuel consumption ; vol. η

A four stroke-petrol engine having six cylinders of 9 cm bore and stroke 10 cm runs at 2000 rpm and develops 65 bhp. When one cylinder at a time is 'cut-off' during a Morse test, the mean bhp developed is 51·5. The thermal efficiency on the ihp basis when cylinders are operative is 30 per cent with petrol having a calorific value of 11,000 kcal per kg. The air to fuel ratio is 14·4 to 1. Calculate :

(a) *the mechanical efficiency ;*

(b) *the petrol used per hour in kg ;*

(c) *the volumetric efficiency, if the air is measured at 1·033 kgf/cm² and °C.*

[ihp=81 ; mech. η=80·24% ; petrol/hr=15·52 kg ; air sucked/hr=223·6 kg ; mass equivalent to stroke volume=296 kg ; vol η considering air alone=75·51 % ; volumetric η considering total charge =80·8 %]

9.8 Diesel engine. Unsupercharged : V_s, mep ; Supercharged : increase in pressure to obtain same power.

A Diesel engine operating on the four-stroke cycle is to be designed to operate with the following characteristics at sea level where the mean conditions are : 1·033 kgf/cm² and 7°C.

bhp=350, Volumetric efficiency 78% (at sea level free air conditions) ; specific fuel consumption=0·18 kg/bhp hr ; Air-fuel ratio =17 to 1 ; speed=1500 rev/min. Calculate the required engine capacity (stroke volume) and the anticipated brake mean effective pressure.

The engine is fitted with a supercharger so that it may be operated at an altitude of 3,000 m where the atmospheric pressure is 0·73 kgf/cm². The power taken by the supercharger is 8 per cent of the total power produced by the engine and the temperature of air leaving the supercharger is 32°C. The air-fuel ratio and the thermal efficiency remain the same for the supercharged engine as when running unsupercharged at sea level, as does the volumetric efficiency. Calculate the increase of air pressure required at the supercharger to maintain the same net output of 350 bhp. Take R=29·27 kgf m/kg °C for air.

[Air consumption=17·85 kg/min, V_s=0·0242 m³, brake mep =8·66 kgf/cm², supercharged : gross power=3[illegible]9·5 bhp, air =19·4 kg/min, P_2=1·22 kg/cm² ∴ increase in pressure=0·49 kgf/cm²]

9.9. Oil engine trial : heat balance sheet

In a trial of an oil engine the following data were obtained : Duration of trial, 30 min ; speed 1750 rev/min ; Brake torque, 35 kgf m ; Fuel consumption, 9·4 kg of calorific value 10 169 kcal/kg ; Jacket cooling water circulation 499 kg with inlet and outlet temperatures of 15°C and 77°C respectively ; Air consumption 1[illegible]0 kg ; Exhaust temperature, 357°C ; Atmospheric temperature, 15°C.

*Calculate the brake horse-power, the fuel consumption in kg/bhp hr, and the indicated thermal efficiency if the mechanical e[illegible]

is 83 per cent. Assuming that the mean specific heat for the exhaust gas is 0·3, draw up an energy balance expressing the various items in kcal/min and percentages.

[bhp=85 6, sfc=0·22 kg ; indicated thermal η=35 % ; heat balance on minute basis : bhp=9·02 kcal or 28·5 % cooling water =960 kcal or 30·4% ; exhaust=890 kcal or 28·1% ; radiation etc.. (by difference)=413 kcal or 13 %]

9·10. Petrol engine : Air/fuel ratio ; %excess air ; volumetric efficiency

A six-cylinder, four-stroke petrol engine with a bore of 12·5 cm and a stroke of 19 cm was supplied during a test with petrol of composition C=82 per cent and H_2=18 per cent by mass.

The dry exhaust composition by volume was CO_2=11·19 per cent, O_2=3·61 per cent and N_2=85·2 per cent.

Determine the mass of air supplied per kg of petrol, the percentage of excess air and the volume of the mixture per kg of petrol at 17°C and 1 kgf/cm², which were the conditions for the mixture entering the cylinder during the test. Also determine the volumetric efficiency of the engine based on intake conditions when mass of petrol used per hour during the test was 31 kg and the engine speed was 1600 rpm. The petrol is completely evaporated before entering the cylinder and the effect of its volume on the volumetric efficiency should be included.

Take the density of petrol vapour as 3·35 times that of air at the same temperature and pressure. 1 kg of air at 0°C and 1·033 kgf/cm² occupies 0·7734 m³. Air contains 23 per cent oxygen by mass.

[Per kg of petrol, dfg=18·29 kg ; mass of air=18·91 kg ; excess air=19 94% ; volume of mixture=16·354 m³, vol η=75%]

9·11. Oil engine : heat balance sheet ; piston cooling.

A six-cylinder four-stroke diesel engine of 34 cm bore and 38 cm stroke was tested at half load and gave the following information :

rpm=300 ; bhp=200 ; mep=3·8 kgf/cm² ; fuel per minute of CV 11,000 kcal/kg=0·77 kg ; hydrogen content of fuel=14 per cent ;

air consumption = 40 kg/minute ; jacket water = 64 kg/minute with a temperature rise of 11°C ; piston cooling oil (specific heat 0 5) = 34 kg/min with temperature rise of 5°C ; exhaust gas temperature (specific heat 0 24) = 190°C ; room temperature = 21°C.

Draw up a heat balance sheet in kcal/min indicating the items which may include the friction losses. Take the specific heat of steam in exhaust gas 0 48 and partial pressure 0 07 kgf/cm² of steam. Assume efficiency of combustion to be unchanged. Estimate the brake specific fuel consumption at full load.

[ihp=314·7 ; heat balance per min : bhp=24 88%, cooling water=8·312%, cooling oil=1·003%, steam=8·174%, dry exhaust =19 03%, heat unaccounted for=38·5% friction item included in all except bhp ; thermal η and friction same at full load and half load ; heat equivalent of ihp at full load=5,423 kcal/min ; specific fuel consumption=0 189 kg/hr]

9.12. Oil engine : heat balance sheet ; exhaust calorimeter.

In a full load test on a 2-cylinder, 4 stroke oil engine, the following readings were taken :

Mean effective pressure = 7 8 kgf/cm², net brake load = 203 kg ; effective brake diameter = 152 cm, mean speed = 302 rpm ; fuel consumption = 14·1 kg/hr ; quantity of cylinder cooling water = 12 kg/min ; cylinder cooling water inlet temperature = 15°C ; cylinder cooling water temperature = 70°C ; quantity of exhaust calorimeter cooling water = 20 5 kg/min ; exhaust cooling water inlet temperature = 15°C ; exhaust cooling water outlet temperature = 55°C ; temperature of gases leaving calorimeter = 86°C ; air/fuel ratio by mass = 19 ; engine room temperature = 18°C.

The calorific value of fuel oil was 10,850 kcal/kg and the mean specific heat of the exhaust gases 0·24 kcal/kg°C. The stroke volume per cylinder was 15 litre and each cylinder may be assumed to have contributed equally towards the work done. Find the mechanical and brake thermal efficiencies. Draw up a heat balance for the test expressed in kcal per minute.

[bhp=65·01 ; ihp=78·52 ; mech η=82·8% ; brake thermal η=28·44% ; heat balance per minute : heat in fuel=2,409 kcal, bhp =685 kcal, engine cooling water=660 kcal ; exhaust gases=79 kcal, exhaust calorimeter cooling water=820 kcal, difference=165 kcal]

9·13. Two-stroke oil engine : indicated thermal efficiency ; specific fuel consumption ; volumetric efficiency ; heat balance sheet.

A full-load test on a two-stroke oil engine, yielded the following results :

Speed, 450 rpm ; brake load 46 kg ; imep, 2·88 kf/cm² ; oil consumption 5 35 kg per hour ; rise in temperature of jacket water, 18°C ; jacket water flow 440 kg per hour ; air-fuel ratio by mass, 31 ; temperature of the exhaust gass, 355°C ; temperature of laboratory, 20°C, barometric pressure, 760 mm of Hg.

The following data apply to the test :

Cylinder diameter, 216 mm ; stroke, 270 mm ; brake diameter, 147 cm ; calorific value of fuel 10,5000 kcal ; proportion of hydrogen in fuel by mass, 0·15 ; R for air 29·27 metre-kgf/kg°C ; mean specific heat of dry exhaust gases, 0·24 ; specific heat of dry steam 0·49 kcal/kg °C ; partial pressure of steam in exhaust gases 0·07 kgf/cm². Determine (a) the indicated thermal efficiency, (b) the specific fuel consumption in kg/bhp-hr, (c) the volumetric efficiency based on atmospheric conditions.

Draw up a heat balance sheet for the test on a percentage basis, indicating carefully the content of each.

[ihp=28·5 ; indicated thermal efficiency=32·05 % ; fuel consumption/bhp-hr=0·252 kg ; volume of air inhaled=0·0051 m³ ; (volumetric η=51·6%) ; heat supplied by fuel=936 kcal (100 %) ; heat to ihp=300 kcal (32 05%) ; heat in friction=76 kcal (8·12 %) ; heat to bhp=224 kcal (23·93 %) ; heat in cooling water=132 kcal (14·10 %) ; heat in dry exhaust gas=90 kcal (9·61 %) ; other losses (by diff.)=270 kcal (28·85 %)].

10

Reciprocating Air Compressors

10·1. Introduction. Air compressors are used for increasing pressure of air by compression. The compressed air has many practical uses such as (*i*) driving a compressed air engine ; (*ii*) driving pneumatic tools , (*iii*) paint spraying , (*iv*) injecting fuel in Diesel engines ; (*v*) conveying solid and powdered materials in pipe lines ; (*vi*) transmitting control-system pressures to remote locations , (*vii*) in blast furnaces ; (*viii*) cleaning surfaces by air blast , (*ix*) cooling of large buildings, etc.

There are two basic types of air compressors :—

(*a*) Positive displacement compressors, and

(*b*) Non-positive displacement compressors

Reciprocating compressor, Root blower and the vane pump are the examples of positive displacement compressors whereas, centrifugal and axial compressors are the examples of non-positive displacement compressors.

Reciprocating air compressors ensure positive displacement and consist of a cylinder, piston and spring loaded non-return plate valves

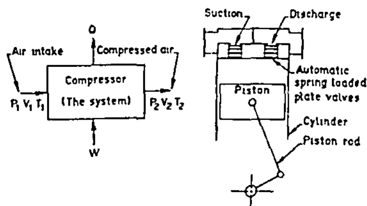


Fig. 10·1. Line diagram of reciprocating air compressor.

(*see* Fig. 10·1) ; the construction is comparable to simple steam engines. The operation of reciprocating *air compressors* is intermittent and *they are best suited* for high pressure ratios and comparatively small mass flow.

10·2. Work Done in Single-stage Compressor Neglecting Clearance Volume. The theoretical air compression cycle neglecting clearance volume is represented in Fig. 10·2 on *P-V* and *T-s* diagrams. Air is sucked in from atmosphere at constant pressure P_1 and is delivered at constant pressure P_2. The area of the *P-V* diagram represents *the work done in the cycle*. The work done is least when the compression is isothermal *i.e.* $n=1$ and is maximum when it is adiabatic *i.e.* $n=\gamma$, because isothermal line has less slope than adiabatic line. Isothermal compression is not possible in practice as the compressor would need to run at very low speed. Generally *the compressors are run at high* speed and hence the *compression* would be nearly adiabatic. However, to approach isothermal compression air or water cooling is done during compression, and in multistage compressors intercooling is done. In practice the compression follows the law $PV^n=C$, where the value of 'n' varies from 1.25 to 1.35.

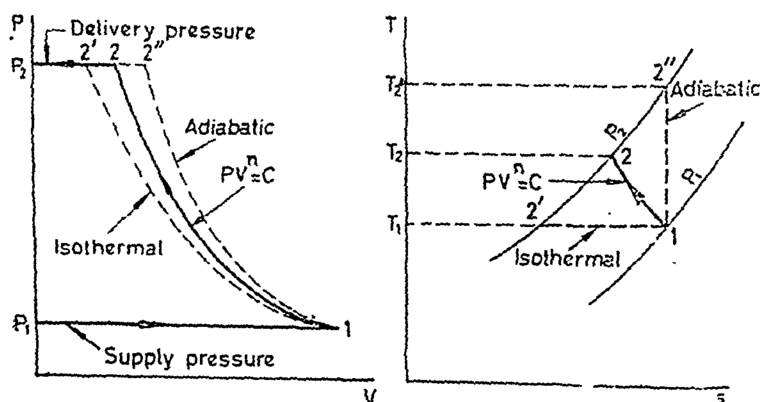


Fig. 10·2. Air compression cycle on *P-V* and *T-s* diagrams.

(*i*) *When compression follows the law* $PV^n=C$

Work done per cycle = area of the *P-V* diagram

$$=\left[P_2V_2+\frac{P_2V_2-P_1V_1}{n-1}\right]-P_1V_1$$

$$=\frac{n}{n-1}\left[P_2V_2-P_1V_1\right] \quad (10\cdot1)$$

$$=\frac{n}{n-1}P_1V_1\left[\frac{P_2V_2}{P_1V_1}-1\right]$$

$$=\frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right] \quad [10\cdot2(a)]$$

$$=\frac{n}{n-1}mRT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right] \quad [10\cdot2(b)]$$

From Eq. (10·1) work done per cycle

$$=\frac{n}{n-1}mR\left[T_2-T_1\right] \quad (10\cdot3)$$

Note. The above formulae are not true when $n=1$ *i.e.* when compression is isothermal.

(*ii*) *When compression is adiabatic*, $PV^{\gamma}=C$

Work done per $\text{kg}=\frac{\gamma}{\gamma-1}R\,(T_2-T_1)$ [as $n=\gamma$

Now, $C_v=\frac{R}{J(\gamma-1)}$ $\therefore$ $R=JC_v\,(\gamma-1)$

Substituting for R,

Work done per kg $=\gamma JC_v(T_2-T_1)$

$$=\frac{C_p}{C_v}\;C_vJdT=C_pdT \text{ heat units} \quad (10\cdot4)$$

(*iii*) *When compression is isothermal*, $PV=C$

Work done per cycle$=P_2V_2+P_1V_1\log_e r-P_1V_1$

Now, $P_1V_1=P_2V_2$ $\therefore$ $W.D=P_1V_1\log_e r$ (10·5)

Note. From equation (10·5) we see that in the above case net-work done in complete cycle is equal to the work done in isothermal process only.

The compression in practice lies between the isothermal and isentropic conditions. Since the object is to approach isothermal compression, as this gives the least power consumed, the performance of a reciprocating air compressor is given by isothermal efficiency which is defined as

$$\text{Isothermal efficiency}=\frac{\text{isothermal work}}{\text{actual indicated work}} \quad (10\cdot6)$$

10·3. Effect of Clearance in Compressors. From practical consideration that the piston does not strike the cylinder end and to provide space for valves, clearance is provided at the end of stroke. The theoretical air compression cycle considering clearance is shown

Fig. 10·3. Theoretical air compressor cycle with clearance.

in Fig. 10·3. At the end of delivery stroke a volume of air represented by V_3 (*i.e.* clearance volume) having a pressure P_2 is left inside the cylinder. This high pressure air will expand to V_4 at the beginning of suction stroke, the law of expansion being $PV^n=C$. This means that the fresh air inhaled in the suction stroke will be only (V_1-V_4). The clearance space air, of course, does work on the piston in expanding, reducing the net work required to area 1234.

Net work done=area 1234

$$=\frac{n}{n-1}\ R(m_1-m_3)(T_2-T_1) \qquad [10{\cdot}7(a)]$$

and, work done per kg of air delivered

$$=\frac{n}{n-1}\ R(T_2-T_1) \qquad [10{\cdot}7(b)]$$

From the above equation, we see that clearance has no effect on work done per kg of air delivered, but it reduces the effective swept volume which means reduction in the mass of air dealt in each cycle and also reduction in volumetric efficiency.

In single-stage compressors the clearance volume is about 4 to 10 per cent of the stroke volume. In multi-stage compressors it is

about 1·5 to 2 per cent in L.P. cylinders and 2·5 to 3 per cent in H.P. cylinders.

10·4. Volumetric Efficiency. As shown in the preceding article the mass of air passing through a compressor is not equal to the mass of air which would fill the complete swept volume at the suction stroke. Their ratio is called the volumetric efficiency and is defined in the following ways :—

(*i*) Overall volumetric efficiency

$$=\frac{\text{mass of air delivered per minute}}{\text{mass of air corresponding to swept volume of L.P. cylinder per minute evaluated at free air conditions}} \quad (10\text{·}8)$$

alternatively,

$$=\frac{\text{volume of free air inhaled}}{\text{swept volume of L.P. cylinder}}$$

(*ii*) Absolute volumetric efficiency

$$=\frac{\text{mass of air delivered per minute}}{\text{mass of air corresponding to swept volume of L.P. cylinder per minute evaluated at N.T.P.}} \quad (10.9)$$

alternatively,

$$=\frac{\text{volume of air inhaled at N.T.P}}{\text{swept volume of L.P cylinder}}$$

The term overall volumetric efficiency is more in common use, its value being about 70 to 90 per cent for reciprocating air compressors.

In practice the temperature at the end of suction, *i.e.* at point 1 is not atmospheric because the fresh air passes over hot valves and mixes with the residual air. Also the pressure at 1 is not atmospheric as shown in Fig. 10 3 as there are obstructions in suction of fresh air. Let suffix *a* refer to atmospheric condition and suffix 1 to condition of air before compression.

$$\frac{P_a V_a}{T_a}=\frac{P_1(V_1-V_4)}{T_1} \qquad \therefore \quad V_a=\frac{P_1 T_a}{P_a T_1}(V_1-V_4)$$

Volumetric efficiency (referred to ambient condition)

$$=\frac{\text{vol. of air sucked referred to ambient condition}}{\text{swept volume}}$$

$$=\frac{P_1 T_a}{P_a T_1}\times\left[\frac{V_1-V_4}{V_1-V_3}\right]$$

$$=\frac{P_1T_a}{P_aT_1}\times\left[\frac{V_s+V_c-V_4}{V_s}\right]$$

$$=\frac{P_1T_a}{P_aT_1}\times\left[1+\frac{V_c}{V_s}-\frac{V_4}{V_c}\times\frac{V_c}{V_s}\right]$$

Assuming the law of expansion to be same as the law of compression, and substituting $\frac{V_c}{V_s}=c$, we get

$$\text{Volumetric efficiency}=\frac{P_1T_a}{P_aT_1}\left[1+c-c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]. \qquad (10\cdot10)$$

From the above expression it is evident that the volumetric efficiency decreases with increasing pressure ratio (there will be no change in the work done per unit mass), and at some pressure ratio the mass flow will be zero. Even before this stage is reached it is more economical to use two or more stages for compression.

10·5. Actual Indicator Diagram. Actual indicator diagrams differ from hypothetical diagrams due to turbulence and eddies in air and inertia of automatic spring-loaded suction and delivery valves as shown in Fig. 10·4. The inlet valve should theoretically

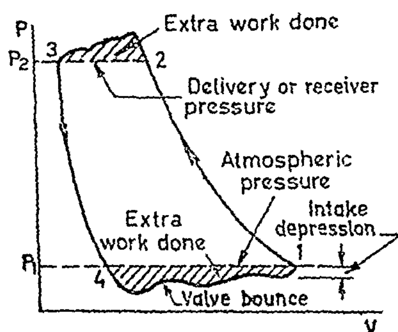


Fig. 10·4. Actual indicator diagram.

open at 4, but actually it does not open because of the valve inertia and because there must be a pressure difference across the valve in order to move it. Thus the pressure drops off below atmospheric until the valve moves off its seat. The opening of the valve is followed by some valve bounce and eventually the intake will become steady at some pressure below atmospheric. This pressure difference is called *intake depression*. Similar situation occurs on the delivery

side. The extra work done due to this may be called the *pumping work*. The area of the indicator diagram is the indicated work required.

Mechanical efficiency of a reciprocating compressor is defined as the ratio of indicated power to the brake power of the shaft supplying power to the compressor.

$$\text{Mechanical efficiency} = \frac{\text{indicated or air hp}}{\text{bhp}} \quad (10\text{·}11)$$

10·6. Work Done in Multi-stage Compressor Neglecting Clearance In a compressor when compression ratio exceeds 5, generally multi-stage compression is adopted to approximate to

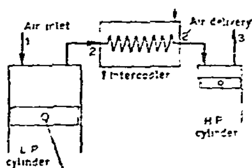


Fig. 10·5. Schematic diagram of a multi-stage compressor.

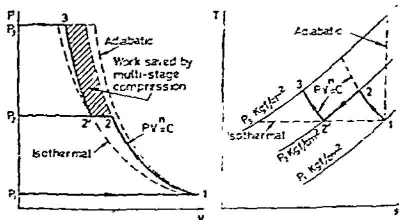


Fig. 10·6. Multi-stage compression cycle on $P-V$ and $T-s$ diagrams.

isothermal compression by cooling the air between the stages and thus reducing the work required for a given ratio. Multi-stage com-

pression also improves the volumetric efficiency. Other incidental advantages of multistage compression are light cylinders, reduced leakage losses, more uniform torque with consequent reduction in the size of the flywheel, improved cooling during compression and better lubrication. A schematic diagram for multi-stage compression is shown in Fig. 10·5.

In multi-stage compressors the air after compression in the first-stage is cooled in an intercooler at constant pressure before passing into the second stage. If the air is cooled to its original temperature the cooling is said to be *perfect* and the point 2′ lies on the isothermal line (*see* Fig. 10·6). If the cooling is not upto the original temperature it is said to be *imperfect* and the point 2 will lie on the right of isothermal line on P-V diagram.

For calculating the work done in multi-stage compressor the following assumptions are made :—

(*i*) Suction and delivery pressures remain constant during each stage.

(*ii*) The index of compression is same in each stage.

(*iii*) The intercooling in each stage is at constant pressure.

(*iv*) There is no pressure drop between stages of a two-stage compressor.

(*v*) The mass of air handled by the L.P. and the H.P. cylinders are same.

Referring to Fig. 10·6

Work required per cycle in L.P. cylinder

$$=\frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right]$$

Work required per cycle in H.P. cylinder

$$=\frac{n}{n-1}P_2V_2\left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}-1\right]$$

Total work required,

W=work per cycle in (L.P. cylinder+H.P. cylinder)

$$=\frac{n}{n-1}\left[P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right\}\right.$$

$$\left.+P_2V_2\left\{\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}-1\right\}\right] \quad (10\cdot12)$$

If the intercooling is complete,

$$P_1V_1 = P_2V_2$$

and
$$W = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 2 \right] \quad (10\cdot13)$$

$$\therefore \text{ hp required} = \frac{W \times N}{75 \times 60} \quad (10\cdot14)$$

10·7. Conditions for Maximum Efficiency in Multi-stage Compressors. For the two-stage air compressor minimum work will be required when the intercooling is perfect, in which case work done per cycle is given by Eq. (10·13). If the initial and final pressures are fixed, the best value for intermediate pressure P_2 can be found by differentiating Eq. (10·13) in terms of P_2 and equating to zero. Noting that P_1, V_1, P_3, and $\frac{n}{n-1}$ are constants and writing

$$\frac{n-1}{n} = a$$

Eq. (10·13) reduces to

$$W = \text{constant} \times \left[\left(\frac{P_2}{P_1} \right)^a + \left(\frac{P_3}{P_2} \right)^a - 2 \right]$$

$$= \text{constant} \times [P_2^a \times P_1^{-a} + P_3^a \times P_2^{-a} - 2]$$

Differentiating and equating to zero, we get

$$\frac{dW}{dP_2} = aP_2^{a-1}P_1^{-a} + P_3^a(-a)P_2^{-a-1} = 0$$

or
$$\frac{P_2^{a-1}}{P_2^{-a-1}} = P_1^a \times P_3^a \qquad \therefore \underline{P_2 = \sqrt{P_1 \times P_3}} \quad (10\cdot15)$$

or $\frac{P_2}{P_1} = \frac{P_3}{P_2}$ *i.e.* pressure ratio in each stage is same. Also, by substituting from Eq. (10·15) in (10·13) it is seen that work done in each cylinder is same. Or

Total work done per cycle

$$= 2 \times \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad (10\cdot16)$$

For 'q' stages $\dfrac{P_2}{P_1}=\dfrac{P_3}{P_2}=\dfrac{P_4}{P_3}=\ldots\ldots\ldots=\dfrac{P_{q+1}}{P_q}$

$$\therefore \qquad \frac{P_2}{P_1}=\left[\frac{P_{q+1}}{P_1}\right]^{\frac{1}{q}}$$

Minimum work for 'q' stages

$$=P_1V_1\frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}+\ldots\ldots+\left(\frac{P_{q+1}}{P_q}\right)^{\frac{n-1}{n}}-q\right]$$

$$=P_1V_1q\frac{n}{n-1}\left[\left(\frac{P_{q+1}}{P_1}\right)^{\frac{n-1}{nq}}-1\right] \qquad (10\cdot17)$$

The assumptions for calculating the work done in multi-stage compressors are given in previous article. From the derivations above, the additional assumptions for minimum work are as follows :—

(*i*) The pressure ratios are the same for each stage.

(*ii*) The intercooling is perfect.

(*iii*) The clearance is neglected or if clearance is considered, n for expansion and compression is equal.

It follows from the above that for minimum work :

(*i*) The work done is the same in each stage.

(*ii*) The temperature ratios and the maximum temperatures are the same in each stage.

(*iii*) The temperature of the gas in either cylinder is minimum when $P_2=\sqrt{P_1\times P_3}$. If the intercooler pressure is greater than P_2, the temperature at the end of compression in the L.P. cylinder would have been greater than T_3, resulting in greater temperature range (T_3-T_1). If the intercooler pressure is less than P_2, the temperature at the beginning of compression in the H.P. cylinder would be less than T_4, resulting in greater temperature range (T_2-T_4),

In practice due to imperfect cooling and pressure drop, a compressor, designed with $P_2=\sqrt{P_1\times P_3}$ would have greater area of H.P. diagram so that the intermediate pressure is raised above the value of P_2.

10·8. Heat Rejected in Compressor and Intercooler. In reciprocating air compressors total heat rejected

=heat rejected during compression+heat rejected in intercooler

$\therefore$ Heat rejected per kg $= \dfrac{\gamma - n}{\gamma - 1} \times \text{W.D.} + C_v(T_2 - T_1)$

$$= \frac{\gamma - n}{\gamma - 1} \times \frac{R(T_2 - T_1)}{J(n-1)} + C_v(T_2 - T_1)$$

Since, $\quad C_v = \dfrac{R}{J(\gamma - 1)}$

Total heat rejected per unit mass

$$= \frac{C_v(\gamma - n)}{(n-1)} \times (T_2 - T_1) + C_v(T_2 - T_1)$$

$$= \left[C_p - C_v \frac{\gamma - n}{n - 1} \right] \left(T_2 - T_1 \right) \quad (10 \cdot 18)$$

10 9. Measurement of Air-flow in an Air Compressor. For measuring the flow of air in a compressor air is sucked through a

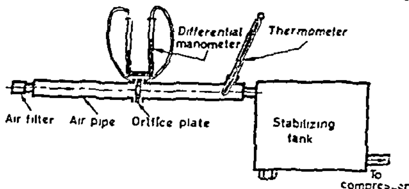


Fig. 10 7. Measurement of air-flow by an orifice

suction pipe with an orifice fitted in it The pressure drop across the orifice is measured by a water manometer. The pressure drop should not exceed about 150 mm in which case we can neglect the effect of compressibility on the two sides of the orifice A stabilising tank of 500 to 600 times the capacity of the compressor cylinder with internal baffles is interposed between this pipe and the compressor to avoid pressure pulsations in the measuring pipe The arrangement for measurement is shown in Fig. 10 7.

Let, $\quad A$ = area of the orifice, in cm²

C_d = coefficient of discharge of the orifice

P_1 = pressure, in kgf/cm² before the orifice

P_2 = pressure, in kgf/cm² after the orifice

v_2 = volume of 1 kg of air at pressure P_2

= volume of 1 kg of free air (approx.)

C_2 = velocity of air after the orifice

H = pressure drop across the orifice, in kgf/cm².

As the air passes through the orifice, which is a small nozzle, there is increase in velocity due to drop in pressure. Assuming initial velocity to be negligible and mass of the air 1 kg

K.E. gained = work done in expanding

$$\frac{C_2^2}{2g} = \frac{n}{n-1} \; P_2 v_2 \left[\left(\frac{P_1}{P_2} \right)^{\frac{n-1}{n}} - 1 \right] \qquad (i)$$

$$\left(\frac{P_1}{P_2} \right)^{\frac{n-1}{n}} = \left(\frac{P_2 + H}{P_2} \right)^{\frac{n-1}{n}}$$

$$= \left(1 + \frac{H}{P_2} \right)^{\frac{n-1}{n}} = 1 + \frac{n-1}{n} \cdot \frac{H}{P_2} + \ldots \qquad (ii)$$

Neglecting higher powers of $\frac{H}{P_2}$, as H is small

$$C_2 = \sqrt{2g \frac{n}{n-1} \; P_2 v_2 \left(\frac{n-1}{n} \cdot \frac{H}{P_2} \right)} \text{ approx.}$$

[From Eqs. (*i*) and (*ii*)]

$$= \sqrt{2gHv_2} = \sqrt{2gHv} \qquad [\text{as } v_2 = v$$

Theoretical flow of air per second, $Q = A\sqrt{2gHV}$

$\therefore$ Actual flow of air/second $= C_d A \sqrt{2gHv}$ [10·19(*a*)]

and Mass flow/second $= C_d A \sqrt{\frac{2gH}{v}}$ [10·19(*b*)]

10·10. Free Air Delivery (F.A.D.). The capacity of a compressor is given in terms of free air delivery. It is the volume in m³ of the delivered air reduced to the ambient temperature and pressure conditions. Theoretically it is equal to the stroke volume but due to clearance, piston leakage and high speed it is less than the stroke volume.

10·11. Control of Delivery of Air Compressors. Generally compressors do not run for the full time at the maximum rated capacity and, therefore, some means may be provided to control the amount of air delivered by it. There are three main methods of controlling the air delivered.

(*a*) Throttle control.

(*b*) Clearance control.

(*c*) Blow-off control.

(*a*) **Throttle control.** In this arrangement the opening of the suction valve of the compressor is controlled by the pressure in the receiver. When the pressure in receiver exceeds the normal pressure the suction valve of the compressor is partly closed thus reducing the quantity of air inhaled. This, however, has the disadvantage that due to higher pressure ratio the temperature of delivery may rise to a dangerous value.

(*b*) **Clearance control.** In this arrangement clearance pockets are provided which are normally not in communication with the cylinder. When the pressure ratio in receiver exceeds the limit these pockets are brought into communication with the cylinder by automatically operated valves. By adding extra clearance space the volumetric efficiency of the plant is reduced.

(*c*) **Blow-off control.** In this arrangement a by-pass valve is provided which opens to the atmosphere in case the receiver pressure exceeds the normal limit.

Various other arrangements which may be the combination of above methods are used in practice. When compressors are driven by prime movers like steam engines which can run on variable speed, the delivery can also be controlled by changing the speed of pr[illegible] mover.

10.12. Compressor Performance. The performance of an air compressor is usually given in terms of delivery pressure, horsepower, volumetric efficiency and capacity. Fig. 10.8 shows the typical performance curves of a water cooled single-stage reciprocating air compressor.

The time required to reach the steady pressure in intercooler and receiver is an indication of the condition of L.P. and H.P. cylinder valves. If the time taken is more than normal, valves are not in perfect condition and need overhaul.

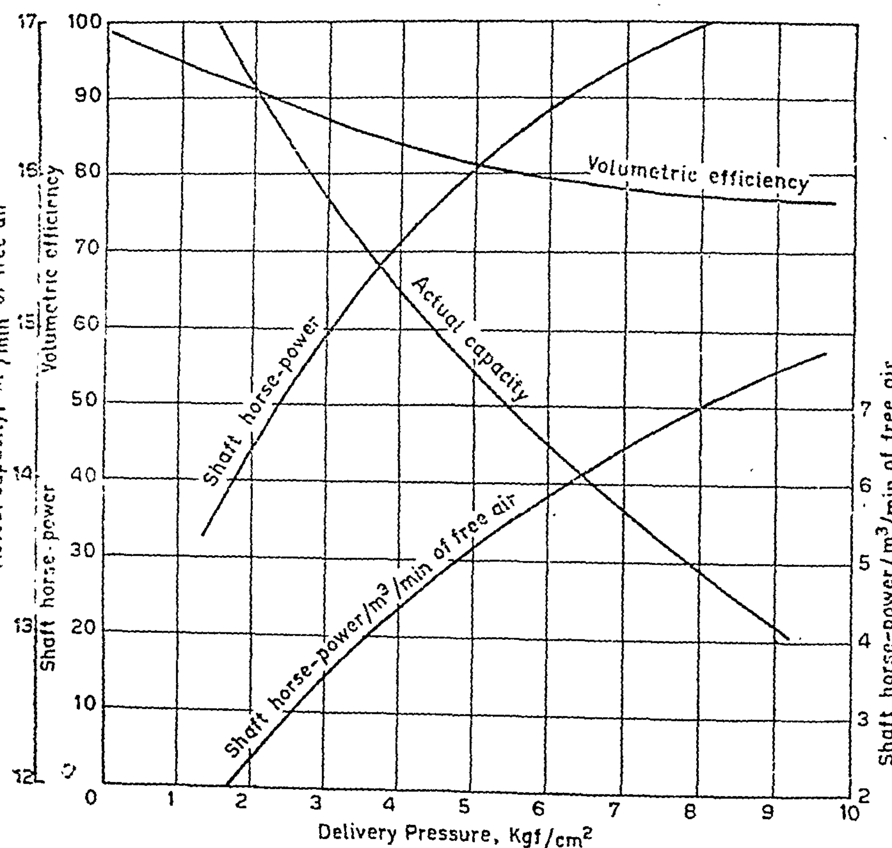


Fig. 10 8. Typical performance curves of a water cooled single-stage reciprocating air compressor.

10·13. Compressors in Mines. Compressed air is used in mining for locomotive, hoisting, pumps, motor generators, coal-cutters, pneumatic drills, picks,s pades, etc. The advantage of the compressed air is that it completely eliminates the fire hazard. In mines compressed air serves two purposes, firstly as a source of power and secondly in supplying air for ventilation.

Explosion in Air Receivers. Sometimes lubricating oil collects in air receivers and under the influence of high pressure and temperature carbon separates out from the oil. Carbon may also collect in the receivers of air compressors installed in mines and boiler-houses as they draw-in finely divided coal dust along with air into the cylinder which finds its way into the pipe line and the receiver. Under such

conditions, if the compressor is delivering high temperature air, explosion is likely to occur. Fusible plugs are provided in air receivers and pipe lines to safeguard against explosion.

Poisoning in Mines. When the compressor is badly chocked up with carbon and the delivery valves are leaking, or when there is an insufficient cooling water supply, air attains the critical temperature. This may cause "cracking" of the lubricant and formation of carbon monoxide, which would be carried to the motors at the coal face and would cause carbon monoxide poisoning in mines. Generally this is accompanied by an explosion in the air receiver due to the ignition of oil and carbonaceous deposits.

Typical Installation. Fig. 10·9 shows the typical installation of a compresed air plant for mines.

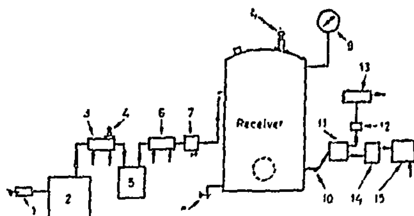


Fig 10 9. Typical installation of a compressed air plant.

INDEX

1. Intake filter and silencer
2. L P. cylinder
3. Intercooler
4. Safety valve
5. H.P. cylinder
6. Aftercooler
7. Condensate trap
8. Drain cock
9. *Pressure gauge*
10. *Fusible plug*
11. *Delivery filter and* [illegible]
12. *Small filters*
13. *Tools*
14. *Preheater*
15. *Motor*

IMPORTANT POINTS

(*i*) In engines mechanical efficiency = $\frac{bhp}{ihp}$ [illegible]

pressors bhp or shaft hp is the input and ihp or air hp is the output ; hence

$$\text{mechanical efficiency} = \frac{\text{ihp}}{\text{bhp}}.$$

(2) The pressure and temperature of air at the beginning of the compression stroke are not atmospheric. The pressure is slightly less than atmospheric due to head loss in obstruction and temperature is more than atmospheric due to fresh air passing over hot valves and mixing with residual air in the cylinder. But in problems, for simplicity, the conditions at point 1 Fig. 10.4, are assumed to be atmospheric unless otherwise stated.

Similarly throughout suction and delivery, temperatures and pressures are assumed constant.

(3) In problems involving clearance, when the index of expansion is not given it is assumed to be the same as that for compression. This assumption has been utilised in the derivation of formula (10.7).

(4) The rate of delivery of compression is generally in m^3 per minute. Hence for applying formula (10.1) or (10.2) corresponding value of v_1 by Boyle's and Charles' law should be found.

(5) While calculating total volume when stroke volume and r.p.m. are given care must be taken to multiply v_s by r.p.m. for a single-acting compressor and by $2 \times$r.p.m. for a double-acting compressor.

(6) Double-acting compressors are usually assumed to have identical indicator diagrams for both sides of piston unless otherwise stated.

(7) The term volumetric efficiency is generally involved in problems having clearance. It should be noted carefully whether the volumetric efficiency is required on F.A.D. basis or N.T.P. basis.

(8) When clearance is taken into account the mass of air delivered per cycle is $(m_1 - m_3)$.

(9) In multi-stage compressors work done in two cylinders is equal only when there is perfect intercooling ; otherwise work done in two cylinders should be evaluated separately.

(10) The total work done in a compressor is equal to heat taken by air + heat taken by cooling medium + radiation and other losses.

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ILLUSTRATIVE EXAMPLES

10.1. Effect of the law of compression on m.e.p and hp ; isothermal efficiency.

A single-stage single-acting air compressor 30 cm diameter, 40 cm stroke makes 100 r.p.m. It takes in air at 1 kgf/cm² and 20°C and compresses it to a pressure of 5 kgf/cm². Find the mean effective pressure and the theoretical horse-power required to drive it when the compression is

(a) *Isothermal,*
(b) *$PV^{1.2} = C$, and*
(c) *Adiabatic.*

Calculate the isothermal efficiency for the cases (b) and (c). Neglect clearance and take $R = 29.27$ kgf m/kg °K.

Why isothermal compression is not possible in actual practice ?

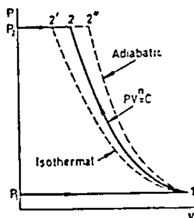


Fig. 10.10

Volume of air compressed per minute $= \frac{\pi}{4} d^2 l \cdot N$

$$= \frac{\pi}{4} \times \left(\frac{30}{100}\right)^2 \times \frac{40}{100} \times 100 = 2{\cdot}828\ m^3$$

Pressure ratio in compression $= \frac{P_2}{P_1} = 5$

(a) *Isothermal compression.*

$$\text{mep} = \frac{\text{area of indicator diagram}}{\text{stroke volume}} = \frac{\text{work done per cycle}}{\text{stroke volume}}$$

$$= \frac{P_1 V_1 \log_e r}{V_1} = 1 \times \log_e 5 = \underline{1{\cdot}61\ \text{kgf/cm}^2} \qquad \text{Ans.}$$

$$\text{hp} = \frac{\text{mep} \times \text{vol/minute}}{4,500}$$

$$= \frac{1{\cdot}61 \times 10^4 \times 2{\cdot}828}{4,500} = 10{\cdot}12 \qquad \text{Ans.}$$

(b) *Compression follows the law* $PV^{1.2} = C$

$$\text{mep} = \frac{\frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]}{V_1}$$

$$= \frac{1{\cdot}2}{1{\cdot}2 - 1} \times 1 \left[5^{\frac{1\cdot2-1}{1\cdot2}} - 1 \right] = 1{\cdot}848 \text{ kgf/cm}^2 \qquad \text{Ans.}$$

$$\text{hp} = \frac{1{\cdot}848 \times 10^4 \times 2{\cdot}828}{4,500} = 11{\cdot}61 \qquad \text{Ans.}$$

(c) *Adiabatic compression*

$$\text{mep} = \frac{\frac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{V_1}$$

$$= \frac{1{\cdot}4}{1{\cdot}4 - 1} \times 1 \left[5^{\frac{1\cdot4-1}{1\cdot4}} - 1 \right] = 2{\cdot}044 \text{ kgf/cm}^2 \qquad \text{Ans.}$$

$$\text{hp} = \frac{2{\cdot}044 \times 10^4 \times 2{\cdot}828}{4,500} = 12{\cdot}84 \qquad \text{Ans.}$$

$$\text{Isothermal efficiency} = \frac{\text{isothermal work required}}{\text{actual work done}}$$

$$\text{Isothermal efficiency, case (b)} = \frac{10{\cdot}12}{11{\cdot}61} = 87{\cdot}2\% \qquad \text{Ans.}$$

$$\text{Isothermal efficiency, case (c)} = \frac{10{\cdot}12}{12{\cdot}84} = 78{\cdot}8\% \qquad \text{Ans.}$$

In isothermal compression, all the heat of compression must be removed so that the temperature remains constant. To achieve this, compression should be extremely slow and the cooling should be perfect, which is not possible in actual practice. Very slow speed would make a compressor bulky.

Note. The work done increases with the index 'n'. It is minimum in isothermal compression and is maximum in adiabatic compression.

10·2. Cylinder dimensions taking volume and pressure losses ; volumetric efficiency ; adiabatic efficiency.

Prove that the volumetric efficiency of a compressor considering

clearance is given by $\frac{P_1T_a}{P_aT_1}\left[1+c-c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$ *where suffix a and 1 represent ambient and before compression conditions respectively and c is the ratio of clearance volume to stroke volume.*

Design a double-acting air compressor to compress 3 m³ of free air per minute at 1·04 kgf/cm² and 20°C to 8·5 kgf/cm² given the following data rev/min=300, mechanical efficiency=0·9 ; pressure loss in passing through intake valves=0·04 kgf/cm² ; pressure loss in passing through discharge valves =0·07 ; kgf/cm² ; clearance volume=4% ; rise in air temperature during suction stroke=10°C ; law of compression $PV^{1\cdot35}=C$; *length of stroke =1·2×diameter.*

What will be the power input ?

For theory—see text.

Pressure at 2, P_2=pressure of air required+pressure loss in discharge valve

$$=8\cdot5+0\cdot07=8\cdot57 \text{ kgf/cm}^2$$

Pressure at suction, P_1 = Atmospheric pressure—pressure loss in intake valve

$$=1\cdot04-0\cdot04=1 \text{ kgf/cm}^2$$

$$\frac{P_aV_a}{T_a}=\frac{P_1V_1}{T_1},\quad \frac{1\cdot04\times3}{293}=\frac{1\times V_1}{303}\ \therefore\ V_1=3\cdot226 \text{ m}^3$$

$$\text{Actual ihp}=\frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right]\times\frac{1}{4{,}500}$$

$$=\frac{1\cdot35}{1\cdot35-1}\times1\times10^4\times3\cdot226\left[\left(\frac{8\cdot57}{1}\right)^{\frac{1\cdot35-1}{1\cdot35}}-1\right]\times\frac{1}{4{,}500}=20\cdot63$$

$\therefore$ Power input (shaft h.p.)= $\frac{20\cdot63}{0\cdot9}$ =22·92 h.p. Ans.

Theoretical piston displacement per stroke

$$=\frac{3}{2\times300}=0\cdot005 \text{ m}^3$$

$$\text{Volumetric efficiency}=\frac{P_1T_a}{P_aT_1}\left[1+c-c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$=\frac{1\times293}{1\cdot04\times303}\left[1+0\cdot04-0\cdot04\left(\frac{8\cdot57}{1}\right)^{\frac{1}{1\cdot3}}\right]$$

$$=0\cdot7845$$

Actual displacement volume (stroke volume)

$$=\frac{\text{theoretical displacement volume}}{\text{volumetric efficiency}}$$

$$=\frac{0{\cdot}005}{0{\cdot}7845}=0{\cdot}006374 \text{ m}^3$$

Also $\frac{\pi}{4}d^2l=0{\cdot}006374$ [$l=1{\cdot}2d$

$\therefore$ $d=18{\cdot}9$ cm, $l=22{\cdot}7$ cm Ans.

10·3. Effect of clearance on free air delivery and output.

What is the effect of excessive clearance on the performance of an air compressor? Show that the cylinder clearance does not effect the theoretical work needed to compress and deliver 1 kg of air, provided that the delivery and suction pressures remain constant and that the indices of compression and expansion have the same value.

An air compressor has a stroke of 96 cm and a clearance equal to 3 per cent of stroke volume. It compresses air to a pressure of 10 kgf/cm². This air compressor was overhauled; during refitting a distance piece of 1 cm thick which was originally inserted between the cylinder head and cylinder was omitted. The air compressor was then worked under this condition.

Determine (a) the percentage change in free air delivered, and (b) the percentage change in h.p. necessary to drive the compressor.

Assume the index of compression as 1·3 and atmospheric pressure as 1 kgf/cm²

Due to clearance, volume of air sucked per stroke is equal to the cylinder volume minus the volume occupied by the residual air. This reduces the mass of air dealt within each cycle and therefore reduces the volumetric efficiency. The effect increases with increasing pressure ratio. There will be no change in work done per kg of air.

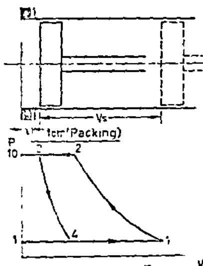
By removing the packing piece the clearance is reduced. As the stroke volume remains same in both the cases, total cylinder volume in the second case is reduced (*see* Fig. 10·12).

Let the area of piston be A cm²

Stroke volume, $V_s=Al=96\ A$ cc

Before overhauling, (*see* Fig. 10·11)

Clearance volume, $V_3=0{\cdot}03\times 96\ A=2{\cdot}88\ A$



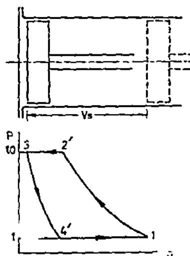


Fig 10.11. Clearance with packing   Fig. 10 12. Clearance without packing

Cylinder volume, $V_1 = V_s + V_3 = 98{\cdot}88\ A$

$$P_3 V_3^n = P_4 V_4^n$$

$$10 \times (A \times 2{\cdot}88)^{1{\cdot}3} = 1 \times V_4^{1{\cdot}3} \quad \therefore \ V_4 = 16{\cdot}92\ A$$

Volume of free air sucked $= (98{\cdot}88 - 16{\cdot}92)A = 81{\cdot}96A$

After overhauling, (*see* Fig. 10 12)

Clearance volume, $V_3' = (2{\cdot}88 - 1)A = 1{\cdot}88\ A$

Cylinder volume, $V_1 = V_s + V_3' = 97{\cdot}88\ A$

$$P_3 V_3'^n = P_4 V_4'^n$$

$$10 \times (1{\cdot}88A)^{1{\cdot}3} = 1 \times V_4'^{1{\cdot}3} \qquad \therefore \ V_4' = 11{\cdot}05\ A$$

Volume of free air sucked $= (97{\cdot}88 - 11{\cdot}05)\ A = 86{\cdot}33\ A$

(*a*) Percentage change in free air delivered

$$= \frac{86{\cdot}33 - 81{\cdot}96}{81{\cdot}96} = 5{\cdot}94\% \qquad \text{Ans.}$$

(*b*) $$\text{h.p.} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] \times K = K_1 V$$

It is seen that the h.p is directly proportional to the free air delivered, hence increase in the h p. will be same as increase in free air delivered.

$\therefore$ Percentage change in h p. $= 5{\cdot}94$ Ans.

Note.—(*i*) The problem illustrates the effect of the clearance volume on free air delivery. With less clearance volume of air delivery is more. It is because less the clearance volume, less is the

volume of residual air after expansion which allows more free air to be sucked.

(*ii*) Horse-power required per kg of air delivered is independent of clearance provided (*a*) the expansion index is same as compression index, and (*b*) the suction and delivery processes are at constant temperature and pressure.

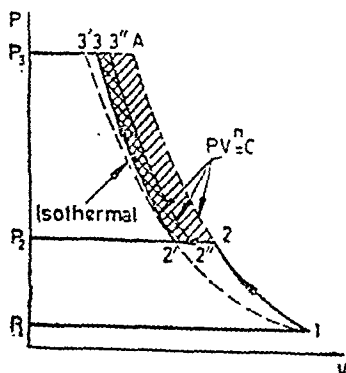
10.4. Single and multi-stage compression : h.p. ; mass of circulating water ; perfect and imperfect intercooling.

Under what circumstances would you recommend the use of a multi-stage compressor ? Explain the underlying principles by means of P-V and T-s diagrams.

Estimate the horse-power required to drive a single-stage, single-acting air compressor to compress 8 m³ of free air per minute at 1 kgf/cm² and 20°C to 7 kgf/cm². The index of compression is 1·3.

Also determine the percentage saving in the indicated horse-power by compressing the same mass of air (a) in two stages, with optimum intercooler pressure and perfect intercooling, (b) in two stages, with imperfect intercooling to 27°C, intercooler pressure remaining same as in the case (a), and (c) in three stages, with optimum intercooler pressure and perfect intercooling.

Aseume for air $C_p = 0.238$, $R = 29.27$ *kgf m/kg°K*



Work saved in two-stage compression with perfect intercooling = Area 22′3A

Work saved in two-stage compression with imperfect intercooling = Area 22″3″A

Fig. 10.13. Two-stage compression with perfect and imperfect intercooling.

For theory—See text.

hp required in single stage compression

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \times \frac{1}{4{,}500}$$

$$=\frac{1 \cdot 3}{1 \cdot 3-1} \times 1 \times 10^4 \times 8 \times \left[\left(\frac{7}{1}\right)^{\frac{1 \cdot 3-1}{1 \cdot 3}}-1\right] \times \frac{1}{4{,}500}$$

$=\underline{43 \cdot 7}$ Ans.

(*a*) Mass of air compressed/minute,

$$m=\frac{PV}{RT}=\frac{1 \times 10^4 \times 8}{29 \cdot 27 \times 293}=9 \cdot 33 \text{ kg}$$

hp required in two-stage compression with perfect intercooling

$$=2 \times \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1}\right)^{\frac{n-1}{2n}}-1\right] \times \frac{1}{4{,}500}$$

$$2 \times \frac{1 \cdot 3}{1 \cdot 3-1} \times 1 \times 10^4 \times 8\left[\left(\frac{7}{1}\right)^{\frac{1 \cdot 3-1}{2 \cdot 6}}-1\right] \times \frac{1}{4{,}500}$$

$=38 \cdot 7$

$\therefore$ Percentage saving in hp $=\frac{43 \cdot 7-38 \cdot 7}{43 \cdot 7}=11 \cdot 5\%$ Ans.

(*b*) Since limits of pressure and compression index are same, hp of low pressure cylinder will remain same as in (*a*)

$$=\frac{38 \cdot 7}{2}=19 \cdot 35 \text{ hp}$$

Intercooler pressure $=\sqrt{7 \times 1}=2 \cdot 646$ kgf/cm²

Volume of air entering hp cylinder

$$V_2''=\frac{P_1 V_1}{T_1} \times \frac{T_2''}{P_2''}=\frac{1 \times 8 \times 300}{293 \times 2 \cdot 646}=3 \cdot 095 \ m^3$$

hp required for compression in high pressure cylinder

$$=\frac{1 \cdot 3}{1 \cdot 3-1} \times 2 \cdot 646 \times 10^4 \times 3 \cdot 095\left[\left(2 \cdot 646\right)^{\frac{1 \cdot 3-1}{1 \cdot 3}}-1\right] \times \frac{1}{4{,}500}$$

$=19 \cdot 8$

Total hp required $=19 \cdot 35+19 \cdot 8=39 \cdot 15$

$\therefore$ Percentage saving in hp $=\frac{43 \cdot 7-39 \cdot 15}{43 \cdot 7}=10 \cdot 43\%$ Ans.

(*c*) hp required for three-stage compression with perfect intercooling

$$=3 \times \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_4}{P_1}\right)^{\frac{n-1}{3n}}-1\right] \times \frac{1}{4{,}500}$$

$$=3 \times \frac{1 \cdot 3}{1 \cdot 3-1} \times 1 \times 10^4 \times 8 \times\left[(7)^{\frac{1 \cdot 3-1}{3 \times 1 \cdot 3}}-1\right] \times \frac{1}{4{,}500}$$

$=37 \cdot 2$

$\therefore$ Percentage saving in hp$=\dfrac{43 \cdot 7-37 \cdot 2}{43 \cdot 7}=14 \cdot 87\%$ Ans.

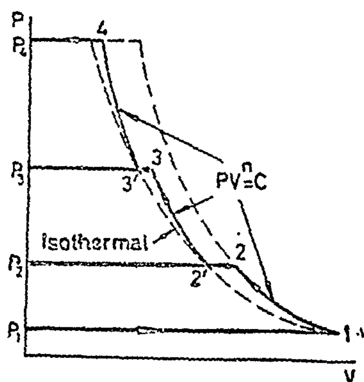


Fig. 10·14. 3-stage compression with perfect intercooling.

Note:. (*i*) The horse-power to drive a compressor of a given capacity decreases with the increase in stages as the air is cooled between the stages. This economy is maximum with perfect intercooling.

(*ii*) With perfect intercooling the work done in each stage is same.

10·5. Four-stage compressor with perfect intercooling : work done ; cooling of receiver.

What is the benefit of using an aftercooler with an air compressor when air under pressure has to be stored over long periods ?

The power cylinder of a naval tarpedo of volume 0·5m³ is charged with compressed air without aftercooling it at 170 kgf/cm² from a four-stage compressor with perfect intercooling between stages and working in the best conditions. What are the most economical intermediate pressures ?

What will be the air pressure in the tarpedo cylinder after long storage when air cools down to initial temperature ? What is the approximate work of charging the cylinder ? Take the index of compression as 1·3, initial pressure and temperature as 1 kgf/cm² and 20°C respectively.

Aftercooler is normally used to cool the compressed air to remove moisture and oil from it before it enters the receiver. In the case of air compressors where air is stored for a long time it serves yet another purpose *i.e.* it avoids large pressure drop in the receiver as there is little further cooling of air in it.

Pressure ratio in each stage for perfect intercooling

$$=\sqrt[4]{\frac{170}{1}}=3{\cdot}611$$

∴ Most economical intermediate pressures are
$P_2=3{\cdot}611$ kgf/cm², $P_3=13{\cdot}04$ kgf/cm², and $P_4=47{\cdot}08$ kgf/cm² Ans.

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}=293\left(3{\cdot}611\right)^{\frac{1\cdot3-1}{1\cdot3}}=394{\cdot}1°\text{K}=T_5$$

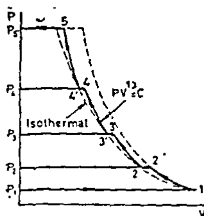


Fig. 10.15. Four-stage compression with perfect intercooling.

The cooling in the tarpedo cylinder is at constant volume.

Let the pressure after cooling be P kgf/cm²

$$P=\frac{P_5}{T_5}\times T_{initial}=\frac{170}{394{\cdot}1}\times293=126{\cdot}1\text{ kgf/cm}^2$$ Ans.

Mass to be charged

$$m_e=\frac{PV}{RT}=\frac{170\times10^4\times0{\cdot}5}{29{\cdot}27\times394{\cdot}1}=73{\cdot}7\text{ kg}$$

Work of charging the cylinder

$$=\frac{n}{n-1}mRT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right]\times N\qquad[N=\text{No. of stages}]$$

$$=\frac{1{\cdot}3}{1{\cdot}3-1}\times73{\cdot}7\times29{\cdot}27\times293\left[\left(3{\cdot}611\right)^{\frac{1\cdot3-1}{1\cdot3}}-1\right]\times4$$

$=37{,}65{,}000$ kgf m Ans.

10·6. Multi-stage compressor : number of stages ; W.D. ; heat rejected to intercooler.

A multi-stage compressor, with intercoolers, takes in air at 1·05 kgf/cm² and 15°C and delivers it at 130 kgf/cm². The compression in each stage may be assumed to obey a law $PV^{1·3}$=constant. For safety reasons it has been decided that the air temperature should not exceed 115°C at any point in the compression. Previous experience suggests that it is uneconomical to attempt to cool the air passing through the intercoolers below a temperature of 43°C. Determine :

(a) the number of stages that are necessary ;

(b) the work done per kg of air (neglecting the effects of clearance) ;

(c) the heat rejected to the intercoolers per kg of air.

For air $R=29·27$ kgf m/kg °K and $C_v=0·17$ kcal/kg °K.

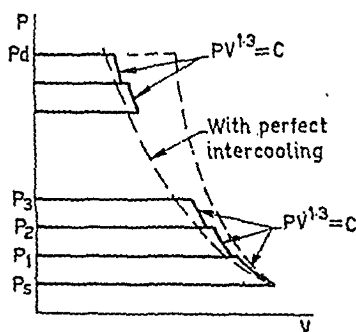


Fig. 10·16.

(a) Suction pressure, $P_s=1·05$ kgf/cm²

and $$T_s=273+15=288°K$$

Maximum discharge temperature in any stage

$$=273+115=388°K$$

Minimum temperature of compressed air leaving intercoolers

$$=273+43=316°K$$

=minimum temperature of entering air in 2nd, 3rd,...nth stages.

Final delivery pressure,

$$P_d=130 \text{ kgf/cm}^2$$

Pressure ratio in 1st stage,

$$\frac{P_1}{P_s}=\left(\frac{388}{288}\right)^{\frac{1.3}{1.3-1}}=3{\cdot}637$$

$\therefore$ $P_1=1.05\times3{\cdot}637=3{\cdot}8$ kgf/cm² (say)

Since temperature of air leaving the intercoolers is limited to 316°K, so the suction temperature of all the subsequent stages will be 316°K and

$$\frac{P_d}{P_1}=(\text{Temperature ratio})^{x\times\frac{1.3}{1.3-1}} \quad [x=\text{no. of subsequent stages}]$$

$$\frac{130}{3{\cdot}8}=\left(\frac{388}{316}\right)^{x\times\frac{1.3}{1.3-1}}$$

$\therefore$ $x=3{\cdot}98$ or say 4

$\therefore$ No of stages $=4+1=5$ Ans.

(b) Pressure ratio in 1st stage $=\dfrac{3{\cdot}8}{1{\cdot}05}$

Pressure ratio in subsequent stages $=\left(\dfrac{130}{3{\cdot}8}\right)^{1/4}=2{\cdot}42$

Temperature leaving 1st stage $=288\times\left(\dfrac{3{\cdot}8}{1{\cdot}05}\right)^{\frac{1.3-1}{1.3}}=387$°K

Temperature of air leaving subsequent stages

$$=316\times(2{\cdot}42)^{\frac{1.3-1}{1.3}}=386\text{°K}$$

Work done/kg of air $=\dfrac{n}{n-1}R(\triangle T)$

Work done in 1st stage $=\dfrac{1{\cdot}3}{1{\cdot}3-1}\times29{\cdot}27(387-288)$

$$=12{,}550 \text{ kgf m}$$

Work done in remaining 4 stages

$$=4\left[\frac{1\cdot3}{1\cdot3-1}\times29\cdot27\times\left(386-316\right)\right]=35,600 \text{ kgf m}$$

$\therefore$ Total work done/kg $=12,550+35,600=$ 48,150 kgf m Ans.

(c) $$C_p=\frac{R}{J}+C_v=\frac{29\cdot27}{427}+0\cdot17=0\cdot2385$$

Heat rejected in intercooler $=4[C_p\times\Delta T]$

$$=4\times0\cdot2385(386-316)=67 \text{ kcal/kg}$$ Ans.

10·7. Two-stage compressor : work done ; stability of inter-cooler pressure.

(a) *Explain the function of intercoolers in a multi-stage compressor.*

(b) *In a two-stage air compressor the air is sucked at 1 kgf/cm² and 20°C and is delivered at constant pressure of 14 kgf/cm². The pressure in intercooler is 4 kgf/cm² and the intercooling is perfect. The law of compression in both cylinders is $PV^{1\cdot3}=C$. The diameter of L.P. and H.P. cylinders are 10 cm and 5 cm respectively and their strokes are equal. Find the work done in compressing 1 kg of air.*

Assuming volumetric efficiency of 85 per cent find whether the intercooler pressure will remain steady or rise or fall as the compressor continues to work. $R=29\cdot27$ kgf m/kg °K.

(a) For theory—See text.

(b) When intercooling is perfect

$$W.D.=\frac{n}{n-1}\cdot RT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}+\left[\frac{P_3}{P_2}\right]^{\frac{n-1}{n}}-2\right]$$

$$=\frac{1\cdot3}{1\cdot3-1}\times29\cdot27\times293\left[\left(\frac{4}{1}\right)^{\frac{1\cdot3-1}{1\cdot3}}+\left(\frac{14}{4}\right)^{\frac{1\cdot3-1}{1\cdot3}}-2\right]$$

$=$ 26,500 kgf m Ans.

Since intercooling is perfect, point 2′ will lie on isothermal line 12′.

$$\therefore\ P_1V_1=P_2'V_2' \quad \text{or} \quad \frac{V_1}{V_2'}=4$$

Ratio of effective cylinder volumes

$$= \frac{\text{effective vol. of L P. cylinder}}{\text{vol. of H P. cylinder}}$$

$$= \frac{\frac{\pi}{4} \times 10^2 \times l \times 0.85}{\frac{\pi}{4} \times 5^2 \times l} = 3 \cdot 4$$

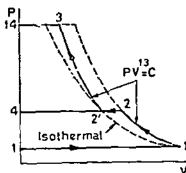


Fig. 10 17.

As the ratio of effective cylinder volumes is less than the ratio of the volumes obtained from P-V diagram, less air is supplied to the high-pressure cylinder than it can hold, therefore H P. cylinder would suck more air from intercooler than received from L. P. cylinder. Thus pressure in intercooler will fall. Ans.

Note. (*i*) As the intercooler pressure is not the geometric mean of P_1 and P_3, the work done in the two cylinders is different and the direct formula for work done, *i e.*

$$W.D. = 2 \times \frac{n}{n-1} mRT_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

cannot be applied.

(*ii*) The effective volume of L P cylinder neglecting clearance is actual volume $\times$ volumetric efficiency It applies to the L.P cylinder only as the air is sucked in this cylinder

10 8. Two-stage double acting compressor : hp.

Deduce an expression for the optimum value of the intercooler pressure in a two stage compressor, stating clearly the assumptions made. Explain why the low pressure cylinder pressure ratio will in practice normally exceed that in the high pressure cylinder.

Also prove that the heat rejected per stage per kg of air in a reciprocating compressor with perfect intercooling is given by

$$\left[C_p + C_v \frac{\gamma - n}{n-1} \right] \left(T_2 - T_1 \right).$$

The L.P. cylinder of a two-stage double-acting air compressor running at 120 rpm has 50 cm diameter and 75 cm stroke. It draws in air at a pressure of 1 kgf/cm² and 20°C and compresses it adiabatically to 3 kgf/cm². The air is then delivered to an intercooler where it is cooled at constant pressure to 35°C, and is then further compressed to 10 kgf/cm² in H.P. cylinder. Determine the required hp of an electric motor to drive the compressor. Assume the mechanical efficiency of the compressor 90 per cent and of the motor 86 per cent.

For theory—See text.

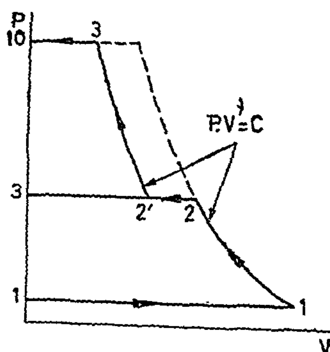


Fig. 10·18.

L.P. cylinder volume $= \frac{\pi}{4} \times (0{\cdot}5)^2 \times 0{\cdot}75 = 0{\cdot}1471$ m³

Mass of air compressed per stroke

$$m = \frac{PV}{RT} = \frac{1 \times 10^4 \times 0{\cdot}1471}{29{\cdot}27 \times 293} = 0{\cdot}1713 \text{ kg}$$

$$\text{hp} = \frac{\gamma}{\gamma - 1} mR \left[T_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \right.$$

$$\left. + T_2' \left\{ \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \right] \times \frac{2N}{4{,}500}$$

$$=\frac{1\cdot4}{1\cdot4-1}\times0\cdot1713\times29\cdot27\left[293\left\{\left(3\right)^{\frac{1\cdot4-1}{1\cdot4}}-1\right\}\right.$$

$$\left.+308\left\{\left(\frac{10}{3}\right)^{\frac{1\cdot4-1}{1\cdot4}}-1\right\}\right]\times\frac{2\times120}{4{,}500}$$

$=220.$

$$\text{Actual } \underline{\text{hp}}=\frac{220}{0\cdot9\times0\cdot86}=\underline{284} \qquad \text{Ans.}$$

10 9. Two-stage air compressor : η_{vol} ; vol. of air ; hp of motor.

A single-acting two-stage air compressor deals with air measured at atmospheric conditions of 1·03 kgf/cm² and 15°C. At suction the pressure is 1 kgf/cm² and the temperature is 30°C. The final delivery pressure is 17 kgf/cm², the interstage pressure is 4 kgf/cm² and perfect intercooling is to be assumed. If the LP cylinder bore is 23 cm, the common stroke is 15 cm and the speed of the compressor is 350 rev/min, calculate (a) the volumetric efficiency of the compressor, (b) the volume of atmospheric air dealt with per minute and (c) the horse-power of the driving motor required Assume, the clearance volume of LP cylinder to be 5% and the indices of compression and expansion in the LP and HP cylinder to be 1·25. The mechanical efficiency being 85%.

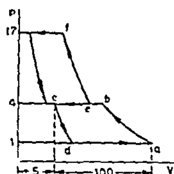


Fig. 10 19

(*a*) LP cylinder stroke volume

$$=\frac{\pi}{4}(0\cdot23)^2\times0\cdot15=0\cdot006225 \text{ m}^3$$

$$V_4=4^{\frac{1}{1\cdot25}}\times5=15\cdot15$$

$\therefore$ Volume inhaled at 1 kgf/cm² and 303°K

$$=105-15{\cdot}15=89{\cdot}85 \text{ m}^3$$

Equivalent atmospheric volume (at 1·03 kgf/cm² and 288° K)

$$=\frac{89{\cdot}85\times 1}{303}\times\frac{288}{1{\cdot}03}=82{\cdot}9$$

$\therefore$ Volumetric efficiency $=\frac{82{\cdot}9}{100}=82{\cdot}9\%$ Ans.

(b) LP cylinder stroke volume per minute

$$=0{\cdot}006225\times 350=2{\cdot}18 \text{ m}^3/\text{min}$$

$\therefore$ Volume of atmospheric air dealt/min

$$=0{\cdot}829\times 2{\cdot}18=1{\cdot}81 \text{ m}^3/\text{min}$$ Ans.

(c) $$\text{W.D.}=\frac{n}{n-1}P_1V_1\left(R^{\frac{n-1}{n}}-1\right)=\frac{n}{n-1}mR(T_2-T_1)$$

Mass of air dealt per minute

$$=\frac{PV}{RT}=\frac{1{\cdot}03\times 10^4\times 1{\cdot}81}{29{\cdot}27\times 288}=2{\cdot}21 \text{ kg}$$

$$T_b=303\times(4)^{\frac{0{\cdot}25}{1{\cdot}25}}=303\times 1{\cdot}32=400°\text{K}$$

and $$T_f=303\times\left(\frac{17}{4}\right)^{1/5}=405°\text{K}$$

$$\therefore \text{Total W.D.}=\frac{1{\cdot}25}{0{\cdot}25}\times 2{\cdot}21\times 29{\cdot}27[(400-303)+(405-303)]$$

$$=5\times 2{\cdot}21\times 29{\cdot}27\times 199=64{,}500 \text{ kgfm/min}$$

Motor hp $=\frac{64{,}500}{4{,}500\times 0{\cdot}85}=16{\cdot}9$ Ans.

10·10. Heat rejected in intercooler ; HP cylinder dia. ; hp, given clearance and pressure drop in intercooler.

A two-stage double-acting air compressor, operating at 150 rpm, takes in air 1 kgf/cm² and a temperature of 27°C. The size of the LP cylinder is 36×38 cm ; the stroke of the HP cylinder is the same as that of the LP cylinder and the clearance of both cylinders is 4 per cent. The LP cylinder discharges the air at a pressure of 4 kgf/cm². The air passes through the intercooler, so that it enters the HP cylinder at 27°C and 3·8 kgf/cm² ; after which it is discharged from the compressor at 15·2 kgf/cm². The value of n in both cylinders is 1·3. Neglect the effect of the piston rods on the crank ends. $C_p=0{\cdot}24$, $R=29{\cdot}27$.

Find (a) the heat rejected in the intercooler, (b) the diameter of the HP cylinder, and (c) the horsepower required to drive the HP cylinder.

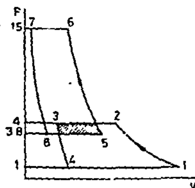


Fig. 10·20.

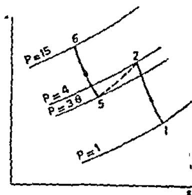


Fig. 10·21.

Swept volume of L.P. cylinder

$$= \frac{\pi}{4}\, d_1{}^2 l \times 2 \times (rpm)$$

$$= \frac{\pi}{4}\left(\frac{36}{100}\right)^2 \times \frac{38}{100} \times 2 \times 150$$

$$= 11{\cdot}6 \text{ m}^3\text{/min}$$

Volumetric efficiency referred to condition at 1

$$= 1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} \quad \left[\; c = \frac{\text{clearance volume}}{\text{stroke volume}}\right.$$

$$= 1 + 0{\cdot}04 - 0{\cdot}04 \times \left(\frac{4}{1}\right)^{\frac{1}{1\cdot3}} = 0{\cdot}924$$

$\therefore$ Volume of air sucked, referred to condition at 1

$$= 0{\cdot}924 \times 11{\cdot}6 = 10{\cdot}72 \text{ m}^3\text{/min}$$

$\therefore$ Mass of air per minute,

$$m = \frac{P_1 V_1}{RT_1} = \frac{1 \times 10^4 \times 10{\cdot}72}{29{\cdot}27 \times 300} = 12{\cdot}21 \text{ kg}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \quad \therefore \quad T_2 = 300\left(\frac{4}{1}\right)^{\frac{1\cdot3-1}{1\cdot3}} = 413^\circ\text{K}$$

(a) Heat rejected in the intercooler $= MC_p(T_2 - T_5)$

$$= 12{\cdot}21 \times 0{\cdot}24(413 - 300) = 331 \text{ K cal/min} \qquad \text{ns.}$$

(*b*) Volume of air drawn in HP cylinder per minute,

$$V_5 = \frac{mRT_5}{P_5} = \frac{12 \cdot 21 \times 29 \cdot 27 \times 300}{3 \cdot 8 \times 10^4} = 2 \cdot 821 \text{ m}^3/\text{min}$$

As the clearance percentage and the pressure ratio in the H.P. and the L.P. cylinders are the same, volumetric efficiency of both cylinders is same referred to condition at the beginning of compression.

∴ Swept volume of HP cylinder

$$= \frac{2 \cdot 821}{0 \cdot 924} = 3 \cdot 053 \text{ m}^3/\text{min}$$

or $$\frac{\pi}{4} \times \left(\frac{d_2}{100}\right)^2 \times \frac{38}{100} \times 2 \times 150 = 3 \cdot 053$$

∴ $d_2 = 18 \cdot 46$ cm Ans.

(*c*) As the initial temperature and pressure ratio in the L.P. and the H.P. cylinders is the same, $T_6 = T_2$

W.D. in the H.P. cylinder $= \frac{n}{n-1} mR(T_2 - T_1)$

Horse-power required for H.P. cylinder

$$= \frac{1 \cdot 3}{1 \cdot 3 - 1} \times \frac{12 \cdot 21 \times 29 \cdot 27(413 - 300)}{4{,}500} = 38 \cdot 9 \quad \text{Ans.}$$

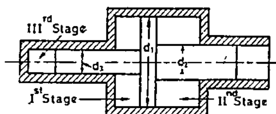
Note. The problem may be solved from fundamentals instead of applying the formula for volumetric efficiency.

10·11. Cylinder dimensions ; three-stage compressor with clearance

In a three-stage air compressor running at 240 rpm with the cylinders arranged as in the diagram below, 12 m³ of free air at 1·033 kgf/cm² and 15°C are to be compressed per minute. The pressure at suction in the L.P. cylinder is 1 kgf/cm², temperature 30°C and the final delivery pressure is 72 kgf/cm². The clearances are 4, 7 and 10 per cent of the displacement volumes for the L.P., I.P. and H.P. cylinders respectively. Assuming perfect intercooling, stage pressures in geometrical progression, $PV^{1.3}$ = constant for compression and expansion curves, and stroke = diameter of L.P. cylinder, calculate suitable diameters for the cylinders.

Mass of air per minute,

$$m=\frac{PV}{RT}=\frac{1{\cdot}033\times10^4\times12}{29{\cdot}27\times288}=14{\cdot}71\text{ kg}$$



$\therefore$ Mass of air per stroke $=\dfrac{14{\cdot}71}{240}=0{\cdot}0613$ kg

Pressure ratio of each cylinder $=\sqrt[3]{\dfrac{72}{1}}=4{\cdot}16$

$\therefore \quad P_2=4{\cdot}16$ kgf/cm², and $P_3=17{\cdot}31$ kgf/cm²

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}, \quad \frac{T_2}{303}=\left(4{\cdot}16\right)^{\frac{1\cdot3-1}{1\cdot3}} \quad \therefore T_2=421\ ^\circ\text{K}$$

As the cooling is perfect and the law of expansion is same in each cylinder,

Temperature after compression in every stage $=421$°K

First cylinder : Mass dealt per 100 m³ of swept volume,

$$M_1-M_3=\frac{P_1V_1}{RT_1}-\frac{P_3V_3}{RT_3}=\frac{10^4}{29{\cdot}27}\left[\frac{1\times104}{303}-\frac{4{\cdot}16\times4}{421}\right]=103{\cdot}8\text{ kg}$$

Since mass of air per stroke is 0·0613 kg

$$\therefore\ V_{s1}=\frac{100}{103{\cdot}8}\times0{\cdot}0613=0{\cdot}0591\text{ m}^3$$

Second cylinder : Mass dealt per 100 m³ of swept volume,

$$M_5-M_7=\frac{10^4}{29{\cdot}27}\left[\frac{4{\cdot}16\times107}{303}-\frac{17{\cdot}31\times7}{421}\right]=403{\cdot}5\text{ kg}$$

$$\therefore\quad V_{s2}=\frac{100}{403{\cdot}5}\times0{\cdot}0613=0{\cdot}0152\text{ m}^3$$

Third cylinder : Mass dealt per 100 m³ of swept volume,

$$M_9-M_{11}=\frac{10^4}{29{\cdot}27}\left[\frac{17{\cdot}31\times110}{303}-\frac{72\times10}{421}\right]=1{,}563\text{ kg}$$

$$\therefore \quad V_{s3} = \frac{100}{1{,}563} \times 0{\cdot}0613 = 0{\cdot}003922 \text{ m}^3$$

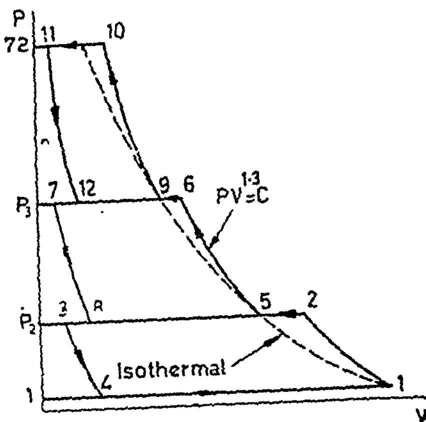


Fig. 10·22.

Now, $$\frac{\pi}{4}(d_1^2 - d_3^2) \times d_1 = 0{\cdot}0591 \quad (1)$$

$$\frac{\pi}{4}(d_1^2 - d_2^2) \times d_1 = 0{\cdot}0152 \quad (2)$$

and $$\frac{\pi}{4}(d_3^2) \times d_1 = 0{\cdot}003922 \quad (3)$$

Solving Eq. (1), (2) and (3), we get

$d_1 = 43{\cdot}12$ cm, $d_2 = 37{\cdot}5$ cm and $d_3 = 10{\cdot}75$ cm Ans.

Note. A three-stage compressor of this type is used for injection of fuel oil in air injection diesel engines.

10·12. Effect of ambient conditions on the output.

What is the effect of intake temperature and pressure on the output of compressors ?

A reciprocating air compressor takes in air at 40°C and 1·033 kgf/cm² in day time.

(a) Find the percentage increase of mass output in night, if night temperature is 10°C.

(b) If this compressor is shifted to a hill station where the barometric pressure is 0·92 kgf/cm², find the percentage decrease in output assuming the suction temperature to be same at two places.

(c) Determine the compression ratio of the compressor at two places if the law of compression is $PV^{1.3} = C$ and the delivery pressure is 7 kgf/cm² at both places.

Lower intake temperature increases the mass output due to the higher density of air

Lower intake pressure increases the mass output due to the lower density of air

For the same delivery pressure and the same law of compression the compression ratio is increased with lower intake pressure.

(a) $$m = \frac{PV}{RT}$$

As the volume of the cylinder and the barometric pressure remain same, $M \propto \frac{1}{T}$

At 313°K, $M_1 \propto \frac{1}{313}$, at 283°K, $M_2 \propto \frac{1}{283}$

<u>Percentage increase in mass output</u>

$$= \frac{M_2 - M_1}{M_1} = \frac{\frac{1}{283} - \frac{1}{313}}{\frac{1}{313}} = \underline{10.61\%} \qquad \text{Ans}$$

(b) $PV = mRT$

If temperature and volume are constant, $M \propto P$

At 1·033 kgf/cm², $M_1 \propto 1.033$, at 0.92 kgf/cm² $M_2 \propto 0.92$

<u>Percentage decrease in mass output</u>

$$= \frac{M_1 - M_2}{M_1} = \frac{1.033 - 0.92}{1.033} = \underline{10.94\%} \qquad \text{Ans}$$

(c) $$P_1 V_1^{1.3} = P_2 V_2^{1.3}$$

Compression ratio, $r = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{1.3}}$

<u>Compression ratio at first place</u> $\left(\frac{7 + 1.033}{1.033}\right)^{\frac{1}{1.3}}$

$$= \underline{4.84} \qquad \text{Ans}$$

$$\therefore \quad V_{s_3} = \frac{100}{1{,}563} \times 0{\cdot}0613 = 0{\cdot}003922 \text{ m}^3$$

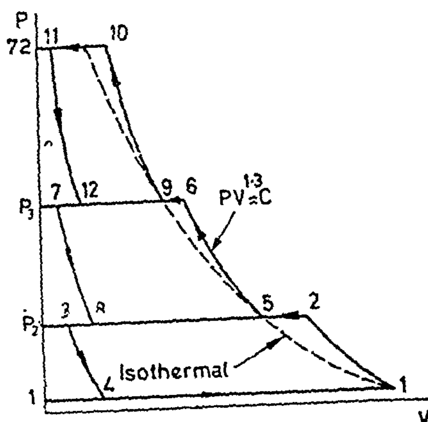


Fig. 10·22.

Now,
$$\frac{\pi}{4}(d_1^2 - d_3^2) \times d_1 = 0{\cdot}0591 \quad (1)$$

$$\frac{\pi}{4}(d_1^2 - d_2^2) \times d_1 = 0{\cdot}0152 \quad (2)$$

and
$$\frac{\pi}{4}(d_3^2) \times d_1 = 0{\cdot}003922 \quad (3)$$

Solving Eq. (1), (2) and (3), we get

$d_1 = 43{\cdot}12$ cm, $d_2 = 37{\cdot}5$ cm and $d_3 = 10{\cdot}75$ cm **Ans.**

Note. A three-stage compressor of this type is used for injection of fuel oil in air injection diesel engines.

10·12. Effect of ambient conditions on the output.

What is the effect of intake temperature and pressure on the output of compressors ?

A reciprocating air compressor takes in air at 40°C and 1·033 kgf/cm² in day time.

(a) Find the percentage increase of mass output in night, if night temperature is 10°C.

(b) If this compressor is shifted to a hill station where the barometric pressure is 0·92 kgf/cm², find the percentage decrease in output assuming the suction temperature to be same at two places.

Compression ratio on hill station $=\left(\frac{7+0{\cdot}92}{0{\cdot}92}\right)^{\frac{1}{1\cdot3}}$

$=5{\cdot}24$ Ans.

10·13. Air motor considering release and compression ; hp ; mass of air used.

Where air motors are used ?

Air enters an air motor of cylinder diameter 25 cm and stroke 40 cm, with a pressure of 6·3 kgf/cm² and temperature 32°C. The cut-off occurs at 0·4 of stroke and the expansion takes place according to the law $PV^{1\cdot25}$=constant. Exhaust takes place at constant pressure of 1·05 kgf/cm². The compression starts at 90 per cent of the return stroke and follows the law $PV^{1\cdot3}$=constant. The clearance volume is one-twentieth of the swept volume. Assuming rpm to be 150 and mechanical efficiency as 85 per cent, determine :

(a) the temperature of exhaust, (b) the bhp of the motor, (c) the mass of the air used per bhp-hr, and (d) the overall efficiency of the compressed air plant if the efficiency of the compressor itself is 65 per cent. Take γ=1·41 and R=29·27.

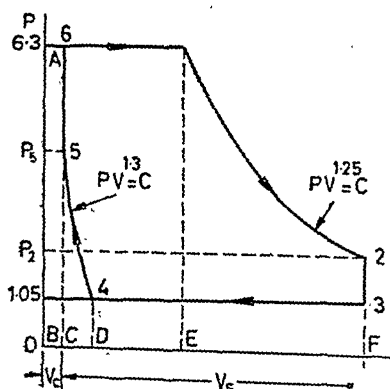


Fig. 10·23.

Stroke volume, $V_s=\frac{\pi}{4}(25)^2\times40=19{,}640$ cc

Clearance volume, $V_c=V_5=V_6=\frac{19{,}640}{20}=982$ cc

$\therefore$ Cylinder volume, $V_2=V_s+V_c=20{,}622$ cc

$V_1=V_6+0{\cdot}4V_s=982+0{\cdot}4\times19{,}640=8{,}840$ cc

$P_1V_1{}^n=P_2V_2{}^n$,

$6{\cdot}3\times8840^{1\cdot25}=P_2\times20{,}622^{1\cdot25}$ $\therefore$ $P_2=2{\cdot}185$ kgf/cm

$$V_4 = V_c + 0{\cdot}1V_s = 982 + 0{\cdot}1 \times 20{,}622 = 3044 \text{ cc}$$

$$P_4V_4^n = P_5V_5^n,$$

$$1{\cdot}05 \times 3044^{1{\cdot}3} = P_5 \times 982^{1{\cdot}3} \qquad \therefore \quad P_5 = 4{\cdot}57 \text{ kgf/cm}^2$$

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}, \ \frac{305}{T_2} = \left(\frac{6.3}{2{\cdot}185}\right)^{\frac{1{\cdot}25-1}{1{\cdot}25}} \quad \therefore \ T_2 = 247°\text{K}$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}, \ \frac{2{\cdot}185}{247} = \frac{1{\cdot}05}{T_3} \ \therefore \ T_3 = \underline{118{\cdot}6°\text{K}} \qquad \text{Ans.}$$

Work done per stroke = area 123456

= area $16CE$ + area $12FE$ − area $54DC$ − area $34DF$

$$\text{Area } 16CE = \frac{P_1(V_1 - V_6)}{J} = \frac{6{\cdot}3 \times 10^4 \times 0{\cdot}4 \times 19{,}640 \times 10^{-6}}{427}$$

$$= 1{\cdot}159 \text{ kcal}$$

$$\text{Area } 12FE = \frac{P_1V_1 - P_2V_2}{J(n-1)} = \frac{10^{-2}(6{\cdot}3 \times 8{,}840 - 2{\cdot}185 \times 20{,}622)}{427(1{\cdot}25 - 1)}$$

$$= 0{\cdot}985 \text{ kcal}$$

$$\text{Area } 54DC = \frac{P_5V_5 - P_4V_4}{J(n-1)} = \frac{10^{-2}(4{\cdot}57 \times 982 - 1{\cdot}05 \times 3{,}044)}{427(1{\cdot}3 - 1)}$$

$$= 0{\cdot}101 \text{ kcal}$$

$$\text{Area } 34DF = \frac{P_3(V_3 - V_4)}{J} = \frac{10^{-2} \times 1{\cdot}05(20{,}622 - 3{,}044)}{427}$$

$$= 0{\cdot}432 \text{ kcal}$$

$\therefore$ Work done per cycle $= 1{\cdot}159 + 0{\cdot}985 - 0{\cdot}101 - 0{\cdot}432$

$$= 1{\cdot}611 \text{ kcal}$$

(*b*) $\underline{\text{bhp}} = \dfrac{1{\cdot}611 \times 427 \times 150}{4{,}500} \times 0{\cdot}85 = \underline{19{\cdot}5}$ Ans.

(*c*) Mass entering the cylinder per stroke,

$$m = \frac{PV}{RT} = \frac{6{\cdot}3 \times 10^4 \times 0{\cdot}4 \times 19{,}640 \times 10^{-6}}{29{\cdot}27 \times 305} = 0{\cdot}0554 \text{ kg}$$

$\underline{\text{Air used per bhp-hr}} = \dfrac{0{\cdot}0544 \times 150 \times 60}{19{\cdot}5} = \underline{25 \text{ kg}}$ Ans.

(*d*) Heat interchange during the process 1-2

$$= \frac{\gamma - n}{\gamma - 1} \times W.D. = \frac{1{\cdot}41 - 1{\cdot}25}{1{\cdot}41 - 1} \times 1{\cdot}611 = 0{\cdot}63 \text{ kcal}$$

Heat interchange during the process 4-5

$$= \frac{1{\cdot}41 - 1{\cdot}3}{1{\cdot}41 - 1} \times 0{\cdot}101 = 0{\cdot}0271 \text{ kcal}$$

$$C_V = \frac{R}{J(\gamma-1)} = \frac{29{\cdot}27}{427(1{\cdot}41-1)} = 0{\cdot}1672$$

Total mass of air inside the cylinder,

$$m_2 = \frac{P_2V_2}{RT_2} = \frac{2{\cdot}185\times10^{-2}\times20{,}622}{29{\cdot}27\times247} = 0{\cdot}0624 \text{ kg}$$

Heat rejected during the process 2-3

$$= mC_V(T_2 - T_3) = 0{\cdot}0624\times0{\cdot}1672(247-118{\cdot}6)$$

$$= 1{\cdot}339 \text{ kcal}$$

Heat supplied = W.D. + Net heat rejected

$$= 1{\cdot}611 - 0{\cdot}63 + 0{\cdot}0271 + 1{\cdot}339 = 2{\cdot}347 \text{ kcal.}$$

$\therefore$ Overall efficiency $= \dfrac{1{\cdot}611\times0{\cdot}65}{2{\cdot}347} = 44{\cdot}6\%$ Ans.

Note. (*i*) The difference between the *P-V* diagram of an air compressor and an air motor should be noted. Release is at higher pressure than atmospheric pressure and there is compression. Such problem should always be attempted from fundamentals.

(*ii*) The exhaust temperature is too low (118·6°K or −154·4°C). This creates trouble in practice as the moisture in air freezes and clogs the valves. Therefore, compressed air is generally preheated before it enters the air motor. A greater part of the heat used in preheating is obtained back in the form of increased work done.

10·14. Air motor (non-expansive) ; hp ; air per bhp-hr.

If the motor used in Question 1-12 is non-expansive type, find

(*a*) *the precentage increase in the hp developed, and*

(*b*) *the percentage increase in the air required per bhp-hr.*

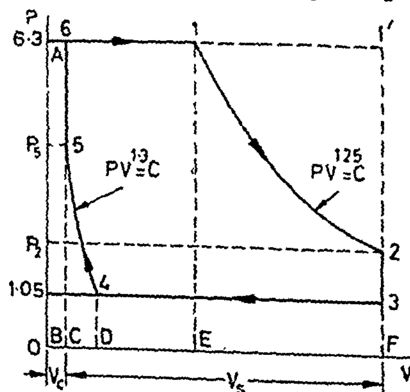


Fig. 10·24.

(a) Work done = Area $61'FC$ − Area $54DC$ − Area $34DF$

$$\text{Area } 61'FC = \frac{PV}{J} = \frac{6{\cdot}3\times10^4\times19640\times10^{-6}}{427} = 2\ 9 \text{ kcal}$$

∴ Work done per stroke = 2·9 − 0 101 − 0 432 = 2·367 kcal

$$\text{bhp} = \frac{2{\cdot}367\times427\times150}{4{,}500}\times0\ 85 = \underline{28{\cdot}6}$$

Percentage increase in bhp $= \dfrac{28\ 6-19{\cdot}5}{19{\cdot}5}\times100 = \underline{45{\cdot}6\%}$ Ans.

(b) Mass entering the cylinder per stroke,

$$m = \frac{PV}{RT} = \frac{6{\cdot}3\times10^4\times19{,}640\times10^{-6}}{29\ 27\times305} = \underline{0\ 1386 \text{ kg}}$$ Ans.

Air used per bhp-hr $= \dfrac{0\ 1386\times150\times60}{28\ 6} = 43\ 6$

Percentage increase in air used per bhp-hr

$$= \frac{43\ 6-25{\cdot}42}{25{\cdot}42} = \underline{71{\cdot}53\%}$$ Ans.

Note. bhp is increased when the motor works non-expansively but the air used per bhp-hr is also increased.

10·15. Measurement of air-flow in an air compressor.

Describe an arrangement for measuring the flow of air in an air compressor.

In a test to measure the air-flow in a compressor the following readings were obtained :—

Barometric pressure = 746 mm ; Room temperature = 24°C ; Orifice diameter = 14mm ; Coefficient of discharge for orifice = 0·6 ; Manometer reading = 147 mm. Determine the volume of free air dealt per minute by the compressor. R = 29 27 kgf m/kg °C.

Derive the formula used.

Flow of air per sec $= C_d A\sqrt{2g\ Hv}$

$$A = \frac{\pi}{4}\times\left(0{\cdot}014\right)^2 = 0\ 000154 \text{ m}^2$$

$$H = 0{\cdot}147\times970 = 142\ 5 \text{ kgf/m}^2$$

Atmospheric pressure

$$= \frac{10{,}332}{760}\times746 = 10{,}170 \text{ kgf/m}^2$$

Volume of air per kg,

$$v = \frac{RT}{P}$$

$$= \frac{29 \cdot 27 \times 297}{10{,}170} = 0 \cdot 856 \text{ m}^3/\text{kg}$$

Free air delivery $= C_d A \sqrt{2g\,Hv}$

$$= 0 \cdot 6 \times 0 \cdot 000154 \sqrt{2 \times 9 \cdot 81 \times 142 \cdot 5 \times 0 \cdot 856} \times 60$$

$= 0 \cdot 293$ m³/min or $0 \cdot 342$ kg/min Ans.

10·16. Two-stage compressor : volumetric efficiency by three methods.

Three methods were used to determine the volumetric efficiency of a two-stage, single-acting reciprocating air compressor running at 370 rev/min.

The first used a metering orifice mounted on a large capacity tank, the air being drawn through the orifice before entering the L.P. cylinder at 1·05 kgf/cm² pressure and 15°C. Using this method the flow was found to be 11·1 kg/min.

In the second method the temperature drop of the air through the intercooler was found to be 58°C. The mass flow rate of the water through the cooler was 10 kg/min with a temperature rise of 15°C.

The third method used a vessel of 3 m³ capacity into which the air could be passed from the compressor. The initial conditions in the vessel were 1·05 kgf/cm² and 15°C. The vessel was charged for four minutes, then isolated and allowed to cool to 15°C ; the pressure in the vessel was then 13·5 kgf/cm².

If the swept volume of L. P. cylinder is 0·0286 m³, determine the volumetric efficiency as given by each of the three methods and comment on their accuracy.

For air $C_p = 0 \cdot 238$, $C_v = 0 \cdot 170$.

(*a*) *Volumetric efficiency by orifice.*

Mass of air/min to fill the swept volume at intake condition,

$$m = \frac{PV}{RT} = \frac{1 \cdot 05 \times 10^4 \times 0 \cdot 0286 \times 370}{29 \cdot 27 \times 288} = 13 \cdot 19 \text{ kg}$$

$$\therefore \text{ Volumetric efficiency} = \frac{11 \cdot 1}{13 \cdot 19} = 84 \cdot 2\% \quad \text{Ans.}$$

(*b*) *Volumetric efficiency by heat transfer in intercooler.*

$(m \times C_p \times \triangle T)_{air} = (m \times C_p \times \triangle T)_{water}$

$$\therefore \quad m_{air} = \frac{10 \times 1 \times 15}{0.238 \times 58} = 10{\cdot}86 \text{ kg}$$

$$\text{Volumetric efficiency} = \frac{10.86}{13.19} = 82{\cdot}5\% \qquad \textbf{Ans.}$$

(*c*) *Volumetric efficiency by vessel on delivery side.*

Mass of air pumped in the vessel in 4 minutes

$$= \frac{P_2 V_2}{RT_2} - \frac{P_1 V_1}{RT_1} = \frac{V_2}{RT}(P_2 - P_1)$$

$$= \frac{3 \times 10^4}{29.28 \times 288}(13{\cdot}5 - 1{\cdot}05) = 44{\cdot}2 \text{ kg}$$

$$\therefore \quad \text{Volumetric efficiency} = \frac{44.2}{4 \times 13.19} = 84\% \qquad \textbf{Ans.}$$

10.17. Test on air compressor : F.A.D. ; $\eta_{vol.}$, η_{iso}, $\eta_{mech.}$

A compound air-compressor is connected to a receiver, previously emptied. Explain how the efficiency of working of the compressor valves may be checked, without taking indicator cards, during the process of charging the cylinder.

In a test of a single-cylinder single-acting air compressor of 10 cm bore and 12·5 cm stroke air is taken from the atmosphere at 1.02 kgf/cm² and delivered through a valve, which maintains a delivery pressure of 8 kgf/cm² to a reservoir of 1·2 m³ of capacity. A motor giving 2·4 bhp drives the compressor at 420 rpm and imep is 2.2 kgf/cm². The reservoir is initially at atmospheric pressure and 18°C and after 19 minutes running reaches 7 kgf/cm² and 63°C. Calculate the delivery in m³ of free air per minute at 18°C, the volumetric efficiency at atmospheric conditions, the isothermal efficiency and the mechanical efficiency.

For theory—see text.

$$\text{Swept volume} = \left(\frac{\pi}{4} \times 10^2\right) \times 12{\cdot}5 \times 10^{-6} = 982 \times 10^{-6} \text{ m}^3$$

The mass of air in the receiver, $m = \frac{PV}{RT}$,

and the mass delivered will be the difference between the final and original mass.

$\therefore$ Mass delivered in 19 minutes $= \frac{V}{R}\left[\frac{P_2}{T_2} - \frac{P_1}{T_1}\right]$

and volume delivered, $F.A.D. = \frac{T_1 V}{P_1}\left[\frac{P_2}{T_2} - \frac{P_1}{T_1}\right]$

$\therefore$ Volume delivered/min,

$$\text{F.A.D.} = \frac{291 \times 1{\cdot}2}{1{\cdot}02 \times 19}\left[\frac{7}{336} - \frac{1{\cdot}02}{291}\right]$$

$$= 0{\cdot}312 \text{ m}^3/\text{min.} \qquad \text{Ans.}$$

$$\text{Volume delivered/stroke} = \frac{0{\cdot}312}{420} = 0{\cdot}000744 \ m^3$$

$$\text{and volumetric efficiency} = \frac{0{\cdot}000744}{0{\cdot}000982} = 75{\cdot}6\% \qquad \text{Ans.}$$

$$\text{ihp} = \frac{p_m V_s N}{75 \times 60} = \frac{2{\cdot}2 \times 10^4 \times 982 \times 10^{-6} \times 420}{75 \times 60} = 2{\cdot}02 \text{ hp}$$

$$\text{Mechanical efficiency} = \frac{\text{ihp}}{\text{bhp}} = \frac{2{\cdot}02}{2{\cdot}4} = 84{\cdot}1\% \qquad \text{Ans.}$$

$$\text{Isothermal hp} = \frac{p_1 V_1}{75 \times 60} \log_e \frac{P_2}{P_1}$$

$$= \frac{1{\cdot}02 \times 10^4 \times 0{\cdot}312}{75 \times 60} \log_e \frac{8}{1{\cdot}02}$$

$$= 1{\cdot}458 \text{ hp.}$$

$$\therefore \text{Isothermal efficiency} = \frac{1{\cdot}458}{2{\cdot}02} = 72{\cdot}1\%. \qquad \text{Ans.}$$

10·18. Test on an air compressor.

The following data refers to a single-acting two-stage air compressor.

Diameter of L.P. cylinder = 9 cm ; Stroke 7·2 cm ; rpm = 880 ; Barometric pressure = 1 kgf/cm² ; Room temperature = 24°C ; Free air delivered = 0·28 m³/min ; Readings of the spring balances of the dynamometer type electric motor = 20 kg and 10 kg ; Arm radius of spring balance = 25 cm (see Fig. given below).

Calculate (a) the volumetric efficiency, and (b) the shaft hp pe m³ of free air per minute.

(c) Draw up a heat balance on one minute basis. Given th mechanical efficiency of compressor = 80 per cent, delivery temperature = 160°C and heat taken away by cooling water = 18 kcal/min. Take fo air, $C_p = 0{\cdot}24$ and $R = 29{\cdot}27$ kgf m/kg °C.

Sketch typical performance curves of an air compressor.

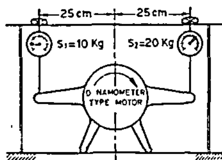


Fig 10 25. Dynamometer type electric motor

(*a*) Stroke volume of the L.P. cylinder

$$= \frac{\pi}{4} \times (9)^2 \times (7{\cdot}2) = 458 \text{ cc}$$

$\therefore$ Volumetric efficiency $= \dfrac{0{\cdot}28 \times 10^6}{458 \times 880} = \underline{69{\cdot}5\%}$. Ans

(*b*) Shaft hp $= \dfrac{2\pi r N T}{4500} = \dfrac{2\pi \times 25 \times 880(20-10)}{4{,}500} = 3{\cdot}07$

Shaft hp/m³ free air per min $= \dfrac{3{\cdot}073}{0{\cdot}28} = \underline{10{\cdot}97}$ Ans

(*c*) Heat equivalent to shaft hp

$$= \frac{3{\cdot}073 \times 4500}{427} = 32{\cdot}38 \text{ kcal/min}$$

Heat equivalent to ihp $= 32{\cdot}38 \times 0{\cdot}8 = 25{\cdot}9$ kcal/min

Mass of air per min.

$$m = \frac{PV}{RT} = \frac{1 \times 10^4 \times 0{\cdot}28}{29{\cdot}27 \times 297} = 0{\cdot}322 \text{ kg}$$

$$\text{Heat in air} = mC_p\Delta T = 0{\cdot}322 \times 0{\cdot}24(160-21)$$
$$= 10{\cdot}52 \text{ kcal/min}$$

Heat lost in friction = heat eq. to (shp − ihp)

= 32·38 − 25·9 = 6·48 kcal/min

Heat balance on one minute basis :

Credit	*Kcal*	%	*Debit*	*Kcal*	%
Input to compressor	32·38	100	ihp	25·9	80
			Friction	6·48	20
			Air	10·52	32·5
			Water jacket	18·00	55·6
			Radiation, convection losses, etc. (By difference)	3·86	11·9
Total	32·38	100		32·38	100·0

EXAMPLES 10

10·1. Volume of air per minute in receiver ; ihp.

Derive an expression for the work done in a single-stage compressor neglecting the effect of clearance (a) when compression is isothermal (b) when compression follows the law PV^n = constant.

Atmospheric air at 1 kgf/cm², and 20°C is taken into a simple compressor having zero clearance. It is compressed according to the law $PV^{1.2}$ = constant to the constant discharge pressure of 4 kgf/cm².

The discharge is taken through a regulating valve into a closed vessel of 3m³ capacity. Here the initial conditions were 1 kgf/cm² and 20°C and after charging for 4·2 minutes were 3·5 kgf/cm² and 25°C. Calculate, neglecting clearance of compressor :—

(a) *the volume of air taken in per minute if measured at atmospheric conditions ;*

(b) *the indicated horse-power required to drive the machine.*

[Mass of air = 2·032 kg/min ; ∴ vol. of air = 1·743 m^3/min ; ihp = 6·04]

10·2. Vacuum pump : ihp ; air per minute.

What is meant by brake horse-power and indicated horse-power in a reciprocating air compressor ? Define mechanical efficiency.

In a certain manufacturing process containers are to be hermetically sealed after partial evacuation of contained air. Average conditions in the working atmosphere are 1·02 kgf/cm² and 17°C. It is required that each container shall be evacuated to 62 cm of mercury (gauge) when the temperature inside the container has returned to 17°C. Assuming that the evacuation process is isentropic, determine the necessary induction pressure of the vacuum pump in cm of mercury (gauge).

If the capacity of each container is 1,000 cc and 2,000 containers are to be evacuated per hour, determine : (a) the mass of air extracted per minute ; and (b) the indicated horse-power of the vacuum pump, assuming that the induction temperature is 17°C and that compression takes place according to the law $PV^{1.28}$*=constant.*

For air $\gamma=1{\cdot}4$ *and* $R=29{\cdot}3$ *kgf m/kg°K.*

[Final pr.=0·0966 kgf/cm² ; induction pr.=68·9 cm Hg. ; mass extracted=0·0414 kg/min ; T_2=486°K ; ihp.=0·241].

10 3. Three-stage compressor ; cylinder diameter neglecting clearance.

What is the object of arranging for compressing air in stages with intermediate water-cooling down to the original atmospheric temperature? Give the usual rules for the values of the intermediate pressures in the case of (i) two-stage, (ii) three-stage compression, and explain the circumstances in which these rules hold.

A three-stage air compressor (with intercoolers) for a Diesel engine of the blast-injection type is to be driven from a crank on the main crankshaft running at 250 rpm. The compressor has to compress 30 m³ air per hour from atmospheric pressure (1 kgf/cm²) to 70 kgf/cm² and the mean piston speed is limited to 130 m/min. Neglecting clearance, obtain the principal dimensions for the compressor cylinders

[Common stroke=0·26 m ; F.A.D.=0·002 m³/stroke , pressure ratio in each cylinder=4·124 ; areas of cylinders ; L.P.=76·9 cm² ; I.P.=18·65 cm² ; H.P.=4·52 cm².]

10 4. Two-stage compressor with imperfect intercooling; hp, given all the efficiences.

Derive an expression for minimum work in terms of initial and final pressures and the volumes of air drawn in per stroke when compres-

sion is adiabatic, State the assumptions made. Hence derive an expression for 'q' stages.

In a two-stage double-acting air compressor running at 120 rpm the low pressure piston is of 75 cm diameter and has a stroke of 30 cm. Air is compressed adiabatically in the L.P. cylinder to 1·75 kgf/cm² gauge delivered to air intercooler where it is cooled at constant pressure to 27°C and then compressed to 6 kgf/cm² gauge in the high pressor cylinder.

Determine the required horse-power to drive the compressor if the mechanical efficiency of the compressor is 88 per cent and of motor 86 per cent.

Assume volumetric efficiency for the compressor 81·5 per cent and atmospheric pressure and temperature as 0·85 kgf/cm² and 20°C. $\gamma=1.4$

[Vol./L.P. stroke=Volumetric $\eta \times V_s = 0 \cdot 108$ m^3 ; ihp of L.P. cy.=64·7 ; Vol. per H.P. stroke=0·03616 m^3 ; ihp of H.P. cy.=55·8 bhp input to motor=(64·7+55·8)/0·88×0·86=159·2.]

10·5. Single-stage double-acting compressor with clearance ; Vs. ; ηvol.

Define volumetric efficiency of a compressor. Explain why the volumetric efficiency of an air compressor is less than unity.

A single-stage double-acting compressor running at 300 rev/min delivers 15 m³ of free air/min at 7 kgf/cm² and 200°C. At the beginning of compression the pressure and temperature of the air are 1 kgf/cm² and 20°C. If the clearance volume is 8 per cent of the swept volume and if the indices of compression and re-expansion are the same, find the swept volume of piston and the volumetric efficiency referred to :

(a) intake conditions, and

(b) free air conditions of 1·03 kgf/cm² and 15°C.

[$n=1 \cdot 33$; effective swept volume=0·0262 m³ ; $V_s=0 \cdot 0356$ m³ ; volumetric efficiency : referred to free air conditions=70·3% ; referred to take conditions=73·5%].

10·6. Cylinder dimensions considering clearance.

Discuss the effect of (a) clearance, (b) altitude on the working of an air compressor.

A single-acting single-stage air compressor is required to compress 2·7 kg of air per minute at 1 kgf/cm² and temperature 24°C to a pressure of 7 kgf/cm². The clearance volume is 6 per cent of the stroke volume and the index of both the expansion and compression curves is 1 3. If the stroke is 1·2 times the bore and the compressor runs at 100 rpm, determine the size of the cylinder. Take the equation for air as $PV=29{\cdot}27\ T$.

What is the net heat transferred per minute during compression and expansion strokes ? Take $\gamma=1{\cdot}41$.

[Let $V_s=100$; $V_4=26{\cdot}8$; $T_s=0\ 0296\ m^3$: $d=31{\cdot}5$ cm ; $l=37{\cdot}8$ cm ; mass, in compression=2 9045 kg/min ; in expansion =0·2045 kg/min ; net $q=27{\cdot}88$ kcal/kg]

10·7 Two-stage compressor with clearance : air taken in L.P. cylinder ; volume of H.P. cylinder.

Discuss concisely the reasons for the use of multi-stage compressor.

The piston displacement of the L P. cylinder of an air compressor is 0·06 m³. Assume that the clearance volume at the stroke is 10 per cent. Air is drawn in at a temperature of 15°C and compressed adiabatically to 2 kgf/cm² at which pressure it is quickly delivered to a receiver. Find the quantity of air taken into the L.P. cylinder per stroke when the compressor is working steadily. Obtain the volume of the H.P. cylinder if the air in the receiver is cooled to 15°C, the clearance volume being 10 per cent of the piston displacement. The final delivery pressure is 4 kgf/cm². Initial pressure 1 kgf/cm².

[Vol of air taken/stroke in L.P cy =0 066—0·0098=0·0562 m^3 ; vol. of compressed air delivered from L P. cy.=0·0281 m^3. Air drawn in H.P. cy./100 m^3 of stroke vol.=93 6 m^3 , H.P cy. vol.= 0 03 m^3]

10 8. Addition of H.P. cylinder : bore of H.P. cylinder.

A single-stage single-acting air compressor has a bore of 13 cm and a stroke volume of 12 cm. The clearance volume is 100 cc. With a suction pressure of 1 kgf/cm² and temperature 17°C it delivers to a pressure of 5 kgf/cm² at a speed of 110 rev/min. Taking the index of expansion and compression as 1·3, calculate the delivery in kg/min and the indicated horse-power.

If delivery pressure be increased to 25 kgf/cm² by the addition of a high pressure cylinder of the same stroke together with an intercooler which reduces the temperature to 40°C, calculate the required bore of the

H.P. cylinder, allowing a clearance equal to 6 per cent of the swept volume and assuming the index 1·3 For air $R=29{\cdot}27$ kgf-m/kg °K.

[$T_2=421$°K ; vol. of air delivered/cycle=0·000393 m³ ; mass delivered=0·1749 kg/cycle ; ihp=0·645 ; vol. of air inhaled by H.P. cy=0·853 V_s ; stroke volume=0·0003415 m³ ; $d=6$ cm]

10·9. Two-stage compressor for two overall pressure ratios: V_c second stage ; ratio of ihp's.

A two-stage reciprocating air compressor has interstage cooling down to the initial air temperature. It is designed so that the first and second stage pressure ratios will be equal to each other at two different overall pressure ratios, 16 and 25. The initial pressure and temperature are the same in each of the two cases. Determine :

(a) the necessary percentage clearance volume in the second stage, and

(b) the ratio of the ihp's of the compressor in the two cases.

Neglect the effect of clearance volume on the volumetric efficiency of the first stage. Assume that compression and expansion are reversible adiabatic process.

[Percentage clearance volume in second stage=0·247 ; ratio of ihp's=0·835]

10·10. Variable clearance : effect on F.A.D. and hp.

(a) Indicate the important uses of compressed air for engineering purposes.

(b) Discuss the merits and demerits of compressed air power and electric power in mining work.

(c) How does the clearance effect the performance of a reciprocating air compressor ?

During the overhaul of an old compressor a distance piece ⅓ cm thick, which was originally inserted between the cylinder head and cylinder, was accidentally ommited. Before overhaul the clearance volume was 3 per cent of the swept volume. The compressor is designed to deliver air at a pressure of 8 kgf/cm² with a stroke of 75 cm. Determine the percentage change in (i) the volume of free air delivered, and (ii) the hp necessary to drive the compressor.

[Before overhaul, V_e=2·25A cc ; air inhaled/stroke=67·31 A cc. After overhaul, V_e=1·25 A cc. ; air inhaled/stroke=70·73 A cc. ; increases in F.A.D.=increase in hp=5·09%]

10 11. Compressor with defective inlet valve : mixing of residual and fresh air ; $m_{atoms.\ air}$; T_{mixing}

In a slow speed reciprocating air compressor, the clearance volume of 110 cc is occupied by the residual air at 4 kgf/cm² and 500°K from the previous cycle. The stroke volume is 1100 cc. The inlet valve is faulty and remains closed during the first half of the 'induction' stroke. The residual air is assumed to expand isentropically. At this point the valve opens suddenly and atmosheric air at 1 kgf/cm² is injected into the cylinder till the pressure becomes 1 kgf/cm². Determine the mass of atmospheric air induced and the temperature of the cylinder contents at the half stroke position. The mixing process takes place adiabatically.

[Temp. after expansion=239°K ; mass of (atmos. air+residual air)×temp. of contents=0 225 kg °K ; mass of residual air= 0·0003 kg ; mass of atmos air=0 000374 kg ; Temp. of contents= 334°K]

10 12. Volumatric efficiency with leakage and clearance.

Define volumetric efficiency of a compressor referred to (a) free air delivered, (b) N.T.P. condition.

An air compressor takes in air with ambient conditions 1 kgf/cm² and 15°C, and compresses it to a pressure of 8 kgf/cm² and the law of compression and expansion is $PV^{1\cdot 3}=C$. The pressure and temperature at the end of suction are 0 9 kgf/cm² and 30°C respectively.

(a) Calculate the volumetric efficiency assuming clearance 5 per cent of the swept volume and no leakage.

(b) If there is a leakage of 6 per cent between piston and valve before air is delivered, what would be the volumetric efficiency?

[V_s=100 ; V_4=26·5 ; free air inhaled=66·42 ; volumetric η= 66·42% ; volumetric η=66·42×0 94=62 7%]

10·13. Effect of intake conditions on output.

A portable single-stage compressor delivers air at 6 kgf/cm² gauge. The clearance volume is 4 per cent of the swept volume and the expansion and compression follow the law $PV^{1\cdot 3}=C$.

H.P. cylinder, allowing a clearance equal to 6 per cent of the swept volume and assuming the index 1·3 For air $R=29{\cdot}27$ kgf-m/kg °K.

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[V_s=100; V_4=26·5; free air inhaled=66·42; volumetric η=66 42%; volumetric η=66·42×0 94=62·7%]

10·13. Effect of intake conditions on output.

A portable single-stage compressor delivers air at 6 kgf/cm² gauge. The clearance volume is 4 per cent of the swept volume and the expansion and compression follow the law $PV^{1.3}=C$.

(a) *If this compressor is shifted from sea-level where the barometer pressure is 1·03 kgf/cm² to a hill station at 2000 m, by what percentage should the swept volume be increased if the free air delivered must remain unchanged. It is assumed that the barometer falls one cm of mercury at every 100 m elevation. Assume temperature at two places same.*

(b) *If the output must remain same without changing the swept volume determine the new clearance volume necessary as a percentage of swept volume.*

[Assuming swept vol. 100, air inhaled=86·47 ; new pressure ratio 8·907 ; new air inhaled=82·5 ; % increase in V_s=4·7% ; clearance vol.=3·12]

10·14. Heat balance of an air compressor.

Give three methods which may be adopted to control the amount of air delivered by an air compressor and point out the main advantages of each.

The following data relate to a two-stage single-acting air compressor : H.P. and L.P. cylinder bores 10·5 and 20 cm and the mep's 3·6 and 1·72 kgf/cm² respectively ; common stroke 15 cm ; rpm 360. The air entering the machine was metered as 0·0232 m³/sec at suction conditions of 1 kgf/cm² and 21°C ; water to jackets and intercoolers (arranged in series) 8·4 kg/min, raised in temperature by δ°C ; output of driving motor 12 horse-power ; temperature of air at discharge 122°C.

Determine the friction horse-power and draw up a heat balance in kcal per minute. Assume Cp for air 0·237.

[hp. of L.P. cy.=6·48, of H.P. cy.=3·74 ; frictional hp=1·78 ; heat balance : heat supplied 126·5 kcal (W.D.=107·7 kcal, friction =18·8 kcal), in coolant=67·2 kcal (53·1% in) ; compressed air= 38·7 kcal (30·6%) ; radiation loss and errors=20·6 kcal (16·3%)]

10·15. Air motor : % cut-off ; η of preheater ; Vs. ; hp.

Explain why compressed air is generally preheated before it enters the air motor.

Air at 7 kgf/cm² and 10°C is heated at constant pressure to 265°C before entering the cylinder of a simple reciprocating air motor. After admission at 7 kgf/cm² in motor the air is expanded isentropically to 1·05 kgf/cm² at which constant pressure it is discharged to atmosphere.

Ignoring the effects of clearance, calculate—

(a) the percentage of the stroke at which cut-off of air supply should occur,

(b) the percentage of the heat supplied to the preheater which is lost to the exhaust,

(c) the swept volume required and the theoretical power developed if the air flow is 2 kg/min at 500 rpm.

[r=3·877 ∴ cut-off=25 8% ; T_2=313°K ; heat in preheater =255 C_p, in exhaust=30 C_p ; loss=11·77% ; V_s=0 00349 m³ ; hp=10 24]

10 16. Air motor considering release and compression : size of cylinder.

A four-cylinder single-acting air motor is supplied with air at 7 kgf/cm² pressure. The clearance volume is 4 per cent of the swept volume. The air is cut-off at half stroke, after which expansion occurs. The exhaust valve is opened at the end of the stroke and the air is exhausted at 1 kgf/cm² pressure. The exhaust valve is closed at 92 per cent of the return stroke, after which compression occurs until the end of the stroke. The expansion and compression are both according to $PV^{1.3}$ =constant.

The motor develops 2·1 hp at 400 rev/min when the mechanical efficiency is 86 per cent. Assuming a diagram factor of 0 72 find the diameter of the cylinder if the bore and stroke are equal.

[Release pr.=2 986 kgf/cm² ; pr. at the end of compression= 4·171 kgf/cm² ; assume stroke vol. 100 m³, W D =467·4×10⁴ kgf-m. ∴ hp=10,29,000 ; when hp 2·1, V_s=0·0002041 m³ ∴ $l=d$=6 38 cm]

11

Refrigeration

11·1. Introduction. A refrigerating machine is an appliance to produce cold. According to the Second Law of Thermodynamics external energy is required to pump out heat from a lower temperature to a higher temperature, and hence any refrigerating machine requires a prime-mover to drive it.

The various applications of refrigeration are cooling of buildings, cooling space so that food or perishable articles be stored and for manufacture of ice.

11·2. Coefficient of Performance. The performance of refrigerators is given by a term *coefficient of performance* which is defined as

$$\text{C.O.P.} = \frac{\text{net refrigerating effect}}{\text{work done}} = \frac{N}{W} \qquad (11{\cdot}1)$$

As the value of C.O.P. is generally more than one it is not termed as efficiency.

The ratio of actual and theoretical coefficient of performance is known as *relative coefficient of performance.*

$$\text{Relative C.O.P.} = \frac{\text{actual coefficient of performance}}{\text{theoretical coefficient of performance}} \qquad (11{\cdot}2)$$

11·3. Unit of refrigeration. In FPS units, *one ton of refrigeration* is defined as a capacity to freeze one short ton (2000 lb.) of water from and at 32°F in 24 hours. The latent heat of ice is nearly 144 Btu per lb ; therefore to freeze one short ton, heat to be abstracted is 2,000 × 144 = 2,88,000 Btu per 24 hours or 200 Btu per minute, which is equivalent to 50 kcal/min or 3000 kcal/hr.

In Europe one unit of refrigeration is equal to 1 kcal/sec or 1·2 tons of refrigeration.

11·4. Carnot or Ideal Refrigerator. A machine working on a reversed Carnot cycle would act as a refrigerator if driven by power from an external source and would be theoretically most efficient. *No such machine has been used in practice* because adiabatic portion of the stroke would necessitate high speed, whilst the isothermal portion of the same stroke would necessitate an extremely slow speed. This variation of the speed during a stroke is not practicable.

The *P-V* and *T-s* diagram of an air refrigerator working on Carnot cycle are shown in Fig. 11·1.

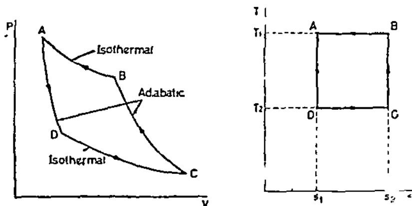


Fig 11·1. Reversed Carnot cycle on *P-V* and *T-s* diagrams.

At point A the cylinder is full of air in clearance volume which expands adiabatically to D causing the fall in temperature from T_1 to T_2. The air is further expanded isothermally to C during which process heat is absorbed from the body to be cooled. The air is then compressed adiabatically to B, raising the temperature to T_1. Finally the air is compressed isothermally to A during which process heat is rejected to the atmosphere.

Assuming 1 kg of working fluid,

Heat abstracted from body to be cooled $= RT_2 \log_e r$

Heat rejected to atmosphere $= RT_1 \log_e r$

$\therefore$ Work done/cycle, $W = RT_1 \log_e r \, (T_1 - T_2)$

Coefficient of performance,

$$COP = \frac{N}{W} = \frac{RT_2 \log_e r}{R(T_1 - T_2) \log_e r} = \frac{T_2}{T_1 - T_2} \qquad (11 \cdot 3)$$

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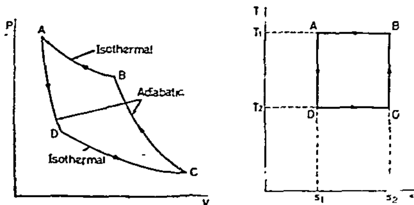


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Coefficient of performance,

$$COP = \frac{N}{W} = \frac{RT_2 \log_e r}{R(T_1 - T_2) \log_e r} = \frac{T_2}{T_1 - T_2} \qquad (11\cdot3)$$

11·5. Bell-Coleman or Reversed Joule Cycle. The earliest type of refrigerator worked on Bell-Coleman or reversed Joule cycle with air as the working substance. Fig. 11·2 shows the schematic diagram of the Bell Coleman cycle refrigerator and Fig. 11·3 its *P-V* diagram.

The air is compressed adiabatically in compression cylinder (operation *bc*) and is cooled at constant pressure in the cooler (operation *cd*). Then it is led into an expansion cylinder where the air expands adiabatically to atmospheric pressure producing power which is utilised, along with external power, to drive the compressor (operation *ef*). The cold air is then led into a chamber where it abstracts heat, thus producing cold (operation *fb*).

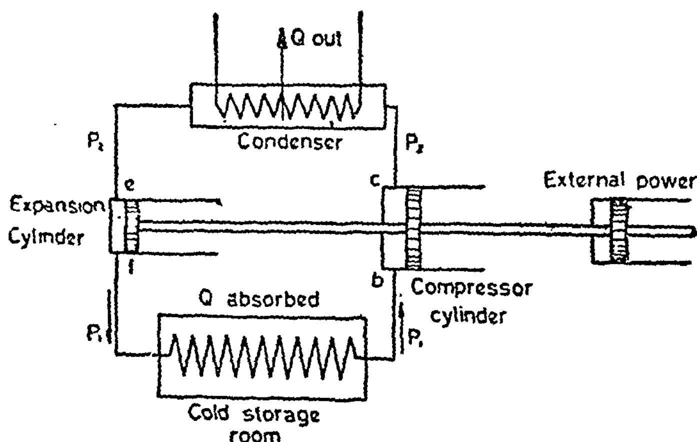
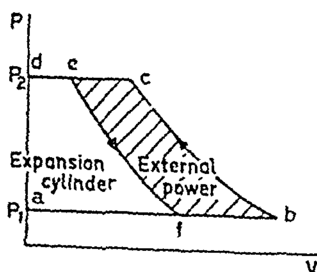


Fig. 11·2. Schematic diagram of Bell-Colemon cycle.



P-V diag. for compression cy. $= abcd$
" " expansion " $= defa$
External power supplied = Area $bcef$.

Fig. 11·3. Bell-Coleman cycle on *P-V* diagram.

Assuming adiabatic compression and expansion and assuming 1 kg of working fluid,

Heat abstracted from cold chamber, $N = C_p(T_b - T_f)$

Heat rejected in condenser $= C_p(T_c - T_d)$

$\therefore$ Work done on refrigerator, $W = C_p[(T_c - T_d) - (T_b - T_f)]$

Since the index of compression and expansion is same

$$\frac{T_c}{T_b} = \frac{T_d}{T_f} = r^{\gamma - 1}$$

Coefficient of performance,

$$COP = \frac{N}{W} = \frac{\left(\dfrac{T_c T_f}{T_d} - T_f\right)}{\left(\dfrac{T_b T_d}{T_f} - T_d\right) - \left(\dfrac{T_c T_f}{T_d} - T_f\right)}$$

$$= \frac{T_f}{T_d - T_f} \qquad (11 \cdot 4)$$

Though theoretically this is more efficient cycle than vapour compression cycle its coefficient of performance is very low, about $\frac{1}{4}$ to $\frac{1}{2}$, which is nearly 1/10th of a vapour compression machine. This is because of low specific heat and poor conductivity of air.

The chief disadvantages of Bell-Coleman cycle refrigerators are :

(1) due to poor conductivity of air the size of machine is bulky resulting in high cost and increased friction ;

(2) At low temperature, moisture in the air drawn freezes and chokes up the valves.

11·6. Vapour Compression Refrigerator. Most of the modern refrigerators work on the vapour compression system using ammonia, carbon dioxide, Freon-12 or other vapours as the working fluid. Fig. 11·4 shows the schematic diagram of such a machine and Fig. 11·5 its cycle on *T-s* diagram.

The refrigerant in the condition of wet vapour is drawn from the evaporator (which is brine tank in the case of ice plant) during the suction stroke of the compressor and compressed adiabatically raising its pressure and temperature. This operation is represented by *AB* on *T-s* diagram. This high pressure vapour is then cooled in a condenser at the same pressure, giving out its latent heat to the coolant. This operation is represented by *BC*. The high pressure liquid is now expanded through a throttle valve lowering its temperature and pressure. As throttling is a constant enthalpy process the refrigerant is partly evaporated as issues out as a very wet vapour and at a very

Fig. 11·4. Schematic diagram for vapour compression refrigerator.

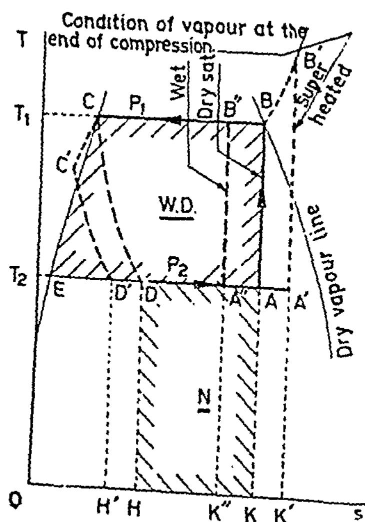


Fig. 11·5. Vapour compression cycle on *T-s* diagram.

low temperature. The expansion operation is represented by CD. The low temperature vapour now abstracts heat from the evaporator at constant temperature producing cold, which results in further evaporation of vapour. The refrigerant thus leaves the evaporator as nearly dry vapour, comlpeting the cycle.

It may be noted the vapour compression cycle is similar to the reversed Rankine cycle except that in this cycle expansion device replaces the pump.

In practice the plant is provided with measuring instruments and other accessories. The oil separator is one of the important accessory. The refrigerant passing through compressor takes up some lubricating oil with it. If this oil is carried to the condenser, it will form a thin film on the tube surface and will reduce the heat transfer efficiency. The oil separator prevents this oil being carried to the condenser.

Work done by compressor, W = area $ABCE$

Net refrigerating effect, N = heat abstracted in evaporator

= total heat at A − total heat at D

= area $ADHK$

∴ Coefficient of performance, $COP = \frac{N}{W}$

$$= \frac{\text{area } ADHK}{\text{area } ABCE} \qquad (11 \cdot 5)$$

Fig. 11·6 shows the vapour compression cycle on h-s and P-h diagrams. The advantage of showing the cycle on these charts is the availability of direct values of enthalpy at the salient points.

Coefficient of performance, $COP = \frac{N}{W}$

$$= \frac{h_b - h_a}{h_a - h_d} \qquad (11 \cdot 6)$$

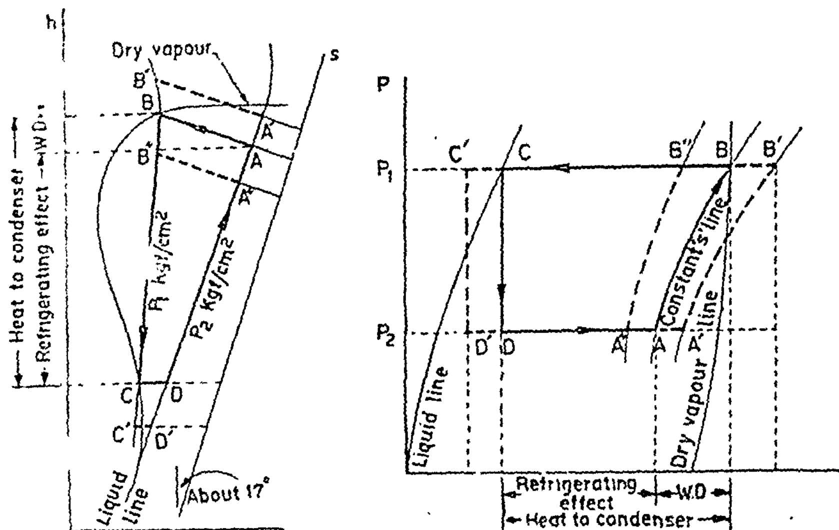


Fig. 11·6. Vapour compression cycle on h-s and P-h diagrams.

11·7. Wet and Dry Compression. In wet compression cycle (Fig. 11·7) vapour enters the compressor with a quality such that

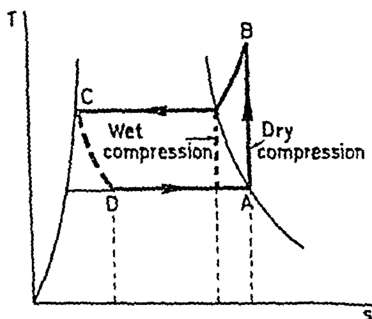


Fig. 11·7. Wet and dry compression.

at the end of compression the vapour is wet or dry and saturated. Thus the whole compression is in wet region. If the vapour enters the compressor in dry and saturated or superheated state the entire compression takes place in the superheated region ; such a compression is called a *dry compression.* The coefficient of performance is higher in wet compression because this cycle approximates more closely to the reversed Carnot cycle than does the dry compression cycle. Other advantages are less cooling water required...

lubrication when using ammonia, as mixture of oil and liquid ammonia clings to the cylinder walls.

In actual practice, however dry compression is preferred as there is higher volumetric and mechanical efficiency and less chances of damage to the compressor due to the pressure of the liquid. In dry compression higher speed can be adopted resulting in low first cost of motor and compressor. Also there is greater refrigerating effect per unit mass of fluid circulating through the system.

11·8. Undercooling. Undercooling refers to reducing the temperature of liquid below saturation temperature before *throttling*

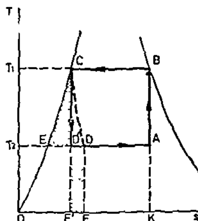


Fig. 11·8. Vapour compression cycle with undercooling.

(*see* Fig. 11·8). Undercooling on h-s and P-h diagrams has been shown in Fig. 11·8. This is one of the methods of reducing the amount of heat which returns via the throttle valve. Work done per lb. with and without undercooling is same and is represented by the area $ABCE$, but the *net refrigerating effect increases* from area $ADFK$ to area $AD'F'K$ and hence the theoretical coefficient of performance increases. However, for undercooling extra coolant is required which may require additional financial outlay.

11·9. Expansion Cylinder Vs. Throttle Valve. In refrigerators the sum of the heat abstracted from the cold chamber and heat equivalent to compression work is thrown into the condenser coolant. Throttling is a constant heat process and therefore when throttle valve is used heat is returned to the evaporator via the throttle valve. *With* expansion cylinder having adiabatic expansion, heat

instead of being returned to evaporator is converted into useful work. The net result is that external work is reduced and net refrigerating effect is increased and hence the coefficient of performance is increased (*see* Fig. 11·9).

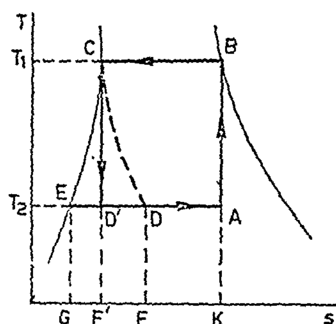


Fig. 11·9. Expansion by cylinder and throttle valve.

$$\text{C.O.P. with throttle valve} = \frac{\text{area } ADFK}{\text{area } ABCE}$$

$$\text{C.O.P. with expansion cylinder} = \frac{\text{area } AD'F'K}{\text{area } ABCD'} \qquad (11\cdot7)$$

11·10. Actual Vapour Compression Refrigeration Cycle. The actual cycle differs from the theoretical cycle mainly because of

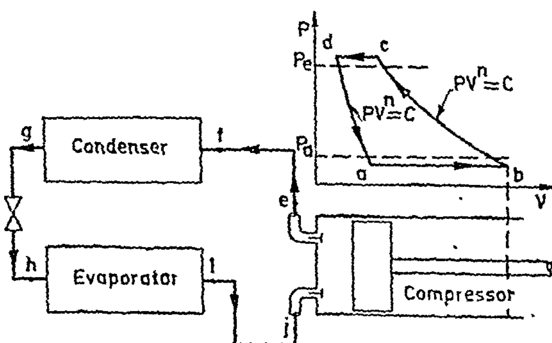


Fig. 11·10. Actual vapour compression cycle on schematic diagram.

pressure drops and heat transfer to or from the surroundings. Fig. 11·10 shows the actual cycle on schematic diagram and Fig. 11·11 shows the corresponding cycle on T-s and P-h diagrams.

The vapour enters the compressor pipe line in state j. Due to suction valve resistance the pressure at entrance drops to state a. During

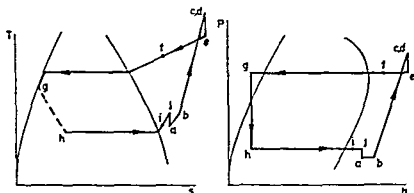


Fig. 11·11. Actual vapours compression cycle on P-h and T-s diagram.

suction stroke the vapour is heated by contact with cylinder walls to b and the compression bc is polytropic. Due to the resistance of delivery valve the vapour pressure leaving the compressor is reduced to point e. The process ef represents the cooling in pipe line connecting compressor and condenser. The process fg represents cooling in condenser. The liquid refrigerant enters the throttle valve at state g and after throttling enters evaporator at state h. The process hi represents heating of vapour heating in pipe line connecting evaporator and compressor.

It may be noted that in the cycle shown the pressure drops in causing refrigeration and ij the condenser, evaporator and piping are neglected.

11·11. Desirable Properties of Refrigerants. The important properties a refrigerant should possess are—

(*i*) low specific heat, and

(*ii*) high latent heat.

Their effects on refrigerating effect and coefficient of performance are explained subsequently. The other properties a refrigerant should have are,

(*iii*) low specific volume to reduce the size of compressor ;

(*iv*) positive evaporating pressures to prevent possible entry of atmospheric air into the system ;

(*v*) moderately low condensing pressures to permit the use of light weight equipment and piping on the high pressure side of the [illegible] system ;

(*vi*) high critical pressure to avoid unduly large power requirements.

(*vii*) high thermal conductivity to reduce heat exchanger surfaces ;

(*viii*) easily miscible in oil and stability after mixing ;

(*ix*) non-poisonous, non-corrosive, non-erosive and non explosive, and

(*x*) cheap.

(*i*) *Low specific heat.* From Fig. 11·10, compared with expansion cylinder or the ideal Carnot cycle the work done per lb with an expan-

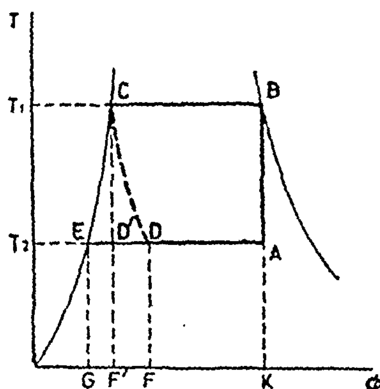


Fig. 11·12.

sion valve (throttle valve) is equal to area *ECBA* instead of area *CBAD'*. At the same time the refrigerating effect is equal to area *ADFK* instead of area *AD'F'K*. Incidentally the extra work done represented by the area *CED'* and the decreased refrigerating effect represented by area *DD'F'F* are equal.

Area *CED'* = Area *DD'F'F* = Area *ECF'G* — Area *ED'F'G*

$$=s(T_1-T_2)-T_2\times s\log_e\frac{T_1}{T_2} \qquad [s=\text{specific heat}$$

$$=s\left[(T_1-T_2)-T_2\log_e\frac{T_1}{T_2}\right]$$

From the above equation it is seen that the extra work done and the decrease in net refrigerating effect are proportional to specific heat, *s*. Hence it is obvious that *s* should be as low as possible in order that C.O.P. is as high as possible.

(*ii*) *High latent heat of vaporisation.*

Refrigerating effect per kg = Area *ADF*

$$=AD\times T_2$$

It has a small specific volume compared to ammonia. Freon is colourless. odourless and non-toxic. It is most suitable for domestic plants for which it is widely used. In commercial plants it competes with ammonia upto 50 H.P. capacity. The disadvantage when compared to ammonia is the larger swept volume of compressor. The major drawback of *F*-12 is its high cost.

(*vi*) *Freon-22 or Difluro-monochloro-methane* ($CHClF_2$). Compared to *F*-12 it has the advantage of less swept volume of compressor (only slightly higher than ammonia) and the evaporating temperatures are low. It is used in small to medium commercial plants. It is even costlier than F-12.

Properties of Refrigerants

S.N.	*Properties*	*Ammonia* NH_3	*Carbon dioxide* CO_2	*Sulphur dioxide* SO_2	*Methyl chloride* CH_3Cl	*Freon-12* CCl_2F_2	*Freon-22* $CHClF_2$
1	Boiling point at atm. pressure °C	−33·3	−78 5	−25·6	−23·8	−29·8	−40·8
2	Critical temperature, °C	132·6	31	157·2	143	111·5	96·2
3	Evaporator pressure at −15°C, kgf/cm²	2·40	23·34	0·822	1·48	1·86	3·01
4	Condenser pressure at 30°C, kgf/cm²	11·88	73·31	4·69	6·65	7·58	12·21
5	Specific volume at −15°C, m³/kg	0·509	0·016	0·405	0·278	0·0928	0·077
6	Latent heat at −15°C, kcal/kg	314·0	65·6			38·4	53·3
7	H.P./ton, −15°C/30°C	0·99	1·84			1·00	1·01
8	Mass flow/ton, −15°C/30°C	0·192	1·632			1·772	1·315
9	C.O.P., −15°C/30°C	4 85	2·56			4·72	4·71

11·13. Vapour Absorption Cycle (*see* Fig. 11·13). The essential difference between compression and absorption systems of refrigeration is in the method of raising the pressure of the vapour. In the compression system a vapour compressor with considerable volumetric capacity is required whereas in the absorption system

much smaller and simpler device, a liquid pump, is used. Thu
the main advantage of the absorption system. For this purpose

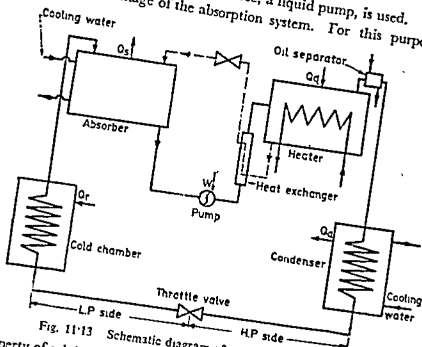


Fig. 11·13 Schematic diagram of vapour absorption cycle.

property of solubility of NH_3 (or any other vapour) in water is utilised. After raising the pressure of liquid ammonia by a pump, it is passed through a heater where ammonia is driven off in the form of vapour by the application of heat. In one of the types of vapour absorption refrigerator (Electrolux Refrigerator), even pump is not required and thus it can be used where electricity is not available. The disadvantage of the absorption cycle is that it has a very low C.O.P. The only possibility of improving the C.O.P. of this cycle is by the use of heat exchangers. In industry absorption machines have applications where thermal energy in the form of waste heat or low cost fuel is available.

The heat balance of the entire cycle is

$$Q_r+W+Q_g=Q_a+Q_s$$

where

Q_r=heat absorbed in cold chamber at temperature T_c

W=work in the pump, (negligible compared to other values)

Q_g=heat received in generator at temperature T_g

Q_a=heat rejected in condenser at temperature T_a

Q_s=heat rejected in absorber at temperature T_a

Ideal Coefficient of Performance for Vapour Absorption Cycle. The absorption system, neglecting the work done by pump, is equivalent to employing heat at higher temperature T_g to elevate heat from lower temperature T_c, the total heat being discharged at intermediate temperature T_a.

Assuming ideal Carnot cycle, work done over the temperature range T_g to T_a is

$$W = Q_g \times \frac{T_g - T_a}{T_g} \qquad (i)$$

This work applied to a reversed Carnot cycle working over the rage T_a to T_c would extract heat Q_c: *i.e.*

$$Q_c = W \times \frac{T_c}{T_a - T_c} = Q_g \times \frac{T_g - T_a}{T_g} \times \frac{T_c}{T_a - T_c} \qquad (ii)$$

and

$$COP = \frac{Q_c}{Q_g} = \frac{T_c}{T_g} \times \frac{T_g - T_a}{T_a - T_c} \qquad (11 \cdot 8)$$

11·14. Electrolux Refrigerator. The electrolux refrigerator is an ingenions absorption refrigerator having no moving machinery (*see* Fig. 11·14). It consists of an absorber A from the bottom of

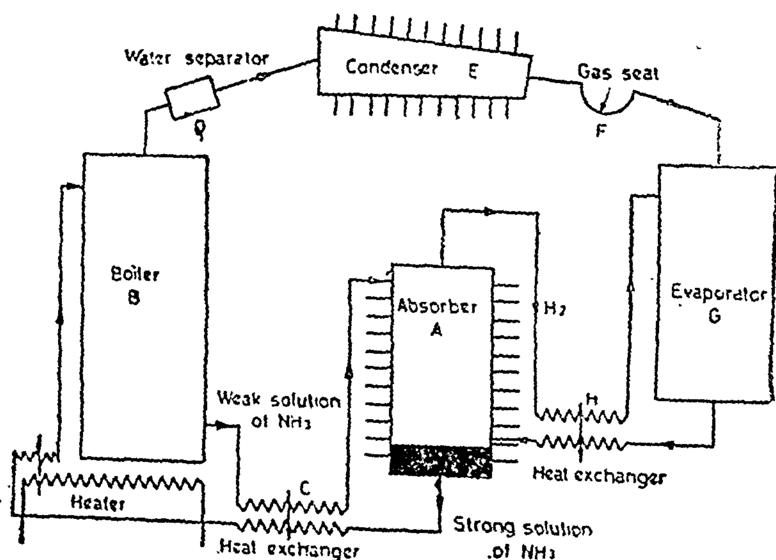


Fig. 11·14. Schematic diagram of Electrolux refrigerator.

which strong solution of ammonia flows to a boiler B through a heat exchanger C. In the heat exchanger the strong solution of ammonia

is heated by the returning weak solution of ammonia from the bottom of the boiler. In the boiler ammonia is driven out from the solution by the application of heat which passes through a water separator *D* into an aircooled condenser *E* where it is liquefied. The liquid ammonia gravitates through the *U*-tube gas seal *F* into the evaporator *G*. The whole plant is charged to 14 kgf/cm² which is the pressure everywhere. The evaporator contains hydrogen at a pressure of 12 kgf/cm², therefore as soon as ammonia enters the evaporator its pressure falls to 2 kgf/cm² according to Dalton's law of partial pressures, the corresponding temperature being −18°C. Due to low temperature ammonia evaporates taking heat from surroundings and produces cooling effect.

The mixture of hydrogen and vapour now flows to absorber through a heat exchanger H, cooling the returning hydrogen from the top of absorber. In the absorber the ammonia vapour is absorbed by the incoming weak solution of ammonia and thus hydrogen is separated which passes to the top. The absorption is accompanied by the evolution of heat, hence the absorber is air cooled.

The chief advantage of Electrolux refrigerator is that it has no moving machinery, no noise and can be used where no electricity is available.

11·15. Steam Jet Refrigeration. This system is primarily used in obtaining chilled water with the application of the principle of the flash cooling. A schematic diagram of steam jet refrigeration system is shown in Fig. 11·15. It consists of two separate circuits of steam and water. The steam circuit consists of a nozzle, a condenser and a boiler. The high pressure steam is supplied to the nozzle which sucks vapour from the flash chamber. It then gets compressed in the diverging portion of the nozzle and passes on to the condenser. The pressure in the condenser is about 5 cm Hg, corresponding to 35°C temperature of saturation.

The water circuit consists of a flash chamber in which low pressure of about 6 mm Hg (corresponding to 5°C temperature of saturation) is maintained by the suction of vapour by steam flow in nozzle. The water is chilled due to the absorption from it of the latent heat

for flashing of water. To make up for the water flashed and any chilled water withdrawn, fresh water is supplied through a spray.

The heat balance sheet for the cycle is

$$Q_r + Q_s + W_1 + W_2 = Q_a$$

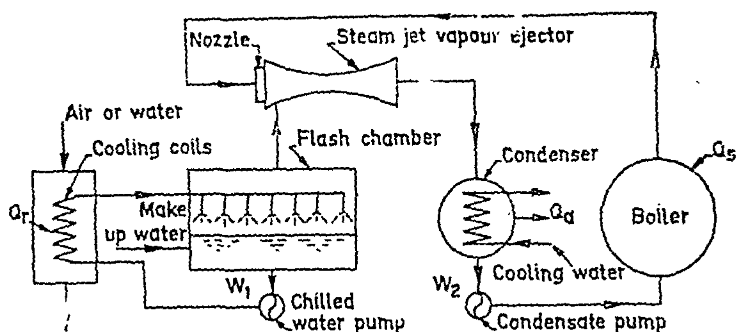


Fig. 11·15. Steam jet refrigeration.

The main advantages of this system are, few moving parts and low maintenance, use of a cheap non-toxic refrigerant, *i.e.*, water, and minimum power requirement. The plant itself is cheap to construct and operate. The disadvantages are, requirements of a large quantity of steam and condensing water and is limited to flash chamber temperatures, of say 5°C. The plant is very economical if waste steam is available in existing boiler plant.

11·16. Important Points. 1. In a Bell-Coleman cycle the formula C.O.P. $= \frac{T_f}{T_e - T_f}$ is only applicable when both compression and expansion are adiabatic.

2. If any of the above processes is not adiabatic the problem should be solved from the fundamentals as follows :—

Heat received = Heat rejected

i.e., $W.D.$ + Net refrigerating effect producted + Heat flow in during expansion stroke = Heat rejected to condenser + Heat rejected during compression stroke.

3. When properties of vapour are given with 0°C as datum the values of liquid enthalpy and liquid entropy may be negative and care should be taken to put proper sign.

Total heat to be abstracted per minute

$$=\frac{78\cdot3\times20{,}000}{10\times60}=2{,}610 \text{ kcal}$$

By definition one ton of refrigeration is equal to 50 kcal per minute,

$\therefore$ Capacity of plant $=\frac{2{,}610}{50}=52\cdot2$ tons   Ans.

Ideal Carnot cycle C.O.P. $=\frac{T_2}{T_1-T_2}=\frac{264}{300+264}=7\cdot32$ Ans.

$$\text{Actual C.O.P.}=\frac{7\cdot32}{3}=2\cdot44$$

$$\text{C.O.P.}=\frac{\text{Net refrigerating effect}}{\text{Work done}}$$

$\therefore$ $$\text{Work done}=\frac{2{,}610}{2\cdot44}=1{,}070 \text{ kcal/min}$$

$$\text{hp required}=\frac{1{,}070\times427}{4{,}500}=101\cdot5$$ Ans.

11·2. Bell-Coleman refrigerator : C.O.P. with adiabatic and polytropic expansion.

Define the term coefficient of performance.

In a Bell-Coleman refrigerating plant air is drawn into the cylinder of the compressor at atmospheric pressure of 1 kgf/cm² and temperature −7°C and it is compressed adiabatically to 5·5 kgf/cm² at which pressure it is cooled to 18°C. It is then expanded in an expansion cylinder to atmospheric pressure and discharged into the refrigerating chamber. Find the coefficient of performance of the plant if (a) the expansion is adiabatic, (b) the expansion follows the law $PV^{1\cdot25}$=constant.

Assume specific heat of air at constant pressure as 0·238 and $\gamma=1\cdot4$.

(a) $$T_C=T_B\times\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=266\times\left(\frac{5\cdot5}{1}\right)^{\frac{1\cdot4-1}{1\cdot4}}=432°\text{K}$$

$$T_F=\frac{T_E}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}=\frac{291}{\left(\frac{5\cdot5}{1}\right)^{\frac{1\cdot4-1}{1\cdot3}}}=179°\text{K}$$

Heat rejected in condenser $= C_p(T_C - T_E)$
Heat absorbed in brine tank $= C_p(T_B - T_F)$

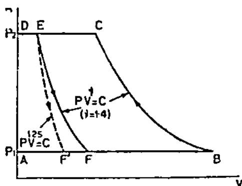


Fig. 11·16. Bell-Coleman cycle on *P-V* diagram.

$$\therefore \quad \underline{\text{C.O.P.}} = \frac{T_B - T_F}{(T_C - T_E) - (T_B - T_F)}$$

$$= \frac{266 - 179}{(432 - 291) - (266 - 179)} = \underline{1{\cdot}6} \qquad \text{Ans.}$$

$$\left[\text{Check: C.O P.} = \frac{T_F}{T_E - T_F} = \frac{179}{291 - 179} = 1{\cdot}6\right]$$

(b) $$T_F' = \frac{T_E}{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}} = \frac{291}{\left(\frac{5{\cdot}5}{1}\right)^{\frac{1{\cdot}25-1}{1{\cdot}25}}} = 207°\text{K}$$

Heat rejected in condenser $= 0{\cdot}238(432 - 291) = 33{\cdot}68$ kcal/kg
Heat absorbed in brine tank $= 0{\cdot}238(266 - 207) = 14{\cdot}04$ kcal/kg
Heat taken in during expansion

$$= \frac{\gamma - n}{n-1} \times C_v(T_E - T_F')$$

$$= \frac{\gamma - n}{n-1} \times \frac{C_p}{\gamma}(T_E - T_F') \qquad \left[C_v = \frac{C_p}{\gamma}\right]$$

$$= \frac{1{\cdot}4 - 1{\cdot}25}{1{\cdot}25 - 1} \times \frac{0{\cdot}238}{1{\cdot}4} \times (291 - 207)$$

$$= 8{\cdot}57 \text{ kcal/kg}$$

$\therefore$ Work done $= 33{\cdot}68 - (14{\cdot}04 + 8{\cdot}57) = 11{\cdot}07$ kcal/kg

$$\therefore \quad \underline{\text{C O.P.}} = \frac{14{\cdot}04}{11{\cdot}07} = \underline{1{\cdot}27} \qquad \text{Ans.}$$

Note.—In the second case, as the expansion is not adiabatic, the problem has been solved from fundamentals.

11·3. NH_3 plant with superheating and undercooling ; condition of vapour at entry to condenser and evaporator : C.O.P. ; hp.

In a vapour compression refrigerator the working fluid is superheated at the end of compression and is undercooled in the condenser before throttling. Sketch a working cycle on a temperature-entropy diagram and show how the theoratical coefficient of performance may be calculated from this diagram.

A food storage locker requires a refrigeration system of 12 tons capacicy at an evaporator temperature of —10°C and a condenser temperature of 25°C. The refrigerant ammonia is subcooled by 5°C before entering the expansion valve and the vapour is 0·97 dry before leaving the evaporator coil. Compression of refrigerant is adiabatic and compressor valve throttling and clearance are to be disregarded. Determine (a) the condition of vapour at entrance to the condenser, (b) the condition of vapour at entrance to the evaporator, (c) the coefficient of performance, and (d) the hp required. Neglect all losses.

Data from NH_3 *table :—*

Sat. temp.	*Enthalpy in Kcal/kg*		*Entropy*		*Specific heat*	
°C	*Liquid*	*Sat. vapour*	*Liquid*	*Sat. vapour*	*Liquid*	*Vapour*
—10°C	89·6	398·7	0·9593	2·1362	—	—
25°C	128·1	406·8	1·0976	2·0324	1·1	0·67

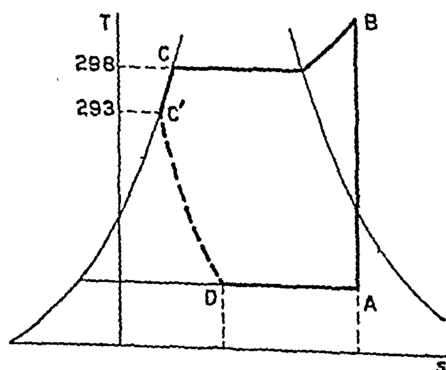


Fig. 11·17.

Entropy at A = Entropy at B

$$0{\cdot}9593+0{\cdot}97(2{\cdot}1362-0{\cdot}9593)=2{\cdot}0324+0{\cdot}67\log_e \frac{T_B}{298}$$

$$\therefore \qquad T_B=329{\cdot}5°K$$

(a) Condition of vapour at entrance to the condenser

$$=329{\cdot}5-298=31{\cdot}5°C \text{ superheat} \qquad \text{Ans.}$$

(b) $h_C'=h_C-C_P(T_C-T_C')=128{\cdot}1-1{\cdot}1\times5=122{\cdot}6$ kcal

Heat before throttling (h_C') = Heat after throttling (h_D)

$$122{\cdot}6=89{\cdot}6+x_D\,(398{\cdot}7-89{\cdot}6)$$

$\therefore$ Condition of vapour at entrance to the evaporator

$$x_D=0{\cdot}1069 \qquad \text{Ans.}$$

$$\text{C.O.P.}=\frac{N}{W}=\frac{h_A-h_C'}{h_B-h_A}$$

$$h_A=89{\cdot}6+0{\cdot}97\times(398{\cdot}7-89{\cdot}6)=389{\cdot}6 \text{ kcal/kg}$$

$$h_B=406{\cdot}8+0{\cdot}67\times31{\cdot}5=427{\cdot}9 \text{ kcal/kg}$$

$$\therefore \quad \text{C.O.P.}=\frac{389{\cdot}6-122{\cdot}6}{427{\cdot}9-389{\cdot}6}=6{\cdot}97 \qquad \text{Ans.}$$

(d) $\quad$ Work done $=\dfrac{N}{\text{C.O.P.}}=\dfrac{12\times50}{6{\cdot}97}$ kcal/min

$$\therefore \quad \text{hp required}=\frac{12\times50}{6{\cdot}97}\times\frac{427}{4{,}500}=8{\cdot}21 \qquad \text{Ans.}$$

11·4. CO_2 refrigerator with heat exchanger ; C.O.P

State the advantages and disadvantages of using carbon dioxide as a refrigerant. A vapour compression CO_2 refrigerator includes a heat exchanger in which the liquid leaving the condenser is undercooled by the vapour leaving the evaporator. The pressure in condenser is 58·46 kgf/cm² and the liquid leaving the condenser at 20°C is further cooled to 7°C in the heat exchanger. The CO_2 is then throttled, passed to the evaporator at 23·34 kgf/cm² and is evaporated at this pressure. The vapour produced is dry and saturated as it leaves the evaporator and passes to the heat exchanger. After leaving the heat exchanger the vapour is compressed isentropically to the condenser pressure.

Show the cycle on a sketch of a T–s. diagram indicating the areas which represent the heat transfer in the heat exchanger. Comment on the relative merit of the above cycle with a simple cycle without undercooling as regards extra work and refrigeration.

Neglecting radiation losses and pressure drops in the heat exchanger, calculate the theoretical coefficient of performance.

Pressure kgf/cm^2	Saturation temperature °C	Enthlapy, kcal/kg		Entropy	
		Liquid	Sat. vap.	Liquid	Sat. vap.
23·34	−15	91·4	156·7	0·969	1·2218
58·46	20	114·0	151·1	1·0468	1·1734

Take C_P for the high pressure liquid as 0·7 and for the superheated vapour as 0·3 kcal/kg°C.

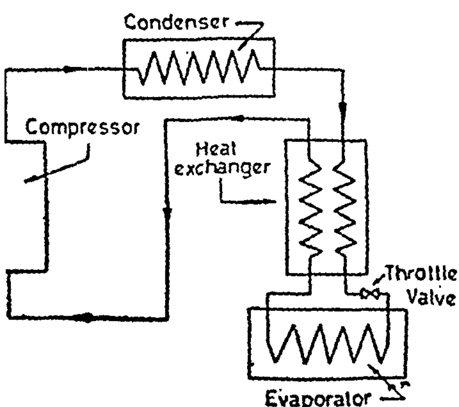


Fig. 11·18. Schematic diagram of CO_2 plant with heat exchanger.

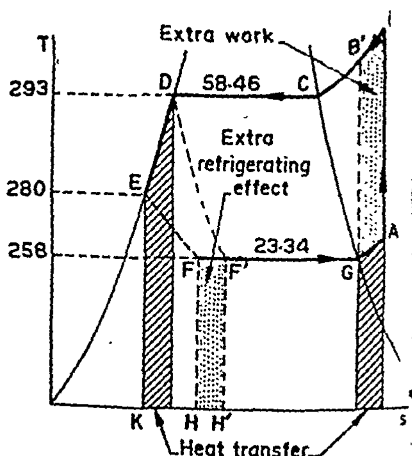


Fig. 11·19. CO_2 plant with heat exchanger on T-s diagram.

$$\text{Heat exchanged in exchanger} = 0{\cdot}7(20-7) = 9{\cdot}1 \text{ kcal}$$

$$H_F = h_E = 114{\cdot}0 - 9{\cdot}1 = 104{\cdot}9 \text{ kcal}$$

Since CO_2 leaves evaporator in dry saturated condition, the heat received in exchanger will superheat it.

$$H_A = 156{\cdot}7 + 9{\cdot}1 = 165{\cdot}8 \text{ kcal}$$

$$0{\cdot}3(T_A - 258) = 9{\cdot}1$$

$$\therefore \quad T_A = 288{\cdot}3°\text{K}$$

Now, Entropy at A = Entropy at B.

$$1{\cdot}2218+0{\cdot}3\log_e\frac{288{\cdot}3}{258}=1{\cdot}1734+0{\cdot}3\log_e\frac{T_B}{293}$$

$$\therefore \quad T_B=385°K$$

$$H_B=151{\cdot}1+0{\cdot}3(385-293)=178{\cdot}7 \text{ kcal}$$

$$\underline{\text{Theoretical C.O.P.}}=\frac{h_G-h_E}{h_B-h_A}$$

$$=\frac{156{\cdot}7-104{\cdot}9}{178{\cdot}7-165{\cdot}8}=\underline{4{\cdot}01} \qquad \text{Ans.}$$

Note. (*i*) The pressures used in the carbon-di-oxide refrigerators are very high.

(*ii*) Undercooling has been achieved by heat exchanger instead of cooling water. Compared to a cycle without heat exchanger there is a greater refrigerating effect shown by the area $FF'H'K$, however, more work is necessary shown by the area $ABB\,G$.

11·5. Freon water cooler : volumetric displacement of compressor ; hp of motor.

Discuss the relative merits and demerits of CO_2, NH_3, SO_2 and F-12 as refrigerants.

A water cooler, using Freon-12 works on the condensing and evaporating temperatures of 26°C and 2°C respectively. The vapour leaves the evaporator saturated and dry. The average output of cold water is 100 kg per hour cooled from 26°C to 6°C.

Allowing 20 per cent of useful heat into the water cooler, and volumetric efficiency of the compressor as 80 per cent, mechanical efficiency of the compressor and the electric motor as 85 per cent and 95 per cent respectively, find (a) the volumetric displacement of the compressor and (b) the hp of the motor. Given for Freon-12

t°C	P kgf/cm²	Enthalpy		Entropy		Specific heat, Cp		Specific volume of vapour m³/kg
		Liquid	Vapour	Liquid	Vapour	Liquid	Vapour	
26	6·82	106 01	139 70	1 0207	1·1334	0·238	0 161	0 027
2 0	3·36	100 45	137 21	1·0016	1 1353	0 225	0·148	0 053

Heat to be abstracted from water, considering leakage, etc.

$$=1{\cdot}20\left[\frac{100(26-6)}{60}\right]=40 \text{ kcal/min.}$$

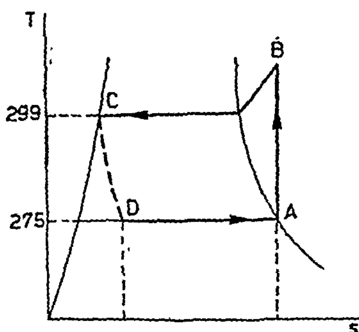


Fig. 11·20.

Entropy before compression = Entropy after compression

$$1{\cdot}1353=1{\cdot}1334+0{\cdot}161 \log_e \frac{T_s}{(273+26)}$$

$$\therefore \quad T_s=302{\cdot}6°\text{K}$$

Heat after compression $=139{\cdot}70+0{\cdot}161(302{\cdot}6-299)=140{\cdot}28$ kcal

Work done per kg $=140{\cdot}28-137{\cdot}21=3{\cdot}07$ kcal

Net refrigerating effect per kg $=137{\cdot}21-106{\cdot}01=31{\cdot}20$ kcal

$$\therefore \quad \text{Mass flow of Freon}=\frac{40}{31{\cdot}20}=1{\cdot}281 \text{ kg/min}$$

(*a*) Volumetric displacement of compressor

$$=\frac{1{\cdot}281\times 0{\cdot}053}{0{\cdot}8}=0{\cdot}085 \text{ m}^3\text{/min} \qquad \text{Ans.}$$

(*b*) Work done $=1{\cdot}281\times 3{\cdot}07$ kcal/min

$$\therefore \quad \text{hp of the motor}=\frac{1{\cdot}281\times 3{\cdot}07\times 427}{4{,}500}\times\frac{1}{0{\cdot}85\times 0{\cdot}95}$$

$$=0{\cdot}464 \qquad \text{Ans.}$$

11·6. Ammonia plant with undercooling : C.O.P.; volume of vapour, given $\frac{dT}{dP}$.

Explain with reference to a T-s diagram, the stages involved in the vapour compression process of refrigeration, and obtain an expression for the coefficient of performance in terms of (a) temperature and entropies (b) enthalpy.

A vapour compression refrigerating plant works between the pressure limits of 1·94 kgf/cm² and 10·23 kgf/cm²; the ammonia

leaves the compressor in dry saturated condition and is condensed at 25°C. Before expansion the liquid ammonia is undercooled to 21·5°C. The average specific heat of ammonia is 1·13.

Assuming compression to be adiabatic, find the work done, the refrigerating effect per kg of ammonia and C.O.P. Given for ammonia

Pressure kgf/cm²	Sat. temp. °C	Enthalpy, kcal/kg		Entropy	
		Liquid	Vapour	Liquid	Vapour
10 23	25	128·1	405 8	1 0976	2 0324
1 94	−20	78 2	395 5	0 9175	2·1710

Find the volume of vapour entering the compressor per minute when the refrigeration produced is 1,00,000 kcal/kg if the relative C.O.P. is 75 per cent. Assume that at lower temperature the pressure-temperature curve has a slope such that $\frac{dT}{dP} = 3\ 04 \times 10^{-4}$, *P being in kgf/m².*

Neglect the volume of the liquid, if any.

Entropy before compression = Entropy after compression

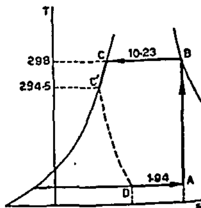
$0\ 9175 + x_A(2{\cdot}1710 - 0{\cdot}9175) = 2{\cdot}0324$

$\therefore \qquad x_A = 0\ 89$

$h_A = 78{\cdot}2 + 0\ 89(395{\cdot}5 - 78{\cdot}2) = 360\ 6$ kcal/kg

$\therefore$ <u>Work done per kg</u> $= h_B - h_A$

$= 406{\cdot}8 - 360\ 6 = \underline{46{\cdot}2 \text{ kcal}}$ Ans.



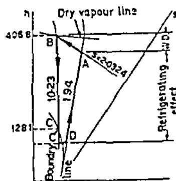


Fig. 11·21.

Refrigeration effect per kg $=h_A-h_C'$

$=360{\cdot}2-[128{\cdot}1-1{\cdot}13(25-21{\cdot}5)$

$=236{\cdot}06$ kcal Ans.

$$\text{C.O.P.}=\frac{N}{W}=\frac{236{\cdot}06}{46{\cdot}2}=5{\cdot}11$$ Ans.

Actual refrigerating effect per kg $=236{\cdot}06\times0{\cdot}75=177$ kcal

$\therefore$ Mass of ammonia $=\dfrac{1,00,000}{60\times177}=9\;4$ kg/min

According to Clapeyron's equation

$$V_s-V_w=\frac{JL}{T}\times\frac{dT}{dP}$$

$$V_s=\frac{427\times(395{\cdot}5-78{\cdot}2)}{(273-20)}\times3{\cdot}04\times10^{-4}$$ [Neglecting V_w

$=0{\cdot}1625$ m³/kg

$\therefore$ Volume of vapour entering the compressor

$=9{\cdot}4\times0{\cdot}89\times0{\cdot}1625=1{\cdot}36$ m³/min Ans.

11·7. Multiple compression and throttling : mass of vapour in L.P. cylinder ; state of vapour in H.P. cylinder.

In a vapour compression refrigeration system using methyl chloride the cycle adopted uses two stages of compression and only one evaporator. After leaving the evaporator as dry saturated vapour at —2·2°C the fluid is compressed isentropically in the L.P. cylinder to a pressure of 5 kgf/cm². The pressure range in the H.P. cylinder is from 5 to 10·08 kgf/cm², and this compression is also isentropic.

The vapour leaving the H.P. cylinder is condensed at a pressure of 5 kgf/cm² and there is no undercooling of the liquid. The liquid is first throttled to 5 kgf/cm², and after throttling, the wet vapour passes to a flash chamber. The liquid from flash chamber is then throttled to 2·41 kgf/cm² and passes to the evaporator whilst the dry saturated vapour is passed to a receiver connecting the L.P. and H.P. cylinders. Determine :

(a) the quantity of vapour compressed in the L.P. cylinder per kg. of vapour compressed in the H.P. cylinder.

(b) the states of the vapour entering and leaving the H.P. cylinder.

P_{sat} kgf/cm²	t_{sat} °C	Enthalpy, kcal/kg		Entropy
		Liquid	Sat. vap.	Sat. vapour
2·41	−2·2	—	111	0·4136
5	20	22·1	113·5	0·3958
10·08	49	32·96	115·7	0·3768

The mean specific heat Cp of the superheated vapour at 10·08 kgf/cm² and at 5 kgf/cm² is 0·17.

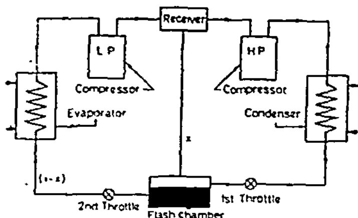


Fig. 11·22 Schematic diagram of plant with multiple compression and throttling.

(a) $h_D = h_E$

$$32{\cdot}96 = 22{\cdot}1 + x_E(113{\cdot}5 - 22{\cdot}10)$$

$\therefore \quad x_E = 0{\cdot}119$

$\therefore$ Quantity of vapour in L.P. cylinder

$= 1 - 0{\cdot}119 \qquad s_A = s_B \qquad = \underline{0{\cdot}881 \text{ kg}}$ Ans.

$$0{\cdot}4136 = 0{\cdot}3958 + 0{\cdot}17 \log_e \frac{T_B}{293}$$

$\therefore \quad T_B = 325°K$

The vapour from L.P. cylinder and separator mixes in receiver.

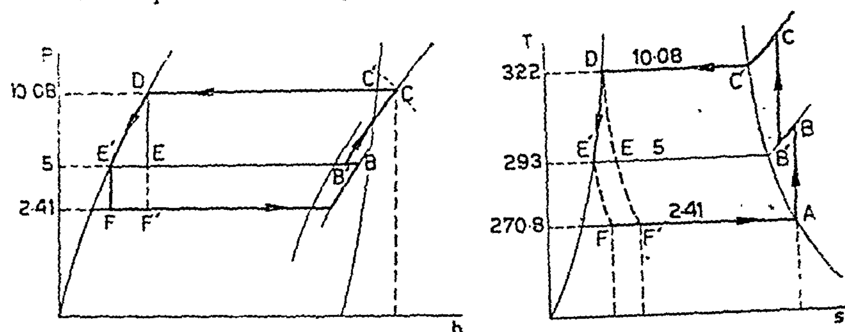


Fig. 11·23. Multiple throttle plant on *P-h* ond *T-s* diagrams.

Total heat before mixing = Total heat after mixing

$$0{\cdot}119\times113{\cdot}5+0{\cdot}881[113{\cdot}5+0{\cdot}17(325-293)]$$

$$=1\times[113{\cdot}5+0{\cdot}17(T_B'-293)$$

$$\therefore \qquad T_B'=322°K$$

i.e. State of vapour entering H.P. cylinder = 49°*C* Ans.

$$s_B'=s_C'$$

$$0{\cdot}3958+0{\cdot}17\log_e\frac{322}{293}=0{\cdot}3768+0{\cdot}17\log_e\frac{T_C}{322}$$

$$\therefore \qquad T_C=396°K$$

i.e. State of vapour leaving H.P. cylinder = 123°*C* Ans.

Note.—Multiple expansion is a method of precooling the liquid and thus improving the coefficient of performance.

11·8. Heat pump using CH_3Cl : W.D. ; C.O.P.

Briefly describe the principle of a heat pump.

River water at 12°C is to be raised to 28°C by means of a heat pump using methyl chloride and working between 7·1 kgf/cm² and 2·8 kgf/cm². The maximum temperature in methyl chloride circuit is 37°C and there is no undercooling.

Neglecting all heat losses and assuming a mechanical efficiency of 85 per cent for the compressor, how many kilowatt hours will be required to deal with 1,00,000 kg and what is the value of the ratio $\dfrac{\text{useful output}}{\text{electrical input}}$ *for this plant ?*

Properties of methyl chloride.

Pressure kgf/cm²	Sat temp. °C	Enthalpy, kcal kg		Entropy vapour	Specific heat vapour
		Liquid	Vapour		
7·1	32·2	268·2	114·4	0·388	0·24
2·8	2·2	15·3	111·7	0·410	—

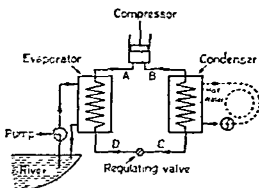


Fig 11·24 Schematic diagrams of heat pump.

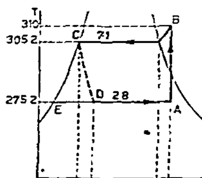


Fig. 11·25

The principle of heat pump is same as that of a refrigerator. In a heat pump heat is drawn in at low temperature *e.g.* from river, atmospheric air, etc. This evaporates a refrigerant which is then compressed and condensed at a higher temperature. The heat given by refrigerant during condensation, which is the sum of the heat taken from external source and heat equivalent to the work done, is used for heating buildings. The liquid refrigerant is then throttled and returned to the evaporator.

Entropy at the end of compression

$$s_B = 0{\cdot}388 + 0{\cdot}24 \log_e \frac{310}{305{\cdot}2} = 0{\cdot}039174$$

Entropy before compression = Entropy after compression

$$\left(0{\cdot}410 - \frac{111{\cdot}7 - 15{\cdot}3}{275{\cdot}2}\right) + x \times \frac{(111{\cdot}7 - 15{\cdot}3)}{275{\cdot}2} = 0{\cdot}039174$$

$$\therefore \qquad x = 0{\cdot}948$$

Enthalpy before compression $= 15{\cdot}3 + 0{\cdot}948(111{\cdot}7 - 15{\cdot}3)$

$= 106{\cdot}7$ kcal/kg

Enthalpy after compression $=114{\cdot}4+0{\cdot}24(37-32{\cdot}2)$

$=115{\cdot}6$ kcal/kg

Work done by the compressor $=115{\cdot}6-106{\cdot}7$

$=8{\cdot}9$ kcal/kg

Input to the compressor $=\dfrac{8{\cdot}9}{0{\cdot}85}=10{\cdot}47$ kcal/kg

Heat supplied to condenser cooling water

$=115{\cdot}6-28$

$=87{\cdot}6$ kcal/kg

Heat required to raise 1,00,000 kg from 12°C to 28°C

$=1{,}00{,}000\times(28-12)$

$=16{,}00{,}000$ kcal

$\therefore$ Mass of refrigerant required $=\dfrac{16{,}00{,}000}{87{\cdot}6}=18{,}270$ kg

Input required $=18{,}270\times10{\cdot}47=1{,}91{,}300$ kcal

$\therefore$ Electrical input $=\dfrac{1{,}91{,}300}{856{\cdot}7}=$ 223 kwh Ans.

$\dfrac{\text{Useful output}}{\text{Electrical input}}=\dfrac{16{,}00{,}000}{1{,}91{,}300}=8{\cdot}36$ Ans.

11·9. Electrolux refrigerator

(a) *Describe with a sketch the principle of an Electrolux refrigerator. What are its merits and demerits ?*

(b) *Obtain an expression for the ideal coefficient of performance of a vapour absorption refrigerator in terms of T_1, the temperature at which the working substance receives heat, T_2, the temperature of the cold body and T_c, the temperature of the circulating water.*

(c) *An ammonia absorption refrigerator works between the pressure limits of 11·9 kgf/cm² and 1·94 kgf/cm², the latent heat at two pressures being 273·6 kcal and 317·3 kcal respectively. If ammonia after throttling is 0·1 dry and after evaporator dry and saturated, calculate the heat liberated in absorbers and the coefficient of performance, given heat of mixing of ammonia is 210 kcal. Assume the work done by pump is negligible.*

For theory—see text.

(c) *Low pressure side*

Heat abstracted from cold chamber $=0{\cdot}9\times317{\cdot}3$

$=285{\cdot}6$ kcal/kg

Heat of mixing =210 kcal

Heat liberated in absorber

=Latent heat at low pressure+heat of mixing

=317·3+210=527·3 kcal

High pressure side

Heat supplied in heater

=Latent heat at high pressure+heat of mixing

=273·6+210=483·6 kcal

∴ Neglecting pump work,

$$\underline{\text{C.O.P.}} = \frac{286 \cdot 5}{483 \cdot 6} = \underline{0.594} \quad \text{Ans.}$$

11·10. Test on vapour compression Freon plant : theoretical and actual C.O.P. ; mass flow of F-12 ; heat balance

Explain how would you find whether a refrigerator is undercharged or overcharged.

A double-cylinder single-acting compressor of a F-12 ice plant has a bore and stroke of 90 mm and 70 mm respectively and runs at 500 rev/min. During a test the following data were obtained :

Pressure limits 11·02 kgf/cm² and 1·86 kgf/cm² ; temperature of refrigerant at entry and exit from condenser 65°C and 32°C respectively ; rate of flow of cooling water 13 kg per minute with a rise of 8°C ; mean effective pressure in compressor (from indicator diagram) 3·1 kgf/cm² ; ice produced in 8 hours from water at 27°C, 360 kg , latent heat of ice 80 kcal.

Find (a) the theoretical coefficient of performance, (b) the actual coefficient of performance, and (c) the mass flow of F-12 per minute.

Draw up a heat balance sheet of the plant on a minute basis. The relevant properties of F-12 are :—

Pressure kgf/cm²	*Sat. temp. °C*	*Enthalpy, kcal/kg*		*Entropy*
		Liquid	*Vapour*	*Vapour*
11·02	45	110·7	141·3	1·1321
1·86	—15	96·7	135·3	1·1373

The average specific heat of liquid is 0·23 and for the superheated vapour 0·154.

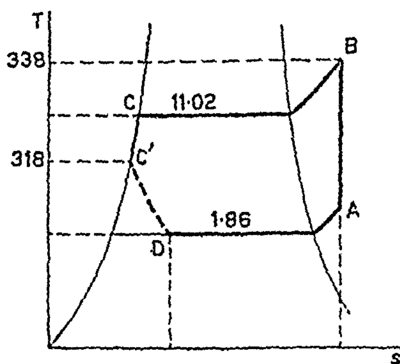


Fig. 11·26.

Undercharged. With too small a charge the compressor suction becomes starved and the delivery may become excessively superheated. The delivery pressure may also be low. There will be large bubbles in the liquid line sight glass. Suction gauge will not rise normally as the throttle valve is opened.

Overcharged. When the compressor is overcharged delivery pressure will be high, delivery temperature will be low and the suction gauge reading will not normally fall with the closing of the throttle.

(*a*) Theoretical C.O.P. $= \frac{N}{W} = \frac{h_A - h_{C'}}{h_B - h_A}$

$$h_B = 141·3 + 0·154 \times (65-45) = 143·6 \text{ kcal/kg}$$

$$h_{C'} = 110·7 - 0·23 \times (45-32) = 107·7 \text{ kcal/kg}$$

Entropy at A = Entropy at B

$$1·1373 + 0·154 \log_e \frac{T_S}{258} = 1·1321 + 0·154 \log_e \frac{338}{318}$$

or $T_S = 265·2°K$

$\therefore$ $h_A = 135·3 + 0·154 \times (265·2 - 258) = 136·4$ kcal

Theoretical C.O.P. $= \frac{136·4 - 107·7}{143·6 - 136·4} = 3·98$ Ans.

(*b*) Actual $N = \frac{360[27+80]}{8 \times 60} = 80·5$ kcal/min

$$\text{Swept volume} = 2\times\frac{\pi}{4}\left(\frac{90}{100}\right)^2\times\frac{70}{100} = 0.00089\ m^3$$

W.D. $= P_m \times$ swept volume

$$= [(3.4\times10^4)\times(0.00089\times500)]\times\frac{1}{427} = 35.4 \text{ kcal/min}$$

Actual C.O.P. $= \frac{80.5}{35.4} = 2.28$ Ans.

(c) Let mass flow of F-12 be m kg/min

Heat taken by cooling water = Heat lost by F-12 in condenser

$$13\times8 = m(143.6-107.7)$$

$\therefore$ $m = 2.9$ kg/min Ans.

Heat balance for 1 minute :

Credit	*kcal*	%	*Debit*	*kcal*	%
1. Heat equivalent to W.D.	35.4	30.6	1. Heat in cooling water.	104	89.7
2. Heat abstracted from water in the formation of ice.	80.5	69.4	2. Net radiation loss to atmosphere (by difference	11.9	10.3
Total	115.9	100		115.9	100

EXAMPLES 11

11.1. Ideal (Carnot) refrigerator : hp, heat rejected by compressor.

Describe with sketches, including T-s and P-v diagrams, the necessary organs and the action of a refrigerator operating with ammonia.

What is the object of undercooling ?

A refrigerator works on the Carnot cycle in reverse, between the temperatures at −7°C and 27°C. It makes 500 kg of ice per hour at −5°C from water at 14°C.

Given that the specific heat of ice is 0.5 and the latent heat 80 kcal/kg determine :—

(a) the horse-power required to drive the compressor ;

(b) the heat rejected by the compressor per minute.

[C.O.P.$=7\cdot82$; heat abstracted$=804$ kcal/min ; hp$=9\cdot76$; heat rejected$=W+N=907$ kcal/min]

11·2. Air refrigerator : C.O.P. ice : capacity ; piston displacement.

The pressure limits in an air-refrigerating plant are 4.3 kgf/cm² and 1 kgf/cm² and the rate of air flow is 13 kg per min. If the temperature at the beginning of compression and the air motor inlet are −1°C and 27°C respectively, find (a) the ideal coefficient of performance assuming isentropic compression and expansion, (b) the ice making capacity in tons of ice per day of 24 hours, and (c) the piston displacement in cu. metre per minute for the air motor and the compressor. Latent heat of fusion of ice is 80 kcal/kg, the isentropic index of expanison or compression for air is 1·4, and C_p is 0·24.

[Temp. at the end of expansion$=197\cdot7°K$; C.O.P.$=1\cdot933$; capacity/day$=4\cdot172$ tons ; piston displacement, air motor$=7\cdot532$ m³/min ; compressor$=10\cdot36$ m³/min]

11·3. Ammonia refrigerator with expansion cylinder ; C.O.P. ; hp ; mass of ammonia.

Why in practice is a throttle valve used in vapour compression refrigerator rather than an expansion cylinder to reduce the pressure between the condenser and the evaporator ?

A refrigerator circulating ammonia operates between pressure limits of 2·41 kgf/cm² and 11·9 kgf/cm². It consists of 4 units, (1) a compressor in which compression is isentropic and the outgoing vapour is dry saturated ; (2) a condenser in which the whole of the latent heat is removed at 11·9 kgf/cm² ; (3) an expansion cylinder into which the condenser discharges and in which expansion is isentropic ; (4) an evaporator operating at 2·41 kgf/cm².

Sketch the temperature-entropy diagram and determine

(a) the coefficient of performance ;

(b) the horse-power input to the machine, if the two cranks are connected and a cooling effect of 250 kcal per minute is required ;

(c) the rate of circulation of ammonia corresponding to (b).

P, kgf/cm²	Saturation temperature, °C	Latent heat, kcal/kg
11 9	30	273 6
2 41	−15	—

[C.O.P $= \frac{T_2}{T_1 - T_2}$ =5 733 ; hp=4 14 , rate of circulation of ammonia=1·072 kg per min.

Note. With expansion cylinder the cycle is same as Carnot cycle]

11 4. F-12 refrigerator ; capacity in tons ; hp

Describe the working of a domestic refrigerator. How is the temperature on the storage compartment and freezing cabinet controlled ?

A Freon-12 compressor 15 cm dia × 15 cm stroke, four cylinder single-acting, running at 970 rev/min is working between −10°C and 30°C. Find out the capacity of the compressor in tons of refrigeration and hp of the electric motor driving the compressor. Assume that the vapour leaves the evaporator saturated and dry. Volumetric efficiency of the compressor is 75 per cent, mechanical efficiency of compressor 95 per cent and efficiency of motor 98 per cent. Given for Freon-12

t°C	P kgf/cm²	Sp. vol m³/kg	Total heat, kcal/kg		Entropy vapour	C_p
			h_f	h_g		
−10	2·236	0 0781	—	135 9	1 1366	0 14
30	7 581	—	107 0	140 1	1·1331	0 16

[Mass flow=98 7 kg , capacity in tons of refrigeration=57·1 tons ; T_B=310°K hp=53 6].

11·5. Carbon-di-oxide refrigerator · theoretical and relative C.O P. ; capacity in tons.

What do you understand by wet and dry compression in a vapour compression plant ? Briefly enumerate the relative merits and demerits of two systems.

A CO_2 plant works between the pressure limits of 65 59 and 31·05 kgf/cm². It produces 1,340 kg of ice in 24 hours fr

at 10°C, the CO_2 flow being 15 kg per minute. The dryness fraction of CO_2 during the beginning of the compression stroke is 0·6. Calculate the theoretical and relative C.O.P. and the capacity of plant in tons of refrigeration. Latent heat of ice is 80 kcal.

Temp., °C	*Pressure kgf/cm²*	*Liquid heat kcal*	*Laten theat kcal*	*Entropy of liquid*
25	65·59	118·8	28·5	1·0628
−5	31·05	96·9	59·5	0·9890

[dryness fraction at the end of compression = 0·621 ; theoretical C.O.P. = 3·48 actual refrigeration = 5·583 kcal per kg ; actual C.O.P. = 1·412 ∴ relative C.O.P. = 40·6% ; capacity of plant = 1·675 tons of refrigeration].

11·6. Ammonia plant : undercooling ; N ; dryness fraction before compression ; volume/min.

A vapour compression refrigerator works between the temperatures of −5°C and 10°C, the vapour being just dry saturated at 10°C at the end of compression. The liquid is cooled to 7°C before being throttled to the evaporator pressure. Assuming that the compression is carried out adiabatically, determine the refrigeration effect produced by 1 kg of the working fluid in passing through the cycle.

The latent heat of the working fluid is 292·8 kcal at 10°C and 305·6 kcal at −5°C and the mean specific heat of the liquid is 0·91 over this range of temperature.

If at the lower temperature, the pressure temperature curve has a slope such that

$$\frac{dT}{dP} = 7 \times 10^{-4}$$

P being in kgf/m², find the volume entering the compressor per minute when the refrigeration produced is 1,00,000 kcal per hour. Neglect the volume of the liquid.

[Taking 0°C datum, $N = h_A - h_C = 286·1 - 6·4 = 279·4$ kcal/kg ; $x_A = 0·951$; $v_s = 0·149$ m³/kg ; mass of fluid per hour = 357·3 kg ∴ vol/min = 1·93 m³]

117. Multiple throttling : superheating ; undercooling : C.O.P.

A vapour compression refrigerating plant uses ammonia as the working fluid and the pressure and the temperature of delivery from the compressor are 11·9 kgf/cm² and 40°C. After condensation the liquid at 25°C is passed through a throttle valve to the evaporator in which the pressure is 2·97 kgf/cm². Using the following information, find the C.O.P. Sketch the total heat-entropy diagram for the cycle.

P kgf/cm²	t°C (sat.)	h (liquid) kcal	Entropy	
			Liquid	Vapour
11 90	30	133·8	1 1165	2 0191
2 97	−10	89 6	0·9593	2·1362

At 11·90 kgf/cm² and 40°C the enthalpy is 414·1 kcal and the liquid heat at the same pressure and 25°C is 128 3 kcal. If the throttling process had been carried out by first throttling to 4 38 kgf/cm² and then throttling the liquid to evaporator, the vapour being passed back to the compressor, find the increase in refrigeration effect per kg of the fluid passing through the evaporator as previously. At 4 38 kgf/cm² the saturation temperature is 0° and the liquid heat is 100 kcal

[x_A=0·919 , C O P.=8 5 ; increase in N=28·3 kcal/kg]

118. Ammonia plant with flash chamber : C.O.P. ; Vs ratio of two compressors.

An ammonia refrigerating plant consists of a main compressor an auxiliary compressor, a condenser, a flash chamber and an evaporator.

The liquid leaves the condenser at 18 17 kgf/cm² as saturated liquid without undercooling and passes to the primary throttle from which it emerges at 7·68 kgf/cm² to enter the flash chamber.

In this the fluid is separated and passed through the main throttle to the evaporator operating at 1 55 kgf/cm². The resulting state at evaporator exit is saturated vapour. This vapour is compressed isentropically in the main compressor to 18 17 kgf/cm²

The saturated vapour resulting from the flash chamber is passed direct to the auxiliary compressor where it is compressed is entropically

to 18·17 kgf/cm², mixed adiabatically and isobatically with the remaining refrigerant from the main compressor and then passed to the condenser. Find : (a) the coefficient of performance, and (b) the ratio of effective swept volumes of main and auxiliary compressors.

[Mass in auxiliary compressor=0·1169 kg ; enthalpy after mixing 480·6 kcal/kg ; C.O.P.=2·86 ; swept volume ratio=34·4]

11·9. Heat pump using SO_2 : thermal efficiency with expansion cylinder and throttle valve ; compressor hp ; quantity of water.

Give a sketch showing the necessary organs of a heat pump and explain the working.

The heating of a block of municipal buildings adjacent to a large river is carried out by means of a heat pump which uses sulphur di-oxide as the working medium. Evaporation takes place at −1°C the temperature of the water drawn from the river meanwhile falling from 5°C to 3°C. After adiabatic compression to 93°C the SO_2 vapour is completely condensed without undercooling at 60°C. The water used for heating the buildings enters the condenser at 44°C and leaves at 50°C. Assuming that specific heat at constant pressure of SO_2=0·154, compute the reciprocal thermal efficiency of the heat pump (a) if adiabatic expansion is performed in a cylinder, (b) if a throttle valve is used to reduce the pressure.

In the latter case determine for a heating effect of 6,700 kcal per min the power input to the compressor and rate of flow of river water and condenser cooling water in kg per hour. Properties of SO_2, measured from the datum of—40°C are given below :

Pressure kgf/cm²	*Temp. °C*	h_f *kcal*	h_g *kcal*	*s*		
				Liquid	*Latent*	*Total*
1·525	−1	12·6	102·8	0·0496	0·3316	0·3812
11·15	60	33·4	100·0	0·1189	0·1999	0·3188

[Adiabatic expansion : heat rejected=71·68 kcal ; work done =13·32 kcal ; reciprocal thermal efficiency=5·38. with expansion valve : heat rejected remains unchanged ; work done=15·27 kcal ;

reciprocal thermal efficiency=4 695, power required=135 4 hp; river water circulation=1,58,000 kg/hr; condenser cooling water =67,000 kg/hr]

11 10. Test on NH_3 plant : theoretical and actual C.O P.; circulation rate of ammonia.

Explain why ammonia is often used in industrial refrigeration plants.

A vapour-compression refrigerator using ammonia as the working fluid operates between pressure limits of 13 77 kgf/cm² and 2·41 kgf/cm² The temperature of the ammonia leaving the compressor is 66°C and of that leaving the condenser is 27°C The pressure drop from the condenser to the evaporator is achieved by throttling. During a trial on this unit it was found that for a condenser cooling water flow rate of 45 kg per minute with a temperature rise of 11°C, the input power was ·5 hp and the ice production rate was 120 kg per hour, from and at °C.

(a) *the theoretical coefficient of performance assuming isentropic compression and constant enthalpy throttling,*

(b) *the circulation rate of ammonia in kg per minute, based on a condenser heat balance assuming no losses,*

(c) *the overall coefficient of performance,*

Data from ammonia tables —

...e	Sat temp °C	Enthalpy		Entropy		Specific heat	
		Liquid	Dry vapour	Liquid	Dry vapour	Liquid	Superheated vapour
	35	139 7	408 0	—	2 0061	1 13	0 73
	−15	83 6	397 1	—	2 1532	1 05	0 55

...atent heat of fusion of ice, 80 kcal/kg.

...halpy : after compression=430 6 kcal, before throttling=130 ... ore compression=377·3, theoretical C O.P.=4 63 ... ...mmonia=1·65 kg/min., overall C.O P.=3·281 ...

12

Steam Nozzle

12·1. Introduction. A steam nozzle is a device for converting heat energy of steam into kinetic energy. It consists of a passage

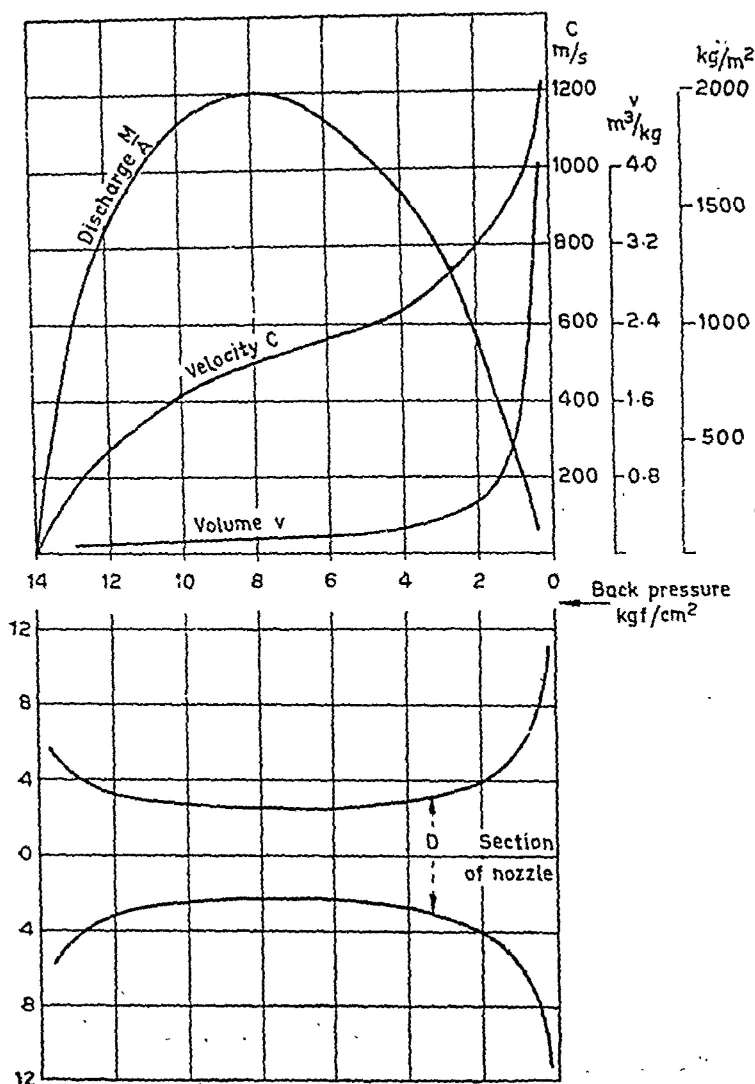


Fig. 12·1. Nozzle shape and its curves for uniform pressure drop.

of varying cross-section. The steam enters the nozzle with a relatively small velocity and high pressure. As the steam flows through the nozzle its pressure falls and the heat drop in expansion is spent in increasing the velocity of steam. The velocity increases continuously from mouth to exit.

Fig. 12·1 represents specific volume, velocity and discharge curves plotted with the axis of steam nozzle assuming uniform pressure drop. It will be seen from it that at the high pressures the specific volume, v, increases at first slowly as the pressure drops while the velocity, C, increases at a geater rate. As the expansion proceeds the increase of specific volume v becomes greater than the increase of velocity C. Hence, as we proceed from the high pressure end to the low pressure end of the nozzle, the area A which is proportion to $\frac{v}{C}$, decreases to a minimum and then increases. (The dryness fraction of the steam also changes during the flow and is taken into account in calculating the size). In other words, the nozzle is *convergent-divergent* in section. The point of the nozzle where the area is a minimum is called the throat and the pressure at the *throat* is called the *critical pressure*.

It is seen from above that a convergent nozzle will be used if the exit pressure is equal to or more than critical pressure and a convergent-divergent nozzle will be used if the exit pressure is less than the critical pressure. Fig. 12·2 and 12·3 show convergent and convergent-divergent nozzles respectively.

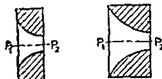


Fig. 12·2. Convergent nozzles.

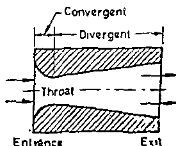


Fig. 12·3. Convergent-divergent nozzle.

The main application of the steam nozzles is in steam turbines for producing high velocity of steam. Other applications are in injectors for pumping feed water into boilers, ejectors for removing air from condensers and in steam flow measuring devices.

12·2. Expansion of Steam in Nozzles. The ideal expansion of steam in a nozzle follows Rankine cycle minus its feed pump term. The steam enters the nozzle at constant pressure, the process being a flow process. During the expansion, work is done in increasing the kinetic energy of the steam. There is no piston but the expansion is resisted by steam itself acting as a piston.

The expansion of steam in a nozzle is a flow process. As the expansion is very rapid it is assumed that the expansion is adiabatic *i.e.*, $q=0$. Since no external work is done w is also zero. The system is shown schematically in Fig. 12·4. Figs 12·5, 12·6 and 12·7 represent the expansion of steam on P-v, T-s and h-s diagrams respectively. AB represents the heat drop when the steam expands adiabatically from pressure P_1 and P_2 and without friction. AD. represents adiabatic expansion with friction.

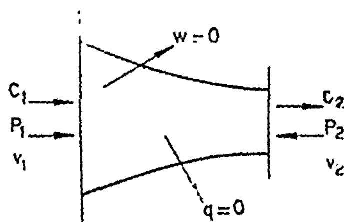


Fig. 12·4. Expansion of steam in a nozzle.

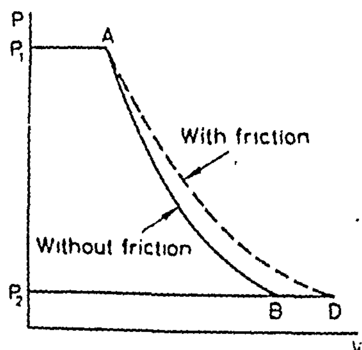


Fig. 12·5. Expansion of steam on P-v diagram.

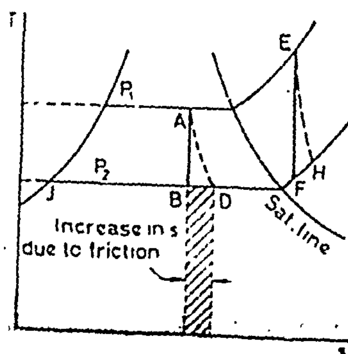


Fig. 12·6. Expansion of steam on T-s diagram.

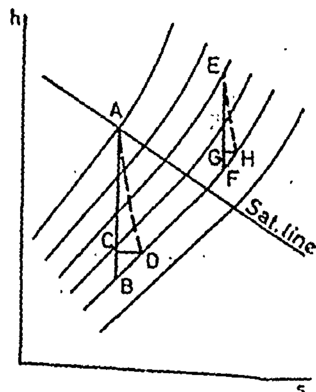


Fig. 12·7. Expansion of steam on h-s diagram.

According to the First Law of Thermodynamics for a flow process, (see Fig. 12·4),

$$\frac{P_1 v_1}{J}+u_1+\frac{C_1^2}{2gJ}+q=\frac{P_2 v_2}{J}+u_2+\frac{C_2^2}{2gJ}+w \qquad (12·1)$$

Substituting $q=0$, $w=0$ and specific enthalpy $h=u+\frac{Pv}{J}$, we get

$$h_1+\frac{C_1^2}{2gJ}=h_2+\frac{C_2^2}{2gJ}$$

or

$$h_1-h_2=\Delta h_t=\left(\frac{C_2^2}{2gJ}-\frac{C_1^2}{2gJ}\right) \qquad (12·2)$$

where Δh_t is the theoretical heat drop.

Neglecting initial velocity, being very small compared to final velocity

$$C_2=\sqrt{2gJ\,\Delta h_t}$$
$$=\sqrt{2\times 9·81\times 427\times \Delta h_t}$$
$$=91·53\sqrt{\Delta h_t} \qquad (12·3)$$

The isentropic expansion of steam through the nozzle may be approximately represented by the equation Pv^n=constant.

where $n=1·135$ for steam initially dry saturated
$=1·3$ for steam initially superheated.

In nozzles the heat drop is equal to the work done in increasing the K.E., which in turn is equal to the Rankine area. Again neglecting velocity of approach i.e., C_1

Heat drop=gain in K.E.=work done during Rankine cycle

$$\Delta h_t=\frac{C_2^2}{2gJ}=\frac{n}{(n-1)J}(P_1 v_1-P_2 v_2)$$

But $\quad P_1 v_1^n=P_2 v_2^n \quad$ or $\quad \frac{v_2}{v_1}=\left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}}$

Substituting this value

$$\frac{C_2^2}{2gJ}=\frac{n}{(n-1)}\times\frac{P_1 v_1}{J}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]$$

hence $$C_2=\sqrt{2g\left(\frac{n}{n-1}\right)P_1 v_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]} \qquad (12·4)$$

Knowing the discharge velocity and exit area, mass of steam, m, flowing per second through the nozzle may be calculated.

$$m=\frac{A_2C_2}{v_2} \qquad [A_2=\text{exit area of the nozzle}$$

$$=\frac{A_2}{v_1\left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}}}\sqrt{2g\left(\frac{n}{n-1}\right)P_1v_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]}$$

$$=\frac{A_2}{v_1}\sqrt{2g\left(\frac{n}{n-1}\right)P_1v_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]} \qquad (12\cdot5)$$

12·3. Condition for Maximum Discharge. Normally a nozzle is designed for maximum discharge. This is achieved by designing for a certain throat pressure which produces this condition.

There is only one value of the ratio $\frac{P_2}{P_1}$ which will produce maximum discharge for the nozzle. This value can be found out by differentiating the value for the mass of discharge given by equation (12·5). All the values of this equation are constants with the exception of ratio $\frac{P_2}{P_1}$; hence, it is only necessary to differentiate the portion of this equation inside the square brackets. Differentiating equation (12·5) and equating to zero

$$\frac{dm}{d\left(\frac{P_2}{P_1}\right)}=\frac{d}{d\left(\frac{P_2}{P_1}\right)}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]=0$$

or $$\frac{2}{n}\left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}}-\frac{n+1}{n}\left(\frac{P_2}{P_1}\right)^{\frac{n+1-n}{n}}=0$$

or $$\left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}}=\left(\frac{n+1}{2}\right)\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

or $$\left(\frac{P_2}{P_1}\right)^{2-n}=\left(\frac{n+1}{2}\right)^{n}\times\frac{P_2}{P_1}$$

or $$\left(\frac{P_2}{P_1}\right)^{2-n-1}=\left(\frac{n+1}{2}\right)^n$$

or $$\frac{P_2}{P_1}=\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \tag{12.6}$$

The nozzle discharge increases as the pressure on the discharge side of a nozzle decreases but once the discharge pressure reaches the critical value given by Eq. (12.6) the discharge reaches a maximum. The throat pressure and mass flow then remains constant even if the pressure at exit is further reduced.

For steam which is initially saturated, $n=1{\cdot}135$

$$\therefore \quad \frac{P_2}{P_1}=\left(\frac{2}{1{\cdot}135+1}\right)^{\frac{1{\cdot}135}{1{\cdot}135-1}}=0{\cdot}58 \tag{12.6(a)}$$

For steam which is initially superheated, $n=1{\cdot}3$

$$\therefore \quad \frac{P_2}{P_1}=\left(\frac{2}{1{\cdot}3+1}\right)^{\frac{1{\cdot}3}{1{\cdot}3-1}}=0{\cdot}545 \tag{12.6(b)}$$

The value of n is found from Zeuner's equation

$$n=1{\cdot}035+0{\cdot}1x \tag{12.7}$$

Substituting $\frac{P_2}{P_1}=\left(\frac{2}{n-1}\right)^{\frac{n}{n-1}}$ in Eq. (12.5), we get

$$\text{Maximum discharge,}\quad m=\frac{A}{v_1}\sqrt{2g\,\frac{n}{n-1}\,P_1v_1\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}} \tag{12.8}$$

From the above equation, we see that the mass flow of steam in a convergent-divergent nozzle depends only on the initial conditions of steam and the throat area and is independent of the final pressure at the exit of the nozzle. Thus the addition of the divergent part of the nozzle after the throat does not affect the discharge of the nozzle. It only affects the final velocity of the steam.

12·4. Expansion of Steam Considering Friction. When steam flows through a nozzle there are frictional losses due to (i) friction between steam and the nozzle surface, (ii) the internal friction of steam itself, and (iii) losses due to shock. The effects of friction losses are:

(1) to reduce the heat drop by 10 to 15 per cent and thus to reduce the exit velocity ;

(2) to increase the final dryness fraction of steam as the energy spent on friction is reconverted into heat in the steam at constant pressure ;

(3) to increase the volume of steam due to increase in dryness fraction. Increase in volume is in the ratio $\frac{JD}{JB}$ (*see* Fig. 12·6).

In h-s diagram (*see* Fig. 12·7) AC represents the useful heat drop taking friction into account. The condition of steam after expansion is represented by D, which is obtained by drawing horizontal CD to meet the constant pressure line P_2. As the friction is through-out, probable expansion is represented by the dotted line AD.

The coefficient of nozzle or nozzle efficiency is defined as

$$\text{Nozzle efficiency, } k=\frac{\text{Useful heat drop}}{\text{Isentropic heat drop}}=\frac{\Delta h_e}{\Delta h_t}=\frac{AC}{AB} \quad (12·9)$$

The exit velocity C_2 considering friction is given by

$$C_2=91·53\sqrt{kh_t} \quad (12·10)$$

Losses near the entrance of the nozzle are much less than at exit owing to the small inlet velocity : hence effect of friction upto throat is usually neglected and the whole of friction loss is assumed between throat and exit.

12·5. Supersaturated or Metastable Flow. The expansion of steam as discussed in §12·2 is in thermal equilibrium conditions. But in actual practice if the initial condition of steam is superheated the expansion is most probably *supersaturated*. Supersaturation means that the steam does not condense at the saturation temperature corresponding to the pressure but continues to expand like a gas with fall in temperature and no condensation, in what is normally the wet region. The limit of supersaturation is upto about 97 per cent dryness and beyond it steam suddenly condenses at constant pressure and enthalpy, and remains in stable condition. This limit line drawn on h-s diagram is known as *Wilson line*.

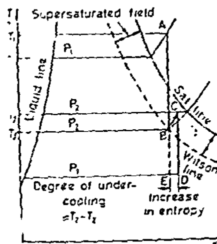
The main reasons for supersaturated flow are :—

(1) The flow of steam is so rapid that it does not allow time for the transfer of heat, and setting up of surface tension, etc. It

may take only 0.001 second for the steam to travel from mouth to nozzle exit.

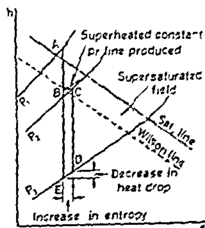
(2) There is absence of dust particles which form a nucleus for condensation.

The phenomenon of supersaturation is shown on T-s diagram in Fig. 12·8 and on h-s diagram in Fig. 12·9 AE represents the normal stable isentropic expansion. If the flow is supersaturated the



Supersaturation on T-s diagram

Fig. 12·8



Supersaturation on h-s diagram.

Fig. 12·9

steam is dry upto the point B on Wilson line. The pressure at B can be found by extending the superheated constant pressure lines upto B, as for all practical purposes Wilson line becomes the saturation line. BC represents stabilisation at constant pressure and total heat and CD is the further normal adiabatic expansion.

The temperature at B is less than the saturation temperature corresponding to the pressure as the gain in the kinetic energy of the steam is at the expense of its sensible heat. The difference of two temperatures is known as *degree of undercooling*. The ratio of pressures corresponding to two temperatures as saturation temperatures is known as *degree of supersaturation*. During the process CD the partial condensation releases sufficient heat to raise the temperature to saturation temeperature.

The effects of supersaturation are :—

1. The discharge is increased by 2 to 5 per cent due to increase in density at the throat which is about eight times the normal $\left[\text{From } PV=mRT, \frac{1}{V} \alpha P \alpha \frac{1}{T}\right]$.

2. The heat drop is slightly *decreased* reducing the exit velocity.

3. The final dryness fraction of steam is slightly increased.

4. The entropy is slightly increased (equal to BC in Fig. 12·8 and 12·9).

In supersaturated flow the law of expansion may be taken as $PV^{1\cdot3}$=constant. Combining this with $\frac{Pv}{T}$=constant, we get the relation $\frac{P}{T^{\frac{13}{3}}}$=constant (12·11)

The specific volume of supersaturated steam may be found by

$$Pv\times10^2=(h-464\cdot1) \qquad (12\cdot12)$$

The problems on supersaturated flow cannot be solved by Mollier chart unless Wilson line is drawn on it.

12·6. Theory of Steam Injectors. A steam injector is a device for feeding water into a boiler, using either live boiler steam or exhaust steam of an engine as the means of operation.

The steam enters a convergent steam nozzle designed for maximum discharge issuing therefrom with a high velocity and mixes with water coming from the feed tank E and condenses in the convergent combining nozzle B (*see* Fig. 12·10). The resulting mixture of water enters the divergent water nozzle D, where the kinetic energy of water is converted into the pressure energy to overcome the boiler pressure. An overflow for excess water is provided at C.

Important Note. It is not easily understood by the student how injector works when the heat energy removed from boiler is returned back in the condensed steam and the hot water boiler feed. Here it should be remembered that the potential energy removed (boiler pressure × volume of steam) is many times greater than the potential energy returned (boiler pressure × volume of condensate and boiler feed). the difference having been utilised for pumping water. Thus

injector operates only due to the large decrease in volume as the steam condenses.

Calculations : Let suffix 1 refer to entrance to steam nozzle, 2 to exit of steam nozzle, 3 to entrance to water nozzle and 4 to entrance to boiler.

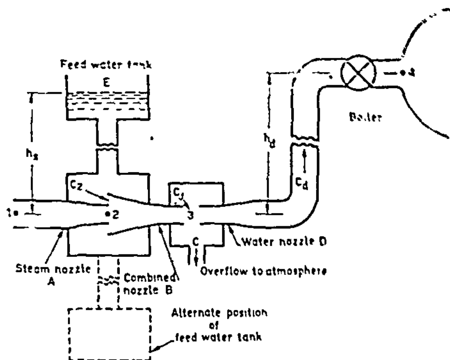


Fig. 12.10. Steam Injector.

C_2 = Steam velocity at exit from steam nozzle A

C_j = Jet velocity entering the water nozzle D.

C_d = Velocity of water in delivery pipe.

m = Mass of water drawn from the feed tank per kg of steam

m_1 = Mass of steam used per second.

h_s = Height of water over the injector.

h_d = Heat of boiler feed check valve above the water nozzle of the injector

P = Absolute pressure inside the boiler

t = Temperature of water in the feed tank,

t_1 = Saturation temperature of the initial steam.

(i) *Mass of water per kg of steam.*

Assuming the condition for maximum discharge,

$P_2 = 0{\cdot}58\, P_1$ (unless given in the problem), from which U_2 is calculated.

Applying Bernoulli's theorem to the points 3 and 4 and taking the centre line of injector as datum

$$0+\frac{C_j^2}{2g}+\frac{1{\cdot}033\times10^4}{10^3}=h_d+\frac{C_d^2}{2g}+\frac{P\times10^4}{10^3} \qquad (12{\cdot}13a)$$

h_d and C_d may be neglected if not given in the problem. In that case $\frac{1{\cdot}033\times10^4}{10^3}$ is also neglected but delivery pressure is assumed 20 per cent greater than the absolute boiler pressure to allow a factor of safety, which gives

$$\frac{C_j^2}{2g}=\frac{1{\cdot}2P\times10^4}{10^3} \qquad [12{\cdot}13(b)]$$

Now, equating momentums

Momentum of 1 kg of steam + momentum of m kg of water = Momentum of the resulting jet

$$\therefore \quad \frac{C_2}{g}+\frac{m}{g}\sqrt{2gh_s}=\frac{m+1}{g}\times C_j$$

[assuming feed tank above]

$$\therefore \quad C_j=\frac{C_2}{m+1}+\frac{m}{m+1}\sqrt{2gh_s} \qquad [12{\cdot}14(a)]$$

If water feed tank is below the injector,

$$C_j=\frac{C_2}{m+1}-\frac{m}{m+1}\sqrt{2gh_s} \qquad [12{\cdot}14(b)]$$

If h_s is not given the term $\frac{m}{m+1}\sqrt{2gh_s}$ is neglected, which gives

$$C_j=\frac{C_2}{m+1} \qquad (12{\cdot}14c)$$

(ii) *Area of steam nozzle.*

$$A=\frac{mx_2v_{g2}}{C_2}=\frac{mv_2}{C_2} \qquad (12{\cdot}15)$$

(iii) *Area of discharge end of combining nozzle or entrance of water nozzle.*

Quantity of water drawn from the feed tank $= m_1\times m$ kg/sec

Mass of mixture $= m_1+m_1\, m$ kg/sec

∴ Area of discharge end of combining nozzle

$$=\frac{m_1+m_1m}{10^3\times C_j} \qquad (12\cdot16)$$

(iv) *Temperature of feed water to the boiler.*

Neglecting losses,

Heat lost by steam = Heat gained by water + *K.E.* of the jet

$$\therefore \quad x_1\, h_{fg1}+(t_1-t_3)=m(t_3-t)+\frac{m+1}{J}\times\frac{C_j^2}{2g} \qquad [12\cdot17(a)]$$

The term $\frac{m+1}{2g}\times\frac{C_j^2}{2g}$, being small, is neglected which gives

$$x_1\, h_{fg1}+(t_1-t_3)=m(t_3-t) \qquad [12\cdot17(b)]$$

IMPORTANT POINTS

1. It is convenient to solve the problem with 1 kg and to introduce mass flow at the end of the problem.

2. Note from the language of the problem whether the flow is in thermal equilibrium (stable) or supersaturated (metastable) condition. Whenever the expansion is given $PV^{1\cdot3}=C$ supersaturated flow is implied.

3. Unless given in the problem, the velocity of approach of the steam is to be neglected.

4. In the equation of mass flow,

$$m=\frac{AC}{v}$$

volume of steam v is found as follows :

For wet steam, $v=x\times v_g$ [x = dryness fraction

For superheated steam v is found by Eq. $Pv\times10^2=h-464\cdot1$ or by gas laws $Pv^n=C$.

It should be checked from Steam Tables that the volume of superheated steam is greater than v_g for the corresponding pressure

The volume can also be read on Mollier chart directly if constant volume lines are drawn on it.

5. If friction is given, the whole of the friction loss is assumed between throat and exit. The enthalpy drop from mouth to throat is assumed independent of friction loss. The dryness fraction at exit is the corrected dryness fraction taking friction into account.

6. If the flow is supersaturated, h-s (Mollier) chart cannot be used. Such problems are solved by taking $n=1\cdot3$ and initial volume by equation given in (4) above.

Final volume is calculated by PV^n=constant and final temperature by

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

7. In supersaturated flow with friction, final temperature is found by adding to the temperature T_2 the rise in temperature due to reheating, assuming a suitable value for specific heat, if not given.

8. In supersaturated flow, unless given, metastable condition is assumed upto throat.

9. The steam flow is same in both thermal equilibrium and supersaturated flow conditions if the throat pressure is in superheated range.

10. When the pressure drop is very small the problems should be solved by Steam Tables as Mollier chart cannot be read accurately in such cases.

11. The problem should generally be solved with unit mass and the mass flow should be introduced at the end of the problem.

12. In problems on steam injector the following assumptions are made :—

(*i*) The steam nozzle is designed for maximum discharge.

(*ii*) The boiler pressure is taken 20 per cent greater if velocity in delivery pipe is not given.

(*iii*) Small values are neglected in Eq. [12·13(*b*)], [12·14(*c*)] and [12·17(*b*)].

ILLUSTRATIVE EXAMPLES

12·1. Throat and outlet areas with and without friction.

Steam expands in a nozzle under the following conditions :—

Initial pressure $=15$ *kgf/cm*^{2}

Initial temperature $=250°C$

Final pressure $=4$ *kgf/cm*^{2}

Mass flow $=1$ *kg/sec*

Calculate the required throat and exit areas, [illegible] diagram,

(a) *when the expansion is frictionless ;*

(b) *when the friction loss at any pressure amounts to [illegible] of the total heat drop down to that pressure.*

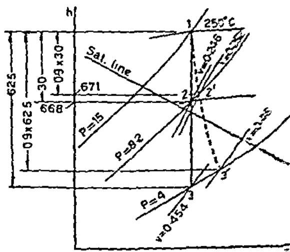


Fig. 12·11.

Throat pressure $P_2 = 0{\cdot}545 \times 15 = 8{\cdot}2$ kgf/cm²

A sketch of the readings taken from [illegible] Fig. 12·11.

(a) *Frictionless expansion.*

From Mollier chart,

Heat drop upto throat, $h_1 - h_2 = 30$ kcal, $h_2 = 66[\text{illegible}]$ kcal

$$v_2 = 0{\cdot}246\ m^3/kg$$

Velocity at throat, $C_2 = 91{\cdot}5[\text{illegible}]\sqrt{30} = [\text{illegible}]$ m/s

$$m = \frac{AU}{v}$$

$$1 = \frac{A_2 \times 10^{-4} \times [\text{illegible}]}{0{\cdot}246}$$

$\therefore$ Throat area, $\underline{A_2 = 4{\cdot}9\ cm^2}$

From Mollier chart, Heat drop upto exit = [illegible]

$$v_3 = 0{\cdot}454\ m^3/kg$$

6. If the flow is supersaturated, h-s (Mollier) chart cannot be used. Such problems are solved by taking $n=1{\cdot}3$ and initial volume by equation given in (4) above.

Final volume is calculated by PV^n=constant and final temperature by

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

7. In supersaturated flow with friction, final temperature is found by adding to the temperature T_2 the rise in temperature due to reheating, assuming a suitable value for specific heat, if not given.

8. In supersaturated flow, unless given, metastable condition is assumed upto throat.

9. The steam flow is same in both thermal equilibrium and supersaturated flow conditions if the throat pressure is in superheated range.

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ILLUSTRATIVE EXAMPLES

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Final pressure $=4$ *kgf/cm*^{2}

Mass flow $=1$ *kg/sec*

Calculate the required throat and exit areas, using the Mollier diagram,

(a) *when the expansion is frictionless ;*

(b) *when the friction loss at any pressure amounts to 10 per cent of the total heat drop down to that pressure.*

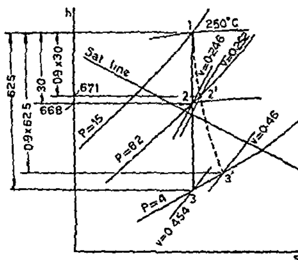


Fig 12·11.

Throat pressure $P_2=0{\cdot}545\times15=8{\cdot}2$ kgf/cm²

A sketch of the readings taken from h-s diagram is given in Fig. 12·11.

(a) *Frictionless expansion.*

From Mollier chart,

Heat drop upto throat, $h_1-h_2=30$ kcal, $h_2=668$ kcal

$$v_2=0{\cdot}246\ m^3/\text{kg}$$

Velocity at throat, $C_2=91{\cdot}53\sqrt{30}=501\ m/s$

$$m=\frac{AU}{v}$$

$$1=\frac{A_2\times10^{-4}\times501}{0{\cdot}246}$$

∴ Throat area, $A_2=4{\cdot}9$ cm² Ans.

From Mollier chart, Heat drop upto exit $=62{\cdot}5$ kcal,

$$v_3=0{\cdot}454\ m^3/\text{kg}$$

Velocity at exit, $C_3 = 91{\cdot}53\sqrt{62{\cdot}5} = 724$ m/s

$$1 = \frac{A_3 \times 10^{-4} \times 724}{0{\cdot}454}$$

$\therefore$ Exit area, $\underline{A_3 = 6{\cdot}27 \text{ cm}^2}$ Ans.

(*b*) *Expansion with 10 per cent friction loss.*

Useful heat drop upto throat $= 0{\cdot}9 \times 30 = 27$ kcal

Velocity at throat, $C_2' = 91{\cdot}53\sqrt{27} = 475{\cdot}5$ m/s

From Mollier chart, $h_2' = 671$ kcal and $v_2' = 0{\cdot}252$ m^3/kg

$$1 = \frac{A_2' \times 10^{-4} \times 475{\cdot}5}{0{\cdot}252}$$

$\therefore$ Throat area, $\underline{A_2' = 5{\cdot}3 \text{ cm}^2}$ Ans.

Useful heat drop upto exit $= 0{\cdot}9 \times 62{\cdot}5 = 56{\cdot}25$ kcal

Specific volume at $3' = 0{\cdot}46$ m^3/kg [From Mollier chart

Velocity at exit $= 91{\cdot}53\sqrt{56{\cdot}25} = 687$ m/s

$$1 = \frac{A_3' \times 10^{-4} \times 687}{0{\cdot}46}$$

$\therefore$ Exit, area, $\underline{A'_2 = 6{\cdot}71 \text{ cm}^2}$ Ans.

Note. Generally friction between mouth and throat is neglected as it is too small and total loss due to friction is assumed in the diverging portion only. However, in this problem friction between mouth and throat has been considered as it is specifically given in the problem.

12·2. Throat and exit areas considering initial velocity and friction.

Distinguish carefully between an adiabatic and an isentropic process using pressure-volume and absolute temperature-entropy diagrams to illustrate your answer.

Steam flows through the nozzle of a turbine at the rate of 4kg per second from a pressure of 8 kgf/cm² and 200°C, to a pressure of 2 kgf/cm². Assuming that (i) the flow between the entry and the throat of the nozzle is frictionless and adiabatic, (ii) there is 10 per cent loss due to friction in the divergent part of the nozzle, and (iii) the

initial velocity of steam entering the nozzle is 70 m/s calculate the areas of the throat and the exit of the nozzle.

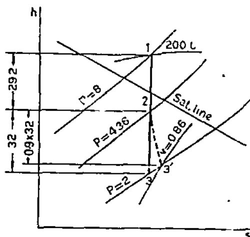


Fig 12 12

Throat pressure, $P_2=0.545\times8=4.36$ kgf/cm²

The diagram shows the data taken from Mollier chart

$$h_1-h_2=29.2 \text{ kcal}$$

$$C_2^2-C_1^2=2gJ(h_1-h_2)$$

$$C_2^2-70^2=2\times9.81\times427\times29.2 \quad \therefore\ C_2=499 \text{ m/s}$$

$$m=\frac{AC}{v}$$

$$4=\frac{A_2\times10^{-4}\times499}{0.43}$$

$\therefore$ Throat area, $A_2=34.5$ cm² Ans.

From Mollier chart, $h_2-h_3=32$ kcal

Useful heat drop upto exit $=29.2+0.9\times32=58$ kcal

From Mollier chart, $v_3'=0.86$ m³/kg

$$C_3'^2-70^2=2\times9.81\times427\times58$$

$\therefore$ $C_3'=698$ m/s

$$4=\frac{A_3'\times10^{-4}\times698}{0.86}$$

$\therefore$ Exit area, $A_3'=49.4$ cm² Ans.

Note.—(*i*) In this problem velocity of approach of the steam is given. Increase in the K.E. is $\frac{C_2^2}{2g}-\frac{C_1^2}{2g}$, which is equal to the heat drop in the same units.

(*ii*) 10 per cent loss is limited to heat drop from throat to exit and not to total heat drop.

12·3. Number of nozzles : exit diameter.

Explain the principle involved in calculation of the velocity with which steam issues from a nozzle, assuming frictionless adiabatic flow.

An impulse turbine is to develop 1,200 k. w. with a steam consumption of 8·4 kg per kw-hour, steam being initially at 20 kgf/cm² and 300°C. If the throat diameter of each nozzle is 1·2 cm, the exhaust pressure is 0·18 kgf/cm², and 12 per cent of the total heat drop is lost in overcoming friction in the diverging portion of the nozzle, calculate (a) the number of nozzles required, and (b) the exit diameter of each nozzle.

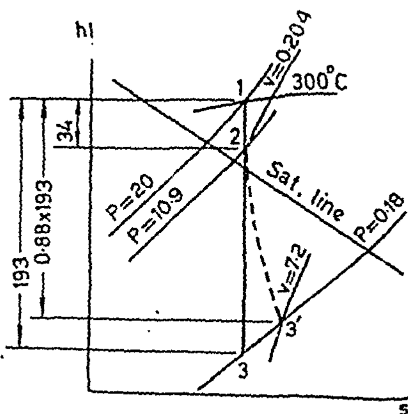


Fig. 12·13.

For theory—See text

(*a*) Throat pressure, $P_2=0{\cdot}545\times 20=10{\cdot}9$ kgf/cm².

The diagram shows the data taken from the h-s diagram.

From Mollier chart, $h_1-h_2=34$ kcal ; $v_2=0{\cdot}204$ m³/kg

Velocity at throat, $C_2=91{\cdot}53\sqrt{94}=534$ m/s

$$m=\frac{AC}{v}=\frac{\frac{\pi}{4}\times(1{\cdot}2)^2\times 10^{-4}\times 534}{0{\cdot}204}=0{\cdot}296 \text{ kg/s}$$

$\therefore$ No. of nozzles$=\dfrac{1{,}200\times 8{\cdot}4}{60\times 60\times 0{\cdot}296}=9{\cdot}46$ or say 10 Ans.

(*b*) From Mollier chart, $h_1-h_3=193$ kcal

Useful heat drop$=0{\cdot}88\times 193=170{\cdot}1$ kcal

Velocity at exit $=91{\cdot}53\sqrt{170{\cdot}1}=1{,}194$ m/s

$$0{\cdot}296=\frac{A_3'\times 10^{-4}\times 1{,}194}{7{\cdot}2} \qquad [v_3'=7{\cdot}2 \text{ m}^3/\text{kg}$$

or Exit area, $A_3'=17{\cdot}8$ cm^2 $\therefore$ Exit diameter $=4{\cdot}75$ cm Ans.

12·4. Temperature and velocity at throat : cone angle.

A nozzle is supplied with steam at 7 kgf/cm² and 275°C. Find the temperature and velocity at the throat.

If the diverging portion is 50 mm long and the throat diameter 6 mm determine the angle of the cone so that steam may leave the nozzle at 1 kgf/cm².

Assume a friction loss of 15 per cent of the heat drop in the diverging part.

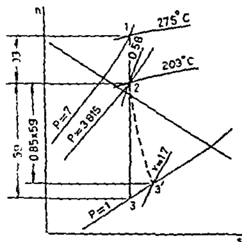


Fig. 12·14.

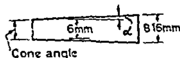


Fig. 12·15.

Throat pressure $=0{\cdot}545\times 7=3{\cdot}815$ kgf/cm^2

From mollier chart, $h_1-h_2=33$ kcal ; $v_2=0{\cdot}58$ m^3/kg .

$t_2=203°C$ Ans.

$\therefore$ Velocity at throat $=91{\cdot}53\sqrt{33}=526$ m/s Ans.

$$m=\frac{AC}{v}=\frac{\frac{\pi}{4}\times(0{\cdot}06)^2\times 526}{0{\cdot}58}=0{\cdot}02565 \text{ kg/s}$$

From Mollier chart, heat drop in diverging part $=59$ kcal

Net heat drop, $h_2-h_3'=0{\cdot}85\times 59=50{\cdot}15$ kcal

From Mollier chart, $v_3'=1{\cdot}7$ m^3/kg

Total useful heat drop $=33+50{\cdot}15=83{\cdot}15$ kcal

$$\text{Velocity at exit} = 91{\cdot}53\sqrt{83{\cdot}15} = 836 \text{ m/s}$$

$$0{\cdot}02565 = \frac{A \times 10^{-4} \times 836}{1{\cdot}7}$$

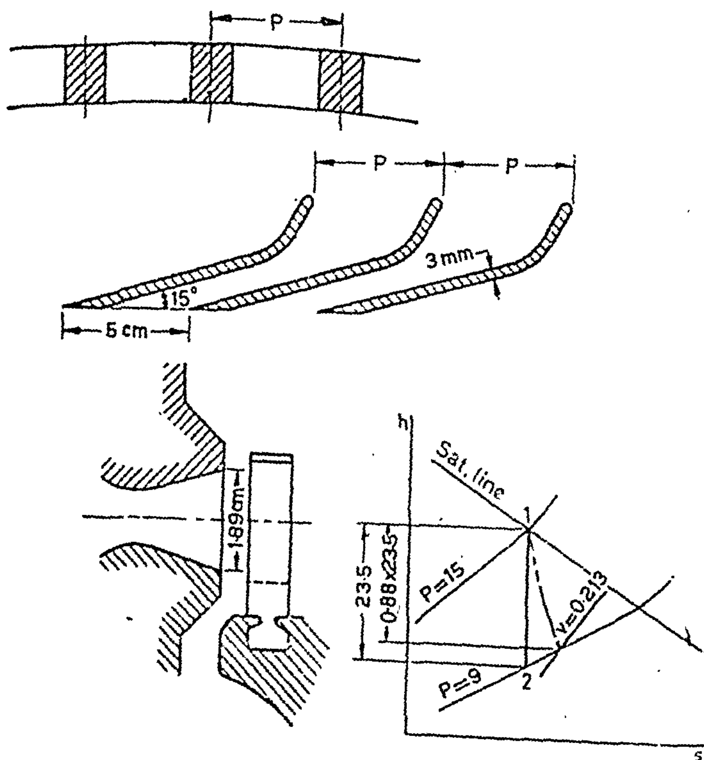
or $\quad A = 0{\cdot}521 \text{ cm}^2 \qquad \therefore\ d = 8{\cdot}16 \text{ mm}$

$$\tan \alpha = \frac{8{\cdot}16 - 6}{2 \times 50} \qquad \therefore\ \alpha = 1{\cdot}25°$$

$\therefore$ Cone angle $= 2 \times 1{\cdot}25 = 2{\cdot}5°$ Ans.

12·5. No. of nozzles ; length of nozzle arc ; nozzle height.

A steam turbine is required to generate 4,000 hp using 4·3 kg of steam per hp-hr, at 15 kgf/cm² dry saturated. The turbine is of the impulse type and in the first stage the steam is expanded through nozzle with an efficiency of 0·88 to a pressure of 9 kgf/cm². The nozzles are to extend over approximately one-third of the circumference with a pitch circle diameter of 60 cm and a pitch of 5 cm. The nozzle angle being 15° to the plane of the wheel and the division plates 3 mm thick, determine the total length of the nozzle arc and the radial height of the nozzles.



From Mollier chart, $h_1 - h_2 = 23 \cdot 5$ kcal

$\therefore$ Useful heat drop $= 0 \cdot 88 \times 23 \cdot 5 = 20 \cdot 68$ kcal

From Mollier chart, $v_2' = 0 \cdot 213$ m³/kg

Velocity of outlet, $C_2 = 91 \cdot 53 \sqrt{20 \cdot 68} = 417$ m/s

$$m = \frac{AC}{v}$$

or $$\frac{4,000 \times 4 \cdot 3}{60 \times 60} = \frac{A \times 10^{-4} \times 417}{0 \cdot 217}$$

$\therefore$ $A = 24 \cdot 4$ cm²

Approximate length of nozzle arc $= \frac{60 \times \pi}{3} = 62 \cdot 8$ cm

$\therefore$ No. of nozzles $= \frac{\text{Length}}{\text{Pitch}} = \frac{62 \cdot 8}{52} = 13$ (say) **Ans.**

Length of nozzle arc $= 13 \times 5 = 65$ cm **Ans.**

Area of flow at exit per jet $= (5 \sin 15° - 0 \cdot 3)$ h

$24 \cdot 4 = (5 \sin 15° - 0 \cdot 3)\, h \times 13$

$\therefore$ Height of nozzle, $h = 1 \cdot 89$ cm **Ans.**

Note.—Circular nozzles are generally not used in practice as the issuing jet is elliptical in cross-section which does not cover the blades completely. Plate nozzles give approximately the same shape as the entrance to the blades *i e*, rectangular, which covers the blades completely. The blade arrangement is shown in Fig. 12·16.

12·6. Measurement of steam flow.

A flow nozzle of 150 mm diameter in a 200 mm diameter pipe is used to meter the steam which arrives at the upstream pressure tap at 3 kgf/cm² 150°C. If the differential pressure is 200 cm of water, find the flow rate in kg per hour. Assume specific heat of superheated steam $= 0 \cdot 52$.

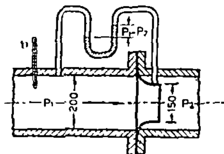


Fig 12 18.

$$\text{Pressure at throat} = 3 - \frac{200}{75{\cdot}056 \times 6{\cdot}8} = 2{\cdot}8 \text{ kgf/cm}^2$$

As the pressure drop is very small, Mollier chart will not give accurate results ; hence the problem is solved by Steam Tables.

From steam tables, at 3 kgf/cm² and 150°C

$$h_1 = 659{\cdot}7 \text{ kcal} ;\ s_1 = 1{\cdot}693 ;$$

$$v_1 = 0{\cdot}6472 \text{ m}^3/\text{kg}$$

Entropy before = Entropy after

$$1{\cdot}693 = 1{\cdot}677 + 0{\cdot}52 \log_e \frac{T}{403{\cdot}6}$$

∴ Temperature after nozzle, $T = 416°K$

Enthalpy after expansion,

$$h_2 = 650 + 0{\cdot}52\ (416 - 403{\cdot}6) = 656{\cdot}45 \text{ kcal}$$

∴ Heat drop, $h_1 - h_2 = 659{\cdot}7 - 656{\cdot}45 = 3{\cdot}25$ kcal

Now, $Pv \times 10^2 = h - 464{\cdot}1$

$$2{\cdot}8 \times v \times 10^2 = 656{\cdot}45 - 464{\cdot}1 \quad \therefore \quad v = 0{\cdot}687 \text{ m}^3/\text{kg}$$

$$C = \frac{mv}{A} \qquad [m = \text{mass in kg}$$

Velocity at entrance to nozzle,

$$C_1 = \frac{m \times 0{\cdot}6472}{\frac{\pi}{4}\ (0{\cdot}2)^2} = 20{\cdot}6m \text{ m/s}$$

Velocity at exit to nozzle,

$$C_2 = \frac{m \times 0{\cdot}687}{\frac{\pi}{4}\ (0{\cdot}15)^2} = 38{\cdot}9m \text{ m/s}$$

$$C_2^2 - C_1^2 = 2gJ(h_1 - h_2)$$

$$m^2[(38{\cdot}9)^2 - (20{\cdot}6)^2] = 2 \times 9{\cdot}81 \times 427 \times 3{\cdot}25$$

∴ Mass flow, $\underline{m = 5 \text{ kg/s or } 18{,}000 \text{ kg/hr}}$ Ans.

$$=\frac{4\cdot5\times10^{-4}\times448}{0\cdot45}=\underline{0\cdot448 \text{ kg/s}}$$ Ans.

(b) Force on plate, $F=\frac{mC}{g}$

$\therefore$ Actual velocity at exit, $C_3'=\frac{35\cdot2\times9\cdot81}{0\cdot448}=771$ m/s

From Mollier chart, $h_1-h_3=79$ kcal

Velocity, $C_3'=91\cdot53\sqrt{K(h_1-h_3)}$ [K=Nozzle efficiency]

$771=91\cdot53\sqrt{K\times79}$

$\therefore$ Nozzle efficiency, $\underline{K=89\cdot7 \text{ per cent}}$ Ans.

Note. The problem shows the method of finding the nozzle efficiency experimentally.

12·8. Supersaturated flow upto throat : exit dia. ; degree of supersaturation ; amount of undercooling.

What are the conditions which produce supersaturation of steam ? How does the area of the throat of a turbine-nozzle for supersaturated flow compare with the area determined for normal flow ?

Three kg of steam per minute are discharged from a convergent divergent nozzle. The pressure of the dry steam supplied to the nozzle chest is 10 kgf/cm² and temperature 200°C, while the discharge pressure is 0.07 kgf/cm². The expansion is supersaturated upto throat and normal afterwards.

Calculate,

(a) *the diameter of the nozzle at exit,*

(b) *the maximum degree of supersaturation,*

(c) *the amount of undercooling at the throat.*

For supersaturated conditions take $PV^{1.3}$=constant and

$$P=\text{constant}\times T^{\frac{1\cdot3}{\cdot3}}.$$

For theory—See text.

(a) The area of throat for supersaturated flow will be less than the area for normal flow for the same discharge.

As the problem on supersaturation cannot be solved by Mollier chart it is solved by Steam Tables.

From Steam Tables, at 10 kgf/cm² and 200°C $h_1=676\cdot4$ kcal/kg ;

$s_1=1\cdot601$; $v_1=0\cdot2103$ m³/kg

Heat drop, $h_1-h_2=\frac{n}{n-1}\times\frac{P_1V_1}{J}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]$

$$=\frac{1\cdot3}{1\cdot3-1}\times\frac{10\times10^4\times0\cdot2103}{427}\left[1-(0\cdot545)^{\frac{1\cdot3-1}{1\cdot3}}\right]$$
$$=28 \text{ kcal}$$

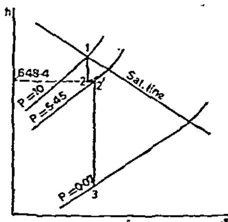


Fig. 12·20.

Enthalpy at 2', $h_2'=676\cdot4-28=648\cdot4$ kcal

Pressure at throat, $P_2=0\cdot545\times10=5\cdot45$ kgf/cm²

Knowing pressure and enthalpy, point 2' is marked on Mollier chart.

From Mollier chart, $h_1-h_3=179\cdot5$ kcal ; $v_3=17$ m^3/kg

Velocity at exit $C_3=91\cdot53\sqrt{179\cdot5}=1227$ m/s

$$m=\frac{AC}{v}$$

$$\frac{3}{60}=\frac{A\times10^{-4}\times1227}{17} \quad \text{or} \quad A=6\cdot94 \text{ cm}^2$$

∴ Diameter of nozzle at exit, $\underline{d=2\cdot98 \text{ cm}}$ Ans.

(b) $$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

∴ $$T_2=473(0\cdot545)^{\frac{1\cdot3-1}{1\cdot3}}=410°\text{K}$$

Pressure corresponding to 410°K saturation temperature
$=3\cdot4$ kgf/cm²

Degree of supersaturation $= \frac{5 \cdot 45}{3 \cdot 4} = 1 \cdot 6$ Ans.

(c) Amount of undercooling $= T_2' - T_2 = 427 \cdot 4 - 410 = 17 \cdot 4°C$ Ans.

12·9. Degree of supersaturation and undercooling ; Δs ; loss due to undercooling.

Steam which is initially dry and saturated expands in a nozzle from 15 kgf/cm² to 6 kgf/cm². The expansion is frictionless throughout. During the expansion upto throat the steam remains in the dry state. What is the degree of supersaturation and of undercooling.

If the steam were to revert instantaneously to the saturated state at constant enthalpy and if the further expansion takes place in thermal equilibrium, calculate the change of entropy, the loss due to undercooling and the percentage loss.

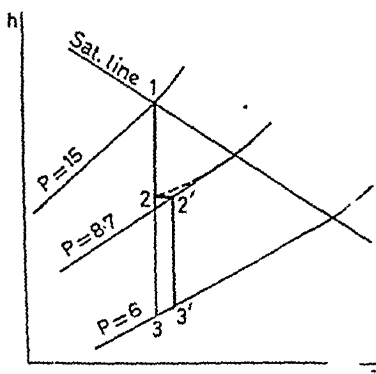


Fig. 12·21.

Throat pressure $= 15 \times 0 \cdot 58 = 8 \cdot 7$ kgf/cm²

From Steam Tables, $T_1 = 470 \cdot 4°K$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{T_2}\right)^{\frac{n-1}{n}}, \quad \frac{470 \cdot 4}{T_2} = \left(\frac{1}{0 \cdot 58}\right)^{\frac{1 \cdot 3 - 1}{1 \cdot 3}} \quad \therefore \quad T_2 = 415°K$$

Saturation temperature corresponding to 8·7 kgf/cm² = 446·1°K

Pressure corresponding to 415°K saturation temperature = 3·898 kgf/cm²

$\therefore$ Degree of supersaturation $= \frac{8 \cdot 7}{3 \cdot 898} = 2 \cdot 235$ Ans.

Degree of undercooling $= T_2' - T_2 = 446 \cdot 1 - 415 = 31 \cdot 1°C$ Ans.

Heat drop, $h_1 - h_2 = (h_1 - 464 \cdot 1)\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]$

$$= (666 \cdot 7 - 464 \cdot 1)\left[1 - (0 \cdot 50)^{\frac{1 \cdot 3 - 1}{1 \cdot 3}}\right] = 24 \text{ kcal}$$

$$h_2' = 666 \cdot 7 - 24 = 642 \cdot 7 \text{ kcal}$$

$$s_2' = 0 \cdot 495 + \frac{642 \cdot 7 - 175}{487} \times 1 \cdot 091 = 1 \cdot 543$$

$\therefore$ Change of entropy $= s_2 - s_1 = 1 \cdot 543 - 1 \cdot 541 = 0 \cdot 002$ **Ans.**

With no supersaturation, $s_1 = s_3$

or $1 \cdot 541 = 0 \cdot 459 + x_3 \times 1 \cdot 157$

$\therefore$ $x_3 = 0 \cdot 9353$

and $h_3 = 159 \cdot 4 + 0 \cdot 9353 \times 498 \cdot 9 = 626$ kcal

$\therefore$ Loss due to undercooling $= h_2' - h_3 = (s_2' - s_2)\, T_3$

$= 0 \cdot 002 \times 431 \cdot 1 = 0 \cdot 8622$ *kcal* **Ans.**

Percentage loss $= \frac{0 \cdot 8622}{666 \cdot 7 - 626} = 2 \cdot 12\%$ **Ans.**

Note.—The problem on supersaturation cannot be solved by Mollier chart

12·10. Supersaturation with friction : actual and isentropic heat drop ; degree of undercooling

Explain what is meant by the supersaturated expansion of steam and give some idea of the limits within which the condition is possible. Steam is expanded from 4 kgf/cm² and 170°C to a pressure of 1·4 kgf/cm². If the expansion is supersaturated, and occurs with a friction loss of 5 per cent, determine the actual heat drop and degree of undercooling.

For supersaturated steam you may use the approximate equation :

$$Pv \times 10^2 = (h - 464 \cdot 1)$$

$$\frac{P}{T^{\frac{13}{3}}} = \textit{constant}\ ,\ PV^{1 \cdot 3} = \textit{constant}$$

Take specific heat as 0·52. Specific volume at 4 ... 170°C = 0·507 m³/kg.

Degree of supersaturation $=\frac{5\cdot45}{3\cdot4}=1\cdot6$ Ans.

(c) Amount of undercooling $=T_2'-T_2=427\cdot4-410=17\cdot4°C$ Ans.

12·9. Degree of supersaturation and undercooling ; Δs ; loss due to undercooling.

Steam which is initially dry and saturated expands in a nozzle from 15 kgf/cm² to 6 kgf/cm². The expansion is frictionless throughout. During the expansion upto throat the steam remains in the dry state. What is the degree of supersaturation and of undercooling.

If the steam were to revert instantaneously to the saturated state at constant enthalpy and if the further expansion takes place in thermal equilibrium, calculate the change of entropy, the loss due to undercooling and the percentage loss.

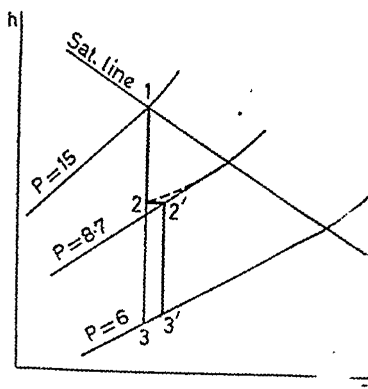


Fig. 12·21.

Throat pressure $=15\times0\cdot58=8\cdot7$ kgf/cm²

From Steam Tables, $T_1=470\cdot4°K$

$$\frac{T_1}{T_2}=\left(\frac{P_1}{T_2}\right)^{\frac{n-1}{n}},\quad \frac{470\cdot4}{T_2}=\left(\frac{1}{0\cdot58}\right)^{\frac{1\cdot3-1}{1\cdot3}}\qquad\therefore\quad T_2=415°K$$

Saturation temperature corresponding to 8·7 kgf/cm² $=446\cdot1°K$

Pressure corresponding to 415°K saturation temperature $=3\cdot898$ kgf/cm²

$\therefore$ Degree of supersaturation $=\frac{8\cdot7}{3\cdot898}=2\cdot235$ Ans.

Degree of undercooling $=T_2'-T_2=446\cdot1-415=31\cdot1°C$ Ans.

Fig. 12·22.

For theory—See text.

$$Pv \times 10^2 = (h - 464{\cdot}1)$$

$$4 \times 0{\cdot}507 \times 10^2 = h_1 - 464{\cdot}1 \quad \therefore \quad h_1 = 666{\cdot}9 \text{ kcal}$$

$$Pv^{1{\cdot}3} = \text{constant.}$$

$$4 \times (0{\cdot}507)^{1{\cdot}3} = 1{\cdot}4 \times v_2^{1{\cdot}3} \quad \therefore \quad v_2 = 1{\cdot}14 \text{ m}^3/\text{kg}$$

$$1{\cdot}4 \times 1{\cdot}14 \times 10^2 = h_2 - 464{\cdot}1 \quad \therefore \quad h_2 = 623{\cdot}7 \text{ kcal}$$

Actual heat drop $= k(h_1 - h_2) = 0{\cdot}95(666{\cdot}9 - 623{\cdot}7)$

$= 41$ kcal **Ans.**

$$\frac{P}{T^{13/3}} = \text{constant,} \quad \frac{4}{(433)^{13/3}} = \frac{1{\cdot}4}{(T_2)^{13/3}} \quad \therefore \quad T_2 = 348°K$$

Rise in temperature due to reheat

$$= \frac{0{\cdot}05 \times (666{\cdot}9 - 623{\cdot}7)}{0{\cdot}52} = 4{\cdot}16°C$$

$$\therefore \quad T_2' = 348 + 4{\cdot}16 = 352{\cdot}16°C$$

$\therefore$ Degree of undercooling $= T_3 - T_2'$

$= 381{\cdot}7 - 352{\cdot}16 = 29{\cdot}5°C$ **Ans.**

12·11. Steam injector : diameter of steam and mixing nozzle ; feed water temperature.

An injector is to deliver 120 kg of water per min from a tank, whose constant water level is 3 m below the level of the injector, into a boiler in which the steam pressure is 15 kgf/cm². The water level in

he boiler is 0·7 m above the level of the injector. The steam for injector is taken from the same boiler and it is found to have a dryness fraction of 0·95. Calculate, (a) the mass of water pumped per kg of steam, (b) the diameter of throat of mixing nozzle, (c) the diameter of the steam nozzle assuming pressure at throat to be 0·6 of the supply pressure, and (d) the temperature of water leaving the injector, if the temperature of supply water is 25°C. The velocity in the delivery pipe may be taken as 15 m/s.

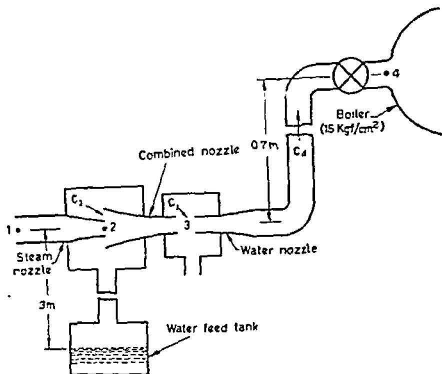


Fig 12·23

(a) Exit pressure of steam $=0{\cdot}6\times15=9$ kgf/cm²

From Mollier chart, $h_1-h_2=22{\cdot}2$ kcal , $v_2=0{\cdot}2$ m³/kg

Steam velocity, $C_2=91{\cdot}53\sqrt{22{\cdot}2}=431{\cdot}2$ m/s

Applying Bernoulli's theorem to the points 3 and 4 and taking centre line of injetor as datum,

$$\frac{C_l^2}{2g}+\frac{1{\cdot}033\times10^4}{10^3}=\frac{15\times10^4}{10^3}+0.7+\frac{15^2}{2g}$$

$$C_l=54{\cdot}6 \text{ m/s}$$

Equating momentums, $\frac{C_2}{g} \pm \frac{m}{g}\sqrt{2gh_s} = \frac{(m+1)}{g} C_j$

$\therefore$ $431 \cdot 2 - m\sqrt{2g \times 3} = (m+1)\ 54 \cdot 6$

[As water is below the injector

$\therefore$ Mass of water per kg of steam, $m = 6 \cdot 05$ kg   Ans.

(b) Mass of mixture passing section 3

$$= \frac{120}{60} + \frac{120}{60} \times \frac{1}{6 \cdot 05} = 2 \cdot 331 \text{ kg}$$

Area of mixing nozzle

$$= \frac{\text{Total discharge}}{\rho \times C_j} = \frac{2 \cdot 331}{54 \cdot 6} = 0 \cdot 427 \text{ cm}^2$$

$\therefore$ Diameter of throat of mixing nozzle $= 0 \cdot 737$ cm   Ans.

(c) Mass of steam per sec, $m_1 = \frac{120}{60} \times \frac{1}{6 \cdot 05} = 0 \cdot 331$ kg

$$m_1 = \frac{AC}{v}$$

$\therefore$ Area of steam nozzle,

$$A = \frac{0 \cdot 331 \times 0 \cdot 2 \times 10^4}{431 \cdot 2} = 1 \cdot 539 \text{ cm}^2$$

$\therefore$ Diameter of steam nozzle $= 1 \cdot 4$ cm   Ans.

(d) Assuming perfect heat interchange

$$x_1 h_{fg1} + (t_1 - t_3) = m(t_3 - t)$$

$$0 \cdot 95 \times 466 + (197 \cdot 4 - t_3) = 6 \cdot 05(t_3 - 25)$$

$\therefore$ Temperature of leaving water,

$$t_3 = 112 \cdot 3°C$$ Ans.

12·12. Exhaust steam injector : water per kg of steam ; area of steam and discharge orifices : feed temperature.

An exhaust steam injector is to be used for feeding a locomotive boiler in which the steam pressure is 15 kgf/cm². If the pressure of the exhaust steam for working the injector is 1·2 kgf/cm² and its dryness fraction is 0·85, estimate the mass of water that can be pumped per kg of steam, the area of steam and water discharge orifices, and the feed temperature, if the mass of water taken from the feed tank is 5,000 kg per hour and its temperature is 10°C.

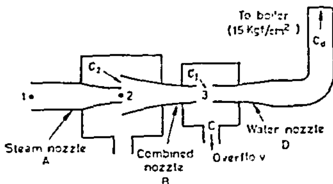


Fig. 12.24.

Pressure at the end of nozzle, $P_2=0.58\times 1.2=0.7$ kgf/cm²

From Mollier chart, $h_1-h_2=18.2$ kcal ; $v_2=0.2$ m³/kg

Steam velocity, $C_2=91.53\sqrt{18.2}=390$ m/s

Neglecting small quantities and assuming delivery pressure 20 per cent greater than the boiler pressure for overcoming frictional losses and having positive delivery

$$\frac{C_f^2}{2g}=\frac{1.2\times 15}{1,000} \quad \therefore \quad C_f=59.4 \text{ m/s}$$

Neglecting momentum of water, $C_f=\dfrac{C_2}{m+1}$

$$59.4=\frac{390}{m+1} \quad \therefore \quad m=5.57 \text{ kg per kg of steam}$$ Ans.

Mass of steam per sec $=\dfrac{5,000}{3,600\times 5.57}=0.2497$ kg

Area of steam nozzle $=\dfrac{mv}{C}$

$$=\frac{0.2497\times 0.2}{390}=12.78 \text{ cm}^2$$ Ans.

Total discharge from injector $=\dfrac{5,000}{3,600}+\dfrac{5,000}{5.57\times 3,600}=1.637$ kg

Area of discharge orifice $=\dfrac{\text{Total discharge}}{C}=\dfrac{1.637\times 10^3}{59.4\times 10^2}$

$=0.276$ cm² Ans.

Heat lost by steam = Heat gained by water + K.E. of jet

$$560.15-t_3=5.57(t_3-10)+\frac{6.57}{2\times 9.81}\times\frac{59.4^2}{427}$$

∴ Feed temperature, $t_3=93.4°C$ Ans.

Note.—In this problem heights of the feed tank and boiler from the injector level are not given. Also the velocity of water in delivery pipe is not given. Such problems are solved by neglecting the small quantities.

EXAMPLES 12

12·1. Throat and exit diameter considering friction.

Derive the expression relating the critical pressure ratio to the index of expansion, n, for expansion in a nozzle.

Steam at a pressure of 10 kgf/cm² and a dryness of 0·96 is to be discharged at a rate of 100 kg/hr through a convergent-divergent nozzle to a back pressure of 1·4 kgf/cm².

Find suitable diameters for the throat and exit assuming that 10 per cent of the overall isentropic enthalpy drop reheats the steam in the divergent portion of the nozzle. For wet steam take n as 1·13 for the purpose of determining the critical pressure. The chart may be used, if desired

[$P=5{\cdot}8$ kgf/cm² : $\triangle h_{1-2}=23{\cdot}5$ kcal ; $v_2=0{\cdot}31$ m³/kg ; throat dia $=0{\cdot}497$ cm : $\triangle h_{1-3}=69{\cdot}75$; $v_3=1{\cdot}1$ m³/kg ; exit dia$=0{\cdot}711$ cm]

12·2. Frictionless flow : throat area ; speed at throat, given initial velocity.

Assuming that the law of isentropic expansion of steam in a nozzle is PV^n=*constant, prove that the critical pressure ratio is given by*

$$\frac{P_{at\ throat}}{P_{at\ exit}}=\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

Use the chart to find the necessary area of throat of a nozzle supplied with steam at 10 kgf/cm² and 200°C, if the critical pressure ratio is 0·56 and the rate of flow is 1·2 kg per second, assuming the speed at inlet to be small and neglecting friction.

What will be the speed at the throat if the steam enters the nozzle at 100 m per second?

[$\triangle h=27{\cdot}4$ kcal ; $v_2=0{\cdot}34$m³/kg ; $C=479$ m/s ; area of throat $=8{\cdot}52$ cm² : with initial velocity, $C=489$ m/s]

12·3. No. of nozzles : corresponding outlet diamensions.

Explain precisely effect of friction on the flow of steam through a nozzle.

At one stage of a steam turbine, the nozzles expand 9 kg of steam per second from a pressure of 15 kgf/cm² with 250°C to 6 kgf/cm². The actual heat drop in the nozzles is 42 kcal. Calculate the number of nozzles required to give an outlet for each nozzle approximately 3·5 cm² and adjust the outlet dimensions to suit this number.

[C_2=594 m/s ; x_2=0·997 ; m=0·65 kg/s ; no of nozzles=14 ; actual size=3·47 cm²]

12 4. Convergent-divergent nozzles : number required throat and exit areas.

When steam flows through a smooth nozzle, remaining dry throughout the expansion, the mass of steam passing per unit time is found to be independent of the steam pressure downstream of the nozzle, if that pressure is less than 0·545 of the steam supply pressure. Explain this result.

A group of convergent-divergent nozzles is required to expand 5 kg steam per sec, from 14 kgf/cm², 0·98 dry to 1 5 kgf/cm². The velocity at the throat pressure of 8 kgf/cm² is 3 per cent below the theoretical value, and the complete expansion takes place with an efficiency ratio of 0·9. If the throat section of each nozzle is to be about 1·7 cm² determine the suitable number of nozzles and specify the proper throat and outlet areas.

[$\Delta h_{1\text{-}2}$=25 kcal ; C_2=444 m/s , x_2=0·94 : no. of nozzles=16 ; throat area=1·61 cm² ; $\Delta h_{1\text{-}3}$=81·27 kcal , C_3=826 m/s : x_3=0 872 ; exit area=3 9 cm²]

12 5. Velocity of jet : mass of steam ; force on the plate.

Steam at 2 kgf/cm², 150°C is admitted to a convergent nozzle of 6·5 cm² throat area. The discharge pressure is 1·4 kgf/cm². Calculate the velocity of the jet issuing from the nozzle and the mass of steam discharged per second.

If the jet from this nozzle is made to impinge upon a flat plate placed at right angles to the path of the jet, what force is exerted upon the plate ?

[$\Delta h_{1\text{-}2}$=17 kcal ; C_2=378 m/s ; m=0·1905 kg ; force=7·34 kg]

12·6. Measurement af steam flow.

Dry saturated steam at 7 kgf/cm² is measured by means of a well-lagged venturi or tapered passage fitted in the pipe line.

The inlet and throat diameters are 100 mm and 75 mm respectively. The pressure at the reduced section is 6·6 kgf/cm².

Calculate, using the steam tables, the rate of flow in kg per second, neglecting friction.

[$s_1=s_2$; $x=0{\cdot}994$; $\Delta h=3{\cdot}6$ kcal ; $C_1=35{\cdot}3$ m/s ; $C_2=66$ m/s ; $m=3{\cdot}12$ kg/s]

12·7. Mass flow in supersaturated and thermal equilibrium conditions.

Sketch the type of diagram one would obtain by varying the discharge pressure P and plotting P against the rate of flow of steam in kg per second. Comment on this diagram.

Steam expands through a nozzle under supersaturated conditions from an initial pressure of 7 kgf/cm² and 250°C to a final pressure of 4 kgf/cm². (a) Compare the mass flow through the nozzle with one in which the expansion takes place under thermal equilibrium conditions. (b) If the initial condition of steam is dry saturated instead of superheated, compare the mass flow in superheated and thermal equilibrium conditions.

[(a) No difference as critical pressure lies in the superheat region ; (b) supersaturated flow ; $\Delta h_{1-2}=23{\cdot}2$ kcal ; $v_2=0{\cdot}4315$ m³/kg ; mass flow $\propto$ 1022 ; flow in thermal equilibrium : $\Delta h_{1-2}=23{\cdot}9$ kcal; $v_2=0{\cdot}45$ m³/ kg ; mass flow $\propto$ 995 $\therefore$ increase in mass flow $=2{\cdot}71\%$]

13·8. Supersaturated and thermal equilibrium conditions with friction : mass flow.

Describe the changes which occur in a convergent-divergent nozzle as the back pressure is slowly increased from the design pressure upto the pressure at entry.

Steam initially dry and saturated at 7 kgf/cm² flows through a nozzle of throat area of 0·65 cm². The outlet pressure is 4 kgf/cm². Assuming that 5 per cent of the K.E. is lost by friction in the nozzle, calculate the mass of steam discharged per hour when the expansion is (a) supersaturated, (b) there is no supersaturation.

[(a) $v_2=0{\cdot}426$ m³/kg ; $C_2=440$ m/s , $m=242$ kg/hr (b) $v_2=0{\cdot}455$ m³/kg : $C_2=441$ m/s ; $m=228$ kg/hr]

12 9. Supersaturated flow ; degree of supersaturation ; gain in entropy ; loss in heat drop.

Write a note, with the aid of a suitable diagram, on the phenomenon of supersaturation. Explain the physical significance and state the effects on the heat drop during expansion, the quality of steam at the end of expansion and the mass flow.

Dry saturated steam at 8 kgf/cm² expands adiabatically in a nozzle to 5 kgf/cm². If the steam remains in a supersaturated state throughout the expansion, what is the degree of supersaturation and undercooling at the end of expansion ?

If the steam reverts to the state of thermal equilibrium at the final pressure, what is the gain in entropy and the loss in heat drop due to undercooling ? The index for adiabatic expansion for saturated steam may be assumed as 1 3.

[Supersaturated state : $T_2=397°K$, degree of supersaturation $=2{\cdot}18$; degree of under cooling$=27{\cdot}1$°C ; $\Delta h_{1-2}=20\ 3$ kcal : gain in entropy$=0\ 002$; thermal equilibrium state : loss in heat drop $=0\ 2$ kcal].

12 10. Supersaturated flow . throat and exit areas ; change in exit area considering friction.

Under what conditions does supersaturated expansion of steam take place ?

A convergent-divergent nozzle expands steam from a pressure and temperature of 10 kgf/cm², 300°C to 1 2 kgf/cm². The final condition is supersaturated and friction may be neglected. Taking n = 1 3 throughout, determine, by calculation from the tables, the throat and exit areas required per kg per second.

If the nozzle efficiency were 90 per cent, find the change in the required exit area.

[$P_2=5\ 45$ kgf/cm² ; $\Delta h_{1-2}=34{\cdot}4$ kcal ; $A_2=7{\cdot}82$ cm² ; $\Delta h_{1-3}=102$ kcal ; $v_2=1{\cdot}349$ m³/kg ; $A_3=14{\cdot}58$ cm² ; $v_3=1{\cdot}434$ m³/kg ; $A_3=16{\cdot}38$ cm² ; increase in exit area$=12{\cdot}35\%$]

12·11. Exhaust steam injector with water level above : water/kg of steam ; injector dimensions ; feed temperature.

An exhaust steam injector is used for supplying feed to a locomotive boiler in which the steam pressure is 10 kgf/cm². If the pressure of exhaust steam for working the injector is 1·5 kgf/cm² and its dryness fraction is 0 9, estimate,

(a) *the mass of water that can be pumped per kg of steam ;*

(b) *the areas of water and steam discharge orifices ;*

(c) *the feed temperature.*

6000 kg of water per hour is taken from a feed tank at 24°C and the level of water in boiler and feed tank is 1·5 and 2 metre respectively above the centre of the injector. Assume velocity of water exit from the injector 15 m per second and the pressure at exit from the discharge orifice 0·6 that at entrance.

[Δh_{1-2}=18·5 kcal C_2=39 m/s ; C_j=454 m/s ; water/kg of steam =9·02 kg ; areas, water orifice=0·412 cm² ; steam orifice=7·85 cm² feed temp.=80·5°C]

12·12. Steam injector ; diameter of orifices ; pressure at feed-check valve, given feed temperature.

Calculate the diameter of the orifices for an injector to deliver 6000 kg of water per hour into a boiler containing steam at 4 kgf/cm². The steam supplied to the injector may be assumed dry, the pressure in the steam orifice 0·6 of the absolute boiler pressure, the temperature of water in the suction tank 35°C and temperature of the feed water 80°C. Check the pressure at the feed-check valve on the boiler.

[Δh_{1-2}=21·5 kcal ; water/kg of steam=12·88 kg ; C_j=30·65 m/s diameter, steam orifice=1·69 cm ; water orifice=0·863 cm ; pressure at valve=4·8 kgf/cm²]

13

Steam Turbine Velocity Diagrams

13 1. Introduction. In reciprocating steam engines the pressure energy of the steam is utilised and the dynamic action of the steam is negligible. Steam engines may be operated without any expansion or drop in pressure in the cylinder. Steam turbines proper cannot be operated in such a manner. The operation of steam turbine depends wholly on the dynamic action of the steam

In a steam turbine steam is passed through nozzle or fixed blades where the heat drop takes place increasing the velocity of steam. This high velocity steam impinges on the curved vanes which causes the direction of the steam to be changed. Due to this change of momentum motive force is exerted on the moving blades and power is obtained (*see* Fig. 13·1).

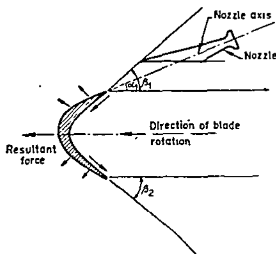


Fig. 13 1. Force exerted on moving blades.

13 2. Advantages of Stem Turbines. A steam turbine offers the following advantages over the conventional steam engine :—

(1) The thermodynamic efficiency of steam turbines is higher than that of steam engines because it works on Rankine cycle, whereas steam engine works on modified Rankine cycle. A steam turbine can thus take advantages of expansion upto the lowest pressure.

(2) The mechanism is simple as intermediate links like piston, piston rod, cross-head, etc., are absent.

(3) There is no initial condensation as the parts are subjected to constant temperature and pressure at constant loads.

(4) Power is generated at uniform rate, hence no flywheel is necessary.

(5) No internal lubrication is necessary, which reduces the cost of lubrication and supplies a purer feed to the boiler.

(6) Due to absence of reciprocating parts, perfect balance is possible which avoids heavy foundations.

(7) Much higher speed and greater range of speed is available. Because of higher speed the power produced is large per unit volume as well as per unit weight.

(8) Steam turbines can be made in very large sizes and hence are very suitable for large thermal stations. Steam turbines have been manufactured to develop 250,000 hp and even more.

(9) Steam consumption does not increase with the years of service.

(10) Steam turbines can carry considerable overloads with only a slight reduction in efficiency.

Only in the small power range a steam engine can compete with a steam turbine.

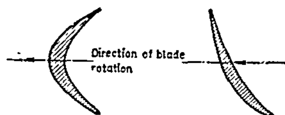
13·3. Types of Turbines. Steam turbines are of two types:

(*i*) Impulse, and

(*ii*) Impulse-reaction (in practice known as reaction turbines).

In an impulse turbine the steam is expanded causing pressure drop and heat drop in nozzles only and the moving blades merely deflect the steam through an angle. The pressure on two sides of the blades remains constant. The impulse blades are symmetrical as shown in Fig. 13·2 (*a*).

I.. an impulse reaction turbine the steam is expanded both in fixed and moving blades continuously as the steam passes over them.



(*a*) Impulse blade (*b*) Reaction blade
Fig. 13·2. Turbine blades

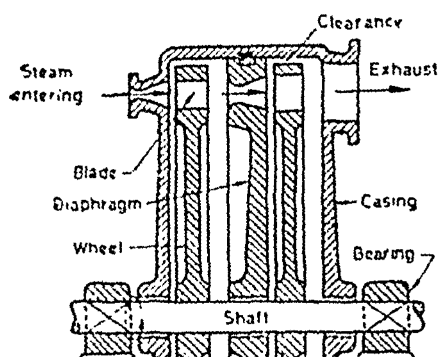
Therefore, the pressure drops gradually and continuously over both moving and fixed blades. The reaction blade is shown in Fig. 13 2 (*b*). It is asymmetrical and is thicker at one end which provides a suitably shaped passage for steam to expand.

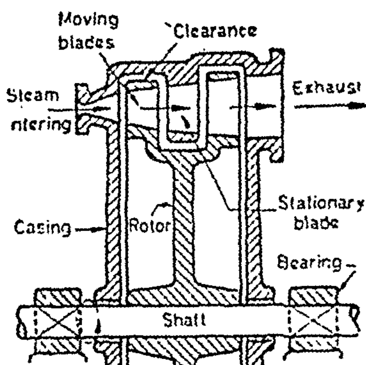
Mostly the steam turbines are of axial flow type. The only important radial flow turbine is Ljungstorm double-flow reaction turbine.

13 4 Thermodynamic Means of Reducing Rotor Speed The earliest type of single wheel impulse turbine *i.e* De-lavel turbine has the disadvantage of very high speed upto 30,000 r.p.m Various thermodynamic means have been adopted for reducing the speed of turbine by compounding.

(1) *Pressure-Compounded Turbine* : (Rateau and Zoelly turbines) In this type the wheel or the rotor having rings of moving blades are keyed to the turbine shaft in series Each ring of moving blades has a ring of fixed nozzles (*see* Fig. 13·3). The partition between the stages is by means of a diaphragm fixed on the casing and having fine clearance with the shaft. The ring of fixed nozzles is made in the diaphragm itself. The pressure drop is divided up equally between all the nozzle rings. The steam from the boiler enters the first ring of nozzles where its pressure is partially reduced and its velocity is increased. It then passes over the first moving blade ring where nearly all of its velocity is absorbed. The exit angle of the nozzle is so made that the steam enters the moving blades without shock. The exhaust from the first moving blade ring enters the next nozzle ring

and is again partially expanded and its velocity is again increased which is absorbed in the second ring of moving blades, and so on. Since only part of the pressure drop occurs in each stage the steam velocities will not be very high and hence the turbine velocity will be restricted.





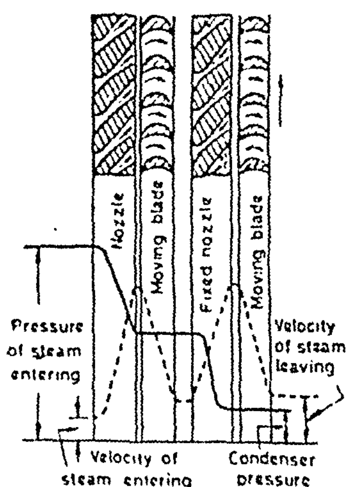


Fig. 13·3. Pressure-compounded turbine.

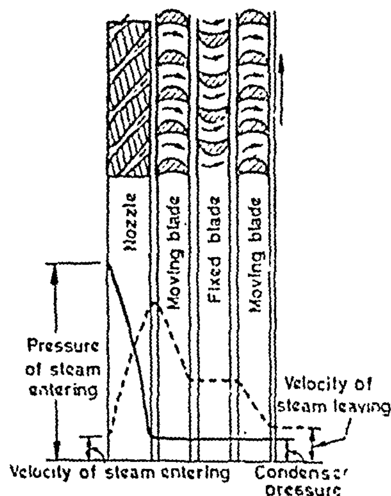


Fig. 13·4. Velocity-compounded turbine.

It may be noted that all pressure drop occurs in the nozzles, the pressure remaining constant over each ring of moving blades.

✓This is the most efficient type of turbine because the ratio of blade velocity to steam velocity remains constant, but it has the disadvantage of large number of stages ; hence it is most expensive. Therefore pressure-compounded turbine has now become out of date.

(2) *Velocity-Compounded Turbine* (*i.e.* Curtis turbine). In this type the total heat drop takes place in the nozzles, but the velocity generated is utilised in two or more rings of blades (*see* Fig. 13·4).

The high velocity of steam leaving the nozzles passes on to the first ring of moving blades where only a portion of high velocity is absorbed. The steam leaving the first ring of moving blades passes on to a ring of fixed blades which are mounted in the turbine casing. *The ring of fixed blades serves to redirect the steam* (without appreciably altering the velocity) such that the steam enters the next ring of moving blades without shock.

Since only part of the velocity of steam is used up in each ring of blades the speed of the turbine will be restricted.

It may be noted that all pressure drop occurs in the nozzles, the pressure remaining constant over each ring of moving and fixed blades.

✓The advantage of velocity compounded turbine is relatively fewer number of stages and hence less initial cost. However, the dis-

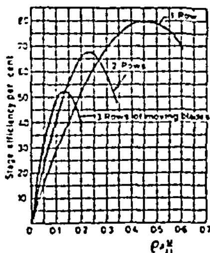


Fig. 13·5. Efficiency of velocity compounded turbine

advantage is low efficiency as the ratio of blade velocity to steam velocity is not optimum for all the wheels. Therefore, efficiency goes on decreasing for subsequent stages (*see* Fig. 13·5). Hence, large velocity compounded turbines are obsolete. But *velocity compounding* has one important application. It is often used in the first stage of multi-stage impulse turbines, the remaining stages having single row

of wheels. In this first velocity-compounded stage there is large pressure drop and therefore the remainder of the turbine stages are subjected to a lower pressure range resulting in fewer stages. In reaction turbines also sometimes a impulse velocity-compounded first stage is added for the same reason.

The velocity compounded first stage is on high pressure end as the reheat due to low efficiency is more recoverable on the high pressure side.

(3) *Pressure-Velocity Compounded Turbine.* Pressure-velocity compounding was an effort in the direction of combining the advan-

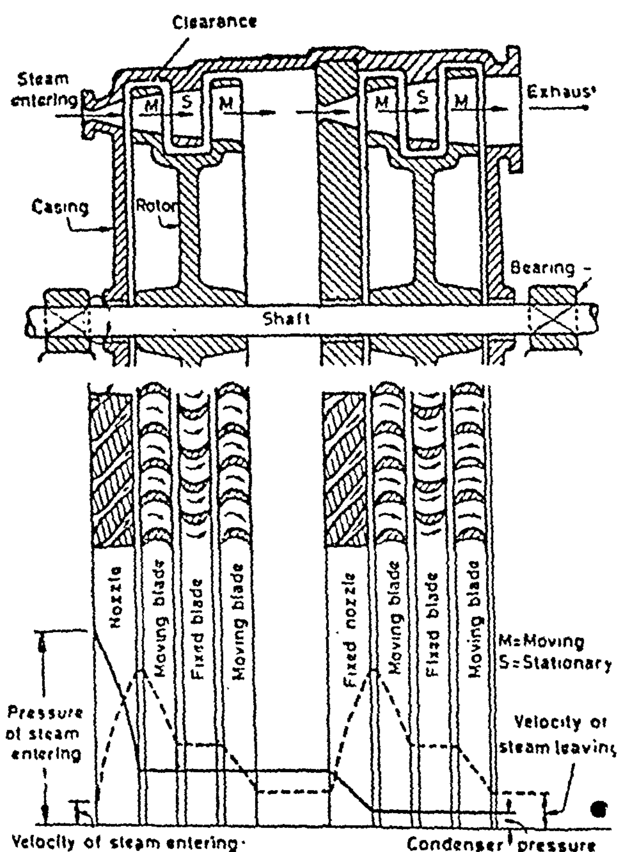


Fig. 13·6. Pressure-velocity compounded turbine.

tages of previous two types (*see* Fig 13·6). The total pressure drop of the steam is divided into stages and the velocity obtained in each stage

is also compounded. It results in bigger pressure drop in each stage and hence less number of stages which means a more compact turbine than a pressure compounded turbine. However, the efficiency is lower than a pure pressure compounded turbine.

Fig. 13 6 shows that the diameter of the pressure velocity compounded turbine is increased at each stage to allow for the increasing volume of the steam at low pressures. Each stage has a ring of nozzles followed by moving and fixed blades.

The pressure-velocity compounded turbine as an impulse turbine because the pressure is constant during each stage.

(4) *Reaction Turbine* (Parson's turbine). It consists of large number of stages, each consisting of fixed and moving blades. The heat drop takes place throughout in both fixed and moving blades (*see* Fig. 13·7).

Unlike the impulse turbine no nozzles as such are provided in a reaction turbine. The fixed blades act both as nozzles in which the velocity of the steam is increased, and as the means of directing the steam so that it enters the ring of moving blades without shock. The turbine derives its name of 'reaction' because the steam expands over the moving blades also giving a reaction to the moving blades. This reaction force is not there in impulse turbines. The steam velocity in the reaction turbine is not very high due to division of pressure drop over fixed and moving blades. Because the pressure drop takes place both in the fixed and moving blades all blades, (fixed and moving) are nozzle shaped.

To accomodate increase in the specific volume at lower pressures the drum diameter is stepped up which allows greater area without unduly increasing the blade height. The increased drum diameter also increases the torque due to steam pressure.

Modern turbines generally have the first stage velocity compounded (Curtis stage) and subsequent stages are either pressure compounded (Rateau stages) or reaction stages. Reaction blading is inefficient in the high pressure region because the clearance between the tip of the blade and casing is relatively large percentage of blade height which will cause excessive tip leakage. The tip leakage is also

accelerated because of the greater pressure drop in high-pressure region for a particular enthalpy drop. Another reason for using impulse stage at the beginning of expansion is that best method of regulation of steam flow is by opening or closing of nozzles.

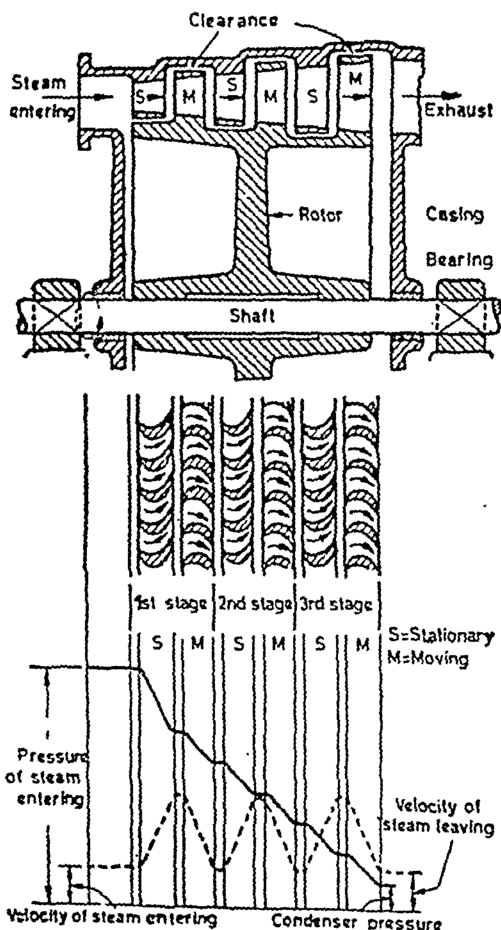


Fig. 13·7. Reaction turbine.

13·5. Notations and Velocity Diagrams. The following nomenclature has been used in the turbine problems.

α=Angle of absolute velocity measured to the direction of the blade motion.

α_1=Angle of absolute velocity at inlet (Nozzle angle).

α_2=Angle of absolute velocity at outlet (Inlet angle of fixed blade).

β=Angle of relative velocity measured to the direction of motion.

β_1=Angle of relative velocity at inlet (Entrance angle of fixed blade).

β_2=Angle of relative velocity at outlet (Entrance angle of moving blade).

U=Linear velocity of blade.

C_1=Absolute velocity of steam entering moving blade.

C_2=Absolute velocity of steam leaving moving blade.

C_{1w}=Velocity of whirl at entrance of moving blade,
i.e. tangential component of $C_1=C_1 \cos \alpha_1$.

C_{2w}=Velocity of whirl at exit of moving blade,
i.e. tangential component of $C_2=C_2 \cos \alpha_2$.

C_{1f}=Velocity of flow at entrance of moving blade,
i.e. axial component of $C_1=C_1 \sin \alpha_1$.

C_{2f}=Velocity of flow at exit of moving blade,
i.e. axial component of $C_2=C_2 \sin \alpha_2$.

V_1=Relative velocity of steam to moving blade at entrance.

V_2=Relative velocity of steam to moving blade at exit.

m=Mass of steam flowing over blades.

d=Mean diameter of blade drum

h=Height of blade.

$$\rho=\frac{\text{Linear velocity of blade}}{\text{Absolute velocity of steam entering moving blade}}=\frac{U}{C_1}$$

$$k=\text{Blade velocity coefficient}=\frac{V_2}{V_1}$$

Fig. 13·8 (a) and (b) represent inlet and outlet velocity diagrams for a simple impulse turbine. AB represents the linear velocity of blade, U. Steam enters the blade with an absolute velocity of C_1 inclined at nozzle angle α_1 to the plane of rotation. The relative velocity of steam with respect to blade is V_1 inclined at an angle β_1 to the plane of rotation. The inlet angle of the blade should be equal to β_1 so that the steam enters the blade without shock. C_{1w} the component of C_1 in the direction of U imparting motion to the blade

is known as *velocity of whirl*. C_{1a} the component of C_1 at right angles to U imparting axial thrust, is known as *velocity of flow*. It is C_a which is responsible for flow over turbine blades.

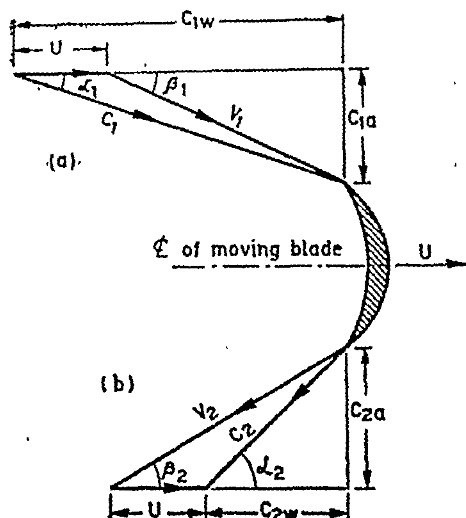


Fig. 13·8. Velocity diagrams.

The velocity at outlet V_2 is equal to V_1 if there is no friction. Due to friction $V_2 = k.V_1$ where k is the blade velocity coefficient. V_2 is inclined at β_2 to U where β_2 is the outlet angle of moving blade for no shock at exit. The absolute velocity C_2 is inclined at α_2 to U which should be the inlet angle of next blade (or nozzle) if the steam is to enter it without shock. C_{2w} and C_{2a} are the velocity of whirl and velocity of flow respectively at outlet.

Fig. 13·9 shows the combined inlet and outlet velocity diagrams. Note that, (*i*) both fixed blades angle are at A and both moving

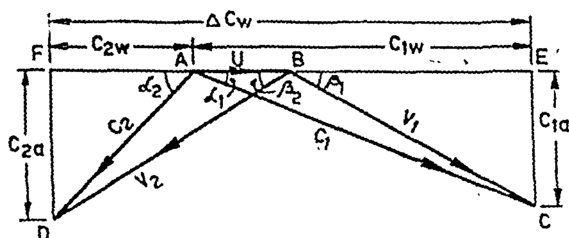


Fig. 13·9. Combined inlet and outlet velocity diagram.

blade angles are at B. Also absolute velocity at inlet and outlet are

drawn from A and relative velocity at inlet and outlet are drawn from B.

(ii) α_1 and β_2 are "must angles" as C_1 and V_2 must be inclined according to the magnitudes of these angles. However β_1 and α_2 are "may" angles because V_1 and C_2 may be inclined to U by the angle β_1 and α_2 respectively, only if steam enters the blades without shock.

Fig. 13·10 show velocity diagrams for a two-stage velocity compound turbine.

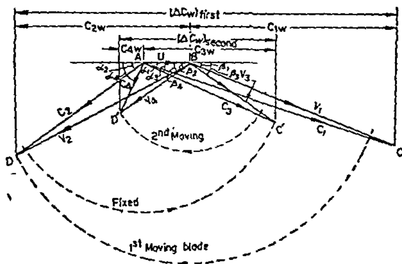


Fig 13 10 Velocity diagrams for a two-stage velocity compounded turbine.

13 6 Calculations.

(A) IMPULSE TURBINE (see Fig. 13 9)

(1) Work done per kg of steam

= force in the direction of motion × distance travelled.

= rate of change of momentum × distance travelled

$$=\frac{(C_{1w}-C_{2w})U}{g}=\frac{U\times\Delta C_w}{g}=\frac{AB\times EF}{g} \qquad (13\cdot1)$$

(2) Diagram or blading efficiency

$$=\frac{\text{work done on blades}}{\text{energy supplied to blades}}$$

$$=\frac{AB\times EF}{g}\times\frac{1}{C_1^2/2g}=\frac{2AB\times EF}{C_1^2}$$

(3) Gross or stage efficiency

$$=\frac{\text{work done on blades}}{\text{total energy supplied per stage}}$$

$$=\frac{AB\times EF}{g\times \Delta hJ} \qquad [13\cdot3(a)]$$

This efficiency takes into account the losses in nozzles also.

$\therefore$ Stage efficiency = blade efficiency × nozzle efficiency [13·3(*b*)]

(4) Net efficiency $=\frac{\text{net work per stage obtained at shaft}}{\text{total energy supplied per stage}}$ (13·4)

The net efficiency allows for friction, windage and other losses of turbines.

(5) Axial thrust per kg of steam

= mass × change of velocity in axial direction

$$=\frac{1}{g}\ (C_{1a}-C_{2a}) \qquad (13\cdot5)$$

(6) Energy lost in blades due to friction per kg of steam

$$=\frac{V_1^2-V_2^2}{2g} \qquad (13\cdot6)$$

(7) Energy lost in exit per kg of steam

$$=\frac{C_2^2}{2g} \qquad (13\cdot7)$$

(B) Reaction Turbine

The so called reaction turbines are essentially "impulse-reaction turbines", the heat drop taking place both on fixed and moving blades. The degree of reaction is defined as the ratio of heat drop over moving blades to total heat drop in the stage.

(1) Degree of reaction

$$=\frac{\text{heat drop on moving blades}}{\text{total heat drop in fixed and moving blades}} \qquad (13\cdot8)$$

Parson's reaction turbine with symmetrical bladings are 50 per cent reaction turbines.

Equations for work done and efficiency from (13·1) to (13·5) are applicable to reaction turbines also.

(2) Heat drop through fixed blades, acting as nozzles, per kg of

$$\text{steam} = \frac{C_1^2 - C_2^2}{2gJ} \quad (13{\cdot}9)$$

(3) Heat drop through moving blades per kg of steam

$$= \frac{V_1^2 - V_2^2}{2gJ} \quad (13{\cdot}10)$$

(4) K.E. supplied per kg of steam

$$= \frac{1}{2gJ}\,[C_1^2 + (V_2^2 - V_1^2)] \quad [13{\cdot}11(a)]$$

(5) Neglecting friction,

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} + \text{heat drop in fixed blades} \quad [13{\cdot}11(b)]$$

(6) Axial thrust on rotor

=pressure drop per moving ring

$$\text{of blades} \times \text{blade area of ring} + \frac{C_{1a} - C_{2a}}{g} \quad (13{\cdot}12)$$

(7) Height of blades is given by (see Fig. 13 11).

Volume of steam $= \pi d b C_a$

$$\therefore \quad b = \frac{m \times x\, v_g}{\pi d\, C_a} \quad (13{\cdot}13)$$

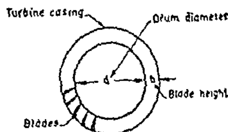


Fig 13 11. Height of blades

13·7. Condition for Maximum Efficiency.

There is a definite value of speed ratio ρ for maximum efficiency in case of impulse and reaction turbines.

(A) *Impulse Turbines* :

Fig. 13·12. Velocity diagram for an impulse turbine.

$$\text{Diagram } \eta = \frac{2AB \times EF}{C_1^2}$$

$$EF = BE + BF$$

$$= V_1 \cos \beta_1 + V_2 \cos \beta_2$$

$$= V_1 \cos \beta_1 \left(1 + K \frac{\cos \beta_2}{\cos \beta_1}\right)$$

$$= (C_1 \cos \alpha_1 - U)(1 + KC) \qquad [C = \cos \beta_2 / \cos \beta_1$$

$$\therefore \text{ Diagram } \eta = \frac{2U(C_1 \cos \alpha_1 - U)(1 + KC)}{C_1^2}$$

$$= 2\rho(\cos \alpha_1 - \rho)(1 + KC)$$

$$= 2(1 + KC)(\rho \cos \alpha_1 - \rho^2)$$

$$\frac{d\eta}{d\rho} = 2(1 + KC)(\cos \alpha_1 - 2\rho) = 0, \text{ for maximum efficiency}$$

$$\therefore \qquad \rho = \frac{\cos \alpha_1}{2} \qquad (13\cdot14)$$

and, maximum efficiency $= (1 + KC) \dfrac{\cos^2 \alpha_1}{2}$ (13·15)

(B) *Reaction Turbines* : The condition for maximum efficiency is derived, assuming the following :—

(*i*) Degree of reaction is half.

(*ii*) The moving and fixed blades are symmetrical.

(*iii*) The velocity of steam at exit from the proceeding stage *i.e.* at entrance to the succeeding stage is same.

On the above assumption the velocity diagram is symmetrical (*see* Fig. 13·13).

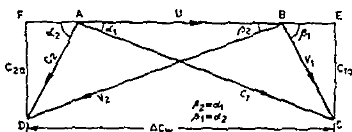


Fig. 13·13. Velocity diagram for 50 per cent reaction.

$$\text{W.D. per kg} = \frac{AB \times EF}{g} = \frac{U(2C_1 \cos \alpha_1 - U)}{g}$$

$$= \frac{C_1^2}{g}\left[\frac{2U}{C_1} \cos \alpha_1 - \frac{U^2}{C_1^2}\right]$$

$$= \frac{C_1^2}{g}[2\rho \cos \alpha_1 - \rho^2]$$

Assuming no loss, energy equivalent to heat drop in the fixed

$$\text{blades} = \frac{(C_1^2 - C_2^2)}{2g}$$

$\therefore$ Total heat drop $= \dfrac{2(C_1^2 - C_2^2)}{2g}$, as the degree of reaction is half.

$$\text{Total energy supplied to the stage} = \frac{2(C_1^2 - C_2^2)}{2g} + \frac{C_2^2}{2g}$$

$$= \frac{2C_1^2 - C_2^2}{2g}$$

From velocity diagram, $C_2 = V_1 = (C_1^2 + U^2 - 2C_1 U \cos \alpha_1)^{\frac{1}{2}}$

$\therefore$ Total energy supplied per kg of steam

$$= \frac{2C_1^2 - C_1^2 - U^2 + 2C_1 U \cos \alpha_1}{2g}$$

$$= \frac{C_1^2 - U^2 + 2C_1 U \cos \alpha_1}{2g}$$

$$= \frac{C_1^2}{2g}[1 - \rho^2 + 2\rho \cos \alpha_1]$$

$$\text{Diagram } \eta = \frac{\text{work done}}{\text{heat supplied}}$$

$$=\frac{\frac{C_1^2}{g}[2\rho\cos\alpha_1-\rho^2]}{\frac{C_1^2}{2g}[1-\rho^2+2\rho\cos\alpha_1]}=2-\frac{2}{1-\rho^2+2\rho\cos\alpha_1}$$

∴ Efficiency is maximum when $1-\rho^2+2\rho\cos\alpha_1$ is maximum

Let, $x=1-\rho^2+2\rho\cos\alpha_1$

∴ $\frac{dx}{d\rho}=-2\rho+2\cos\alpha_1=0$, for maximum efficiency

∴ $\rho=\cos\alpha_1$ (13·16)

and maximum $\eta=\frac{2(2\cos^2\alpha_1-\cos^2\alpha_1)}{(1-\cos^2\alpha_1+2\cos^2\alpha_1)}=\frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$ (13·17)

13·8. Losses in Steam Turbines. The losses in steam turbines may be categorised as follows :—

1. *Loss in the exit velocity of steam.* The loss in the exit velocity of steam is due to blade efficiency not being 100 per cent. This is because of the obliquity of nozzles. If the nozzle angle α_1, is zero blade efficiency would be 100 per cent, but for practical reasons the value is more than 15°. For a nozzle angle of 20° the ideal efficiency is about 85 per cent.

2. *Loss due to friction and turbulence* Friction occurs in nozzles and blades and between steam and rotating discs. Also due to centrifugal action steam is thrown radially towards the casing and dragged along the surface by the moving blades. These losses are called the disc friction and windage losses. Again in impulse turbines due to partial admission there is churning of the steam in the inactive blades of the wheel. The loss due to friction and turbulence is about 10 per cent.

3. *Loss due to leakage.* In impulse turbines leakage occurs between the shaft and the stationary diaphragms carrying nozzles. In the reaction turbines, the leakage is at the blade tips. There is also some leakage of steam to atmosphere through glands at high pressure and the loss due to leakage is about 1 to 2 per cent.

4. *Loss due to mechanical friction to the bearings, etc.* This loss is less than 1 per cent and it decreases with the size of the plant.

5. *The loss due to radiation, etc.*, is negligible.

In addition to above losses the other losses are as follows :

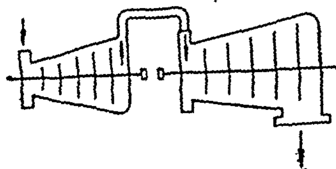
1. *Governor loss.* It is due to throttling. This may be of the order of 5 to 10 per cent.

2. *Exhaust loss.* The steam leaves the turbine with a finite absolute velocity which is partly or wholly lost. This loss is also known as the residual velocity loss.

13 9. Classification of Turbines. Steam turbines may be classified according to principle of action, arrangement of pressure drop, direction of flow, method of governing, etc., as given in the following table.

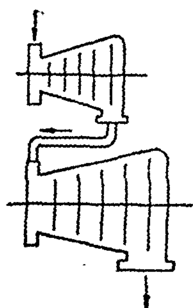
Methods of Classification	*Types*
1. According to principle of action.	1. Impulse. 2. Reaction (Impulse-reaction).
2. According to the arrangement of pressure drop.	1. Single stage turbines. Suitable for small power capacities and are used mostly for driving centrifugal compressors, blowers, etc. (*a*) Single wheel (*b*) Velocity-compounded. 2. Multistage turbines— (*a*) Pressure-compounded. (*b*) Velocity-compounded. (*c*) Reaction.
3. According to the direction of steam flow.	1. Axial flow. 2. Radial flow. 3. Tangential flow.
4 According to the number of cylinders.	1. Single cylinder. 2. Double cylinder. 3 Three cylinder. 4. Four cylinder.
5. According to general arrangement of flow.	1 Single flow 2 Double flow It is generally used for low pressure machines and is completely balanced against end thrust Separate or combined exhaust connections are made with the condenser.

3. Compound flow.



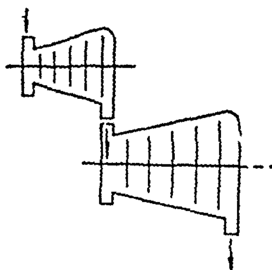
(a) Tandem.

(a) *Tandem.* These are in line on the same shaft and drive the same electric unit.



(b) Cross-compound

(b) *Cross-compound.* These are on separate shafts. The advantage is that they can run at separate speeds.



(c) Vertical compound.

(c) *Vertical compound.* Principally used where an existing H.P. plant is to be superimposed and the available floor space is limited.

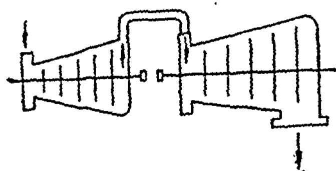
6. According to the method of governing.	1. Throttle. 2. Nozzle. 3. Bypass.
7. According to the heat drop process.	1. High pressure non-condensing. 2. High pressure condensing. 3. *Back pressure.* The exhaust heat is used for process work or heating purposes. 4. *Superposed or topping.* These are of same type as above with the difference that the exhaust steam is utilized in medium or low pressure condensing turbines. 5. *Mixed pressure.* see page 538

	6. *Regenerative.* Most of the turbines have extractions for regenerative feed heating. 7. *Pass-out or extraction.* See page 537. 8. *Reheating* When the steam is heated in between the stages. This is done to increase the output per unit mass and increase dryness condition of exhaust steam. 9. Low pressure or exhaust turbine. 10. *Binary.* In this type two fluids are used.
8. According to the steam pressure	1 Low pressure, 1.2 to 2 kgf/cm^2. 2. Medium pressure, 2 to 40 kgf/cm^2. 3. High pressure above, 40 kgf/cm^2. 4. Supercritical pressure, above 225 kgf/cm^2.
9. According to usage	1. Stationary (*a*) constant speed for driving alternators (*b*) Variable speed for a turbo blower. 2. Non stationary with variable speed. Locomotive, ships, etc

13·10. Radial Flow Turbine (Ljungstrom Turbine). The turbines described in article 13 4 are axial flow type *i.e.* the general direction of the steam flow is parallel to the turbine axis It is however, possible to arrange the blades so that the flow of steam is radial-inwards or outwards. In a outward flow radial turbine the steam is supplied near the axis and expands radially outward through a series of blades which are fixed on two concentric rings. If one set of blades is stationary it is called a single motion radial flow turbine ; if all the rings of blades are moving with alternate rings moving in opppsite direction it is called a double motion radial-flow turbine. An example of the latter arrangement is Ljungstrom turbine shown in Fig. 13·14.

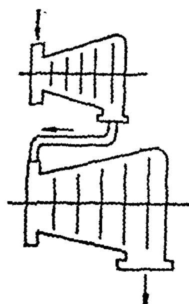
The blade wheels *A* and *B* rotate in opposite direction and their output shafts drive separate alternators (or any other load) at the same speed. The blades are of 50 per cent reaction type.

3. Compound flow.



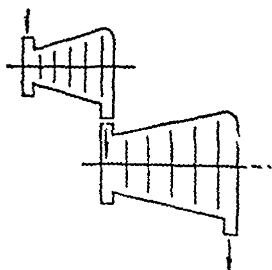
(a) Tandem.

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	9. Low pressure or exhaust turbine.
	10. *Binary.* In this type two fluids are used.
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The blade wheels A and B rotate in opposite direction and their output shafts drive separate alternators (or any other load) at the same speed. The blades are of 50 per cent reaction type.

As the rings of blades rotate in opposite direction their effective speed is doubled. Hence the steam velocity is doubled resulting in a powerful turbine for a given size and weight.

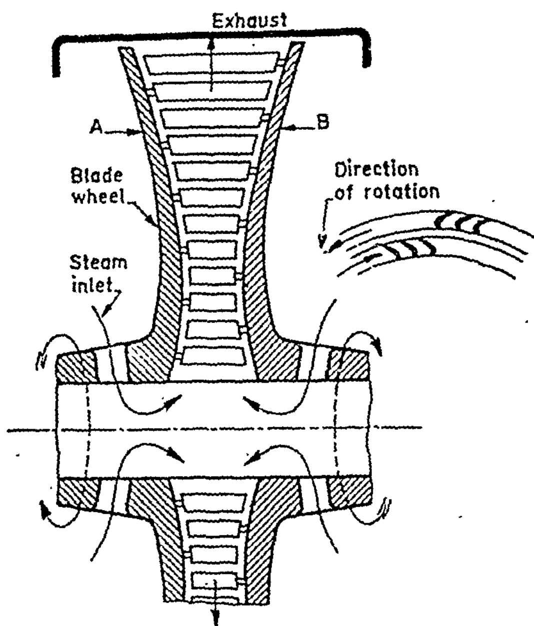


Fig. 13·14. Ljungstrom turbine.

In this type of turbine since the casing is removed from the high pressure and temperature it can be started quickly and easily from cold. Due to this reason it is used as a power plant for 'stand by' or 'peak load' generator sets.

IMPORTANT POINTS

1. In all problems on velocity diagrams, the diagram must invariably be sketched first and given values should be inserted in it ; then the diagram should be drawn to scale.

2. When it is mentioned that axial discharge is zero it means that absolute velocity at outlet makes an angle of 90° with the plane of rotation ($\alpha_2=90$). Axial flow should not be confused with axial discharge. All turbines are axial flow except radial flow Ljungstrom turbine.

3. K.E. supplied in a reaction turbine per stage is not equal to $\frac{C_1^2}{2g}$ but $\frac{C_1^2}{2g}+\frac{V_2^2-V_1^2}{2g}$ [*see* equation 13·11 (*a*)]. This should be kept in mind while calculating diagram efficiency.

4. In impulse turbines $V_2=KV_1$ but in reaction turbines $V_2 \neq KV_1$ as the relative velocity at exit increases due to pressure drop over moving blades [*see* equation 13 11 (*b*)].

5 In problems on reaction turbines, pressure drop in a stage is very small, hence these problems should be solved by calculations rather than drawing the velocity diagrams to scale.

6. In reaction turbines diagram is assumed symmetrical unless otherwise stated.

7. When the diagram is unsymmetrical in reaction turbines, blade height at inlet and exit are different. Blade height given in problems are customarily referred to exit conditions.

8. In reaction turbines change of volume in passage over the moving blades is neglected in calculations

9. For calculating the blade height only actual flow over the blade is taken i.e. total steam flow minus tip leakage.

ILLUSTRATIVE EXAMPLES

(A) Impulse Turbines

13 1. De-laval turbine · rpm , steam consumption ; blade η ; stage η , axial thrust ; exit loss.

A De-laval turbine is supplied with steam at a pressure of 15 kgf/cm² and temperature 250°C. The back pressure is 0 12 kgf/cm². Given coefficient of nozzle, 0 9 , blade velocity coefficient, 0 8 , mechanical efficiency, 90 per cent , nozzle angle 20° and symmetrical blades with an angle of 30°, draw the velocity diagram and calculate :—

(*a*) *the revolutions per minute, if the mean diameter of [illegible] is 75 cm ,*

(*b*) *the steam consumption per bhp-hr ;*

(*c*) *the blade efficiency ;*

(*d*) *the stage efficiency ;*

(*e*) *the axial thrust per kg .*

As the rings of blades rotate in opposite direction their effective speed is doubled. Hence the steam velocity is doubled resulting in a powerful turbine for a given size and weight.

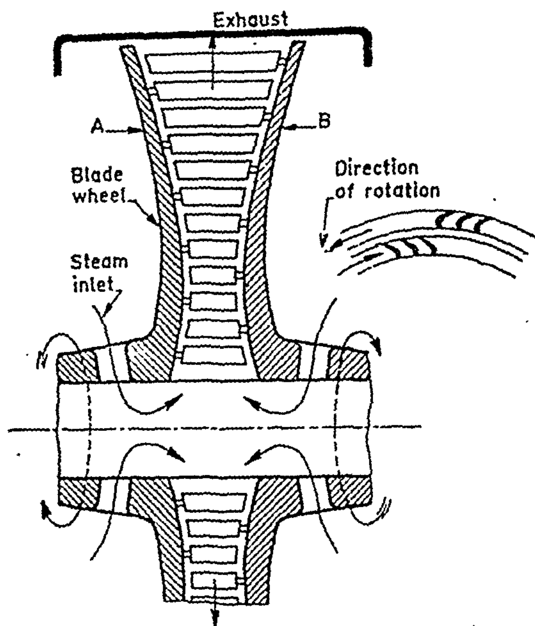


Fig. 13·14. Ljungstrom turbine.

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4 In impulse turbines $V_2=KV_1$ but in reaction turbines $V_2\neq KV_1$ as the relative velocity at exit increases due to pressure drop over moving blades [*see* equation 13 11 (*b*)].

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8. In reaction turbines change of volume in p moving blades is neglected in calculations

9. For calculating the blade height only blade is taken *i.e.* total steam flow minus tip

(*f*) *the percentage loss in exit.*

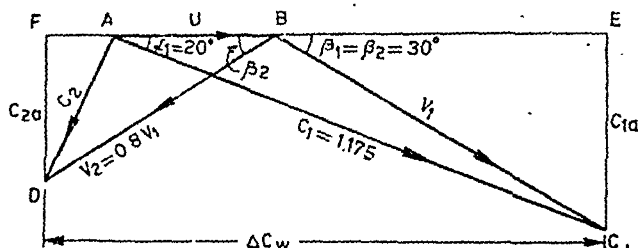


Fig. 13·15.

From mollier chart, Heat drop$=183$ kcal

$\therefore$ Actual velocity of steam, $C_1=91{\cdot}53\sqrt{k\Delta h}$

$$=91{\cdot}53\sqrt{0{\cdot}9\times 183}$$

$$=1175 \text{ m/s}$$

Given : $\alpha_1=20°$; $\beta_1=\beta_2=30°$; $V_2=0{\cdot}8\,V_1$

The velocity diagram is now drawn and the following results are obtained : $U=407$ m/s ; $EF=1252$; $C_{1a}=402$;

$C_{2a}=322$; $C_2=355$.

(*a*) $U=\dfrac{\pi dN}{60}$ $\quad\therefore\quad \underline{N}=\dfrac{407\times 60}{\pi\times 0{\cdot}75}=\underline{10350}$ Ans.

(*b*) W.D. per kg of steam$=\dfrac{AB\times EF}{g}$

$$=\frac{407\times 1252}{9{\cdot}81}=52\ 000 \text{ kgf m}$$

hp per kg of steam per sec$=\dfrac{52{,}000}{75}=694$

bhp per kg of steam per sec$=694\times 0{\cdot}9=624{\cdot}6$

$\therefore$ Steam consumption per bhp-hr

$$=\frac{60\times 60}{624{\cdot}6}=\underline{5{\cdot}77 \text{ kg}}$$ Ans.

(*c*) Blade efficiency$=\dfrac{\text{work done on blade}}{\text{energy supplied to blade}}$

$$=\frac{52{,}000}{(1175)^2/2\times 9{\cdot}81}=\underline{74\%}$$ Ans.

(d) Stage efficiency $= \dfrac{\text{work done on blade}}{\text{total energy supplied to stage}}$

$$= \frac{52{,}000}{183 \times 427} = 66{\cdot}5\%. \qquad \text{Ans.}$$

(e) Axial thrust per kg $= C_{1a} - C_{2a} = \dfrac{402-322}{9{\cdot}81} = 8{\cdot}15$ kgf Ans.

(f) Loss in exit per kg $= \dfrac{C_2^2}{2g \times J} = \dfrac{355^2}{2 \times 9{\cdot}81 \times 427} = 15{\cdot}05$ kcal

$\therefore$ Percentage loss in exit per kg $= \dfrac{15{\cdot}05}{183} = 8{\cdot}23\%$. Ans.

13·2. Impulse turbine : θ ; ϕ ; α ; C_1 ; axial thrust ; hp ; Δh given initial velocity.

A simple impulse turbine has one ring of moving blades running at 150 metres per second , velocity of steam reaching nozzles, 90 metres per second ; nozzle efficiency, 0·85 ; absolute velocity of steam at exit from stage, 85 metres per second at an angle of 80° with tangent of wheel ; blade velocity coefficient, 0·82 , rate of steam flowing, 2 kg per second. Assuming the moving blades to be equiangular find the blade angles, nozzle angle, absolute velocity of steam at entrance, axial thrust, metric hp developed and the heat drop in nozzles in kcal.

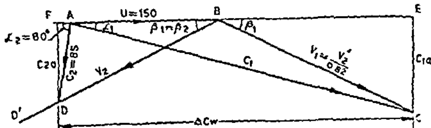


Fig. 13·16

Given : $U = 150$ m/s ; $C_2 = 85$ m/s ; $\alpha_2 = 80°$ $\beta_1 = \beta_2$;
$V_2/V_1 = 0{\cdot}82$

From the available data the velocity diagram is drawn and the following are obtained

$\beta_1 = \beta_2 = 27°$, $\alpha = 16{\cdot}2°$, $C_1 = 366$ m/s Ans.

$C_{1a} = 102$; $C_{2a} = 83{\cdot}5$; $EF = 366$

Axial thrust $= \dfrac{(C_{1a} - C_{2a}) \times m}{g}$

$$= \frac{(102 - 83{\cdot}5) \times 2}{9{\cdot}81} = 3{\cdot}77 \text{ kgf} \qquad \text{Ans.}$$

(f) *the percentage loss in exit.*

Fig. 13·15.

From mollier chart, Heat drop = 183 kcal

$\therefore$ Actual velocity of steam, $C_1 = 91{\cdot}53\sqrt{k\Delta h}$

$$= 91{\cdot}53\sqrt{0{\cdot}9 \times 183}$$

$$= 1175 \text{ m/s}$$

Given : $\alpha_1 = 20°$; $\beta_1 = \beta_2 = 30°$; $V_2 = 0{\cdot}8\ V_1$

The velocity diagram is now drawn and the following results are obtained : $U = 407$ m/s ; $EF = 1252$; $C_{1a} = 402$;

$C_{2a} = 322$; $C_2 = 355$.

(a) $U = \dfrac{\pi d N}{60}$ $\quad\therefore\quad \underline{N} = \dfrac{407 \times 60}{\pi \times 0{\cdot}75} = \underline{10350}$ Ans.

(b) W.D. per kg of steam $= \dfrac{AB \times EF}{g}$

$$= \frac{407 \times 1252}{9{\cdot}81} = 52\ 000 \text{ kgf m}$$

hp per kg of steam per sec $= \dfrac{52{,}000}{75} = 694$

bhp per kg of steam per sec $= 694 \times 0{\cdot}9 = 624{\cdot}6$

$\therefore$ Steam consumption per bhp-hr

$$= \frac{60 \times 60}{624{\cdot}6} = \underline{5{\cdot}77 \text{ kg}}$$ Ans.

(c) Blade efficiency $= \dfrac{\text{work done on blade}}{\text{energy supplied to blade}}$

$$= \frac{52{,}000}{(1175)^2/2 \times 9{\cdot}81} = \underline{74\%}$$ Ans.

(*d*) Stage efficiency $= \dfrac{\text{work done on blade}}{\text{total energy supplied to stage}}$

$= \dfrac{52{,}000}{183 \times 427} = 66.5\%$. Ans.

(*e*) Axial thrust per kg $= C_{1a} - C_{2a} = \dfrac{402-322}{9.81} = 8.15$ kgf Ans.

(*f*) Loss in exit per kg $= \dfrac{C_2^2}{2g \times J} = \dfrac{355^2}{2 \times 9.81 \times 427} = 15.05$ kcal

$\therefore$ Percentage loss in exit per kg $= \dfrac{15.05}{183} = 8.23\%$. Ans.

13·2. Impulse turbine : θ ; ϕ ; α ; C_1 ; axial thrust : hp ; Δh given initial velocity.

A simple impulse turbine has one ring of moving blades running at 150 metres per second ; velocity of steam reaching nozzles, 90 metres per second ; nozzle efficiency, 0.85 ; absolute velocity of steam at exit from stage, 85 metres per second at an angle of 80° with tangent of wheel ; blade velocity coefficient, 0.82 ; rate of steam flowing, 2 kg per second. Assuming the moving blades to be equiangular find the blade angles, nozzle angle, absolute velocity of steam at entrance, axial thrust, metric hp developed and the heat drop in nozzles in kcal.

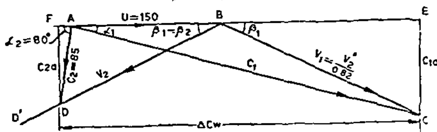


Fig. 13.16

Given : $U = 150$ m/s ; $C_2 = 85$ m/s ; $\alpha_2 = 80°$ $\beta_1 = \beta_2$;
$V_2/V_1 = 0.82$.

From the available data the velocity diagram is drawn and the following are obtained

$\beta_1 = \beta_2 = 27°$; $\alpha = 16.2°$, $C_1 = 366$ m/s Ans.

$C_{1a} = 102$; $C_{2a} = 83.5$; $EF = 366$

Axial thrust $= \dfrac{(C_{1a} - C_{2a}) \times m}{g}$

$= \dfrac{(102 - 83.5) \times 2}{9.81} = 3.77$ kgf Ans.

$$\underline{\text{hp}} = \frac{AB \times EF \times m}{g \times J} = \frac{150 \times 366 \times 2}{9 \cdot 81 \times 75} = \underline{149} \qquad \text{Ans.}$$

Let heat drop in the nozzle be Δh

$$\frac{366^2}{2 \times 9 \cdot 81} - \frac{90^2}{2 \times 9 \cdot 81} = 0 \cdot 85 \times \Delta h \times 427 \quad [\text{nozzle } \eta = 0 \cdot 85$$

$$\therefore \qquad \underline{\Delta h = 17 \cdot 68 \text{ kcal}} \qquad \text{Ans.}$$

13·3. Impulse turbine for max. η conditions ; blade angles ; blade η, stage η, given velocity of approach.

What is the disadvantage of having very small exit angles for nozzles and moving blades of an impulse turbine.

One stage of an impulse turbine consists of a row of nozzles and one row of moving blades. The steam enters the nozzles at a pressure of 15 kgf/cm², dry saturated, with a velocity of 130 m/s. The pressure drops along the nozzle to 9 kgf/cm². The nozzles have discharge angle of 20° and the steam passes into the blades without shock. If the velocity coefficient for nozzle is 0·9, determine for maximum efficiency conditions,

(a) *the blade angles for equiangular blades ;*

(b) *the blading efficiency ;*

(c) *the stage efficiency.*

If exit angle of nozzle is made too small there will be long nozzles which will be accompanied by higher frictional losses. Exit angle of moving blade cannot be reduced beyond a certain limit as it will reduce the area of cross-section and height will have to be increased for no pressure drop in moving blades. Actually there is 5 to 10 per cent pressure drop in moving blades.

From Mollier chart, heat drop = 23 kcal

Useful heat drop = Gain in $K.E.$

$$23 \times 0 \cdot 9 \times 427 = \frac{C_1^2}{2g} - \frac{130^2}{2g}$$

$\therefore$ Velocity from nozzle, $\quad C_1 = 435$ m/s

For maximum efficiency, $\quad U = \dfrac{C_1 \cos \alpha_1}{2}$

$\therefore$ Blade velocity, $\quad U = \dfrac{435 \times \cos 20}{2} = 204$ m/s

(a) $AE=435 \cos 20=408$, $EC=435 \sin 20=148{\cdot}8$

$$\tan \beta_1 = \frac{148{\cdot}8}{408-204} \qquad \therefore \ \underline{\beta_1=36^\circ=\beta_2} \qquad \text{Ans.}$$

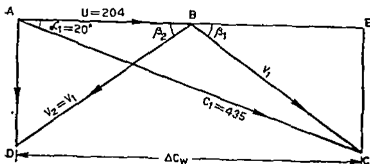


Fig 13·15.

For maximum efficiency, $V_2=V_1$, as [No losses

$$V_2=V_1=\frac{BE}{\cos \beta_1}=\frac{408-204}{\cos 36^\circ}=252 \text{ m/s}$$

$$\text{W.D. per kg of steam}=\frac{2\times 204(408-204)}{9{\cdot}81}=8470 \text{ kgf m}$$

$$\text{K.E. supplied per kg of steam}=\frac{435^2}{2\times 9{\cdot}81}=9620 \text{ kgf m}$$

(b) $\therefore$ <u>Blading efficiency</u> $=\dfrac{8470}{9620}=\underline{88\%}$ Ans.

(c) <u>Stage efficiency</u> $=\dfrac{8\,470}{23\times 427+130^2/2\times 9{\cdot}81}=\underline{79{\cdot}3\%}$

Ans.

13 4. Impulse turbine . η , hp ; heat balance, given no axial thrust.

A single-row impulse turbine stage receives 5 kg of dry saturated steam per second at 10 kgf/cm². The steam is expanded in the nozzle to a pressure of 7 kgf/cm², with an efficiency of 0 94, and discharges at an angle of 20° to the plane of rotation of the blades. Find a suitable exit angle for the blades in order that there shall be no axial thrust on the blades, allowing a blade velocity coefficient of 0 85 and a blade speed of 175 m/s If the internal losses due to disc friction and windage amount to 0·95 kcal/kg of steam, find the efficiency and the hp of the stage. Also draw up a heat balance for the stage.

From Mollier Chart, Heat drop = 16 kcal/kg

Velocity of steam, $C_1=91{\cdot}53\sqrt{0\ 94\times 16}=356$ m/s

Given : $U=175$ m/s ; $\alpha_1=20°$; No axial thrust, *i.e.*
$C_{1a}=C_{2a}$; $K=V_2/V_1=0{\cdot}85$

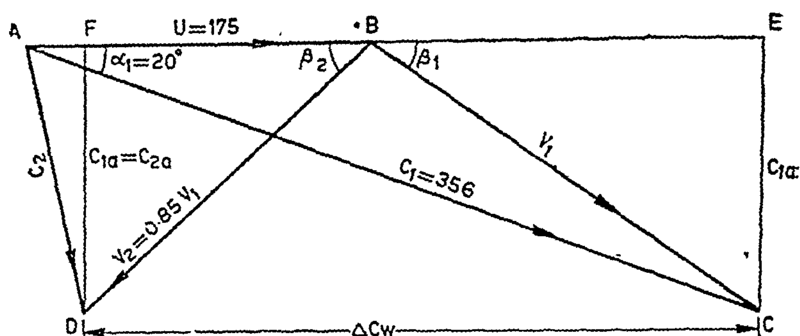


Fig. 13·18.

The velocity diagram is now drawn and the following results obtained : $\beta_o=45{\cdot}5°$ Ans.

$$EF=279 \text{ ; } V_1=200 \text{ ; } C_2=134 \text{ m/s}$$

$$\text{W.D. per kg of steam}=\frac{279\times175}{9{\cdot}81\times427}=11{\cdot}64 \text{ kcal}$$

$$\text{Stage efficiency}=\frac{11{\cdot}64-1}{16}=\frac{10{\cdot}64}{16}=66{\cdot}5\% \quad \text{Ans.}$$

$$\text{hp}=\frac{10{\cdot}64\times5\times427}{75}=303{\cdot}5 \quad \text{Ans.}$$

Energy lost in nozzle $=0{\cdot}06\times16=0{\cdot}96$ kcal/kg

$$\text{Energy lost in blade friction}=\frac{V_1^2(1-K^2)}{2g\times J}$$

$$=\frac{200^2(1-0{\cdot}85^2)}{2\times9{\cdot}81\times427}=1{\cdot}327 \text{ kcal/kg}$$

$$\text{Leaving loss}=\frac{C_2^2}{2g\times J}=\frac{134^2}{2\times9{\cdot}81\times427}=2{\cdot}123 \text{ kcal/kg}$$

Heat balance for stage in kcal per kg

Cr.	*kcal*	%	*Db.*	*kcal*	%
Heat supplied	16	100	(1) Useful work	10·64	66·50
			(2) Nozzle friction	0·96	6·00
			(3) Blade friction	1·327	8·27
			(4) Windage	0·95	5·93
			(5) Leaving loss	2·123	13·30
Total	16	100		16·000	100·00

13 5. Velocity compounding : stage pressure ; diagram η ; hp ; dryness fraction leaving stage.

Enumerate the advantages and disadvantages of velocity compounded impulse stages in steam turbines.

The steam supply to a velocity-compounded impulse stage with two rows of moving blades is at 60 kgf/cm² and 400°C. Find (a) the stage pressure, (b) the diagram efficiency, and (c) the power output from the following data :—

Speed 3000 rev/min, mean diameter of blading 1 2 m, steam flow 5 kg/s, blade speed/steam speed ratio 0·2. Nozzle efficiency 90 per cent, nozzle angle 18°, exit angles 26°, 25° and 40° for the first moving, fixed and second moving respectively. Velocity coefficient 0·86 of all blades. 25 h.p. is used in disc friction and windage.

What is the condition of steam leaving the stage ?

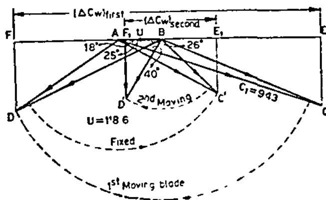


Fig 13 19.

(a) Blade velocity, $U = \dfrac{\pi \times 1{\cdot}2 \times 3000}{60} = 188\ 6$ m/s

Steam velocity, $C_1 = \dfrac{188\ 6}{0\ 2} = 943$ m/s

$\therefore$ Heat drop, $\Delta h = \dfrac{943 \times 943}{2 \times 9{\cdot}81 \times 0\ 9 \times 427} = 118$ kcal

From Mollier chart, for the calculated heat drop

Stage pressure = 6·4 kgf/cm² Ans.

(b) Now the velocity diagram is drawn from the following data available, $U=188\cdot6$ m/s ; $C_1=943$ m/s ; $\alpha_1=18°$; $\alpha_3=25°$;

$$\beta_4=40° \; ; \; V_2/V_1=0\cdot86$$

From the diagram : $EF=1298$; $E'F'=373$; $C_2=148$

$$\text{W.D. per kg of steam}=\frac{AB\times(EF+E'F')}{g\times J}$$

$$=\frac{188\cdot6(1298+373)}{9\cdot81\times427}=75\cdot1 \text{ kcal}$$

$$\underline{\text{Diagram efficiency}}=\frac{75\cdot1}{118\times0\cdot9}=\underline{70\cdot8\%} \qquad \text{Ans.}$$

(c) $$\underline{\text{Net power output}}=\frac{75\cdot1\times427\times5}{75}-25$$

$$=2140-25=\underline{2115 \text{ hp}} \qquad \text{Ans.}$$

$$\text{Loss at exit per kg}=\frac{148\times148}{2\times9\cdot81\times427}=2\cdot61 \text{ kcal}$$

$$\text{Heat equivalent of useful work}=\frac{2{,}115\times75}{427\times5}=74\cdot2 \text{ kcal/kg}$$

Hence, $\Delta h=74\cdot2+2\cdot61=76\cdot81$ kcal/kg

and from Mollier chart,

<u>Final condition of steam $=200°C$ (superheat)</u> Ans.

13·6. Velocity-compounding, axial discharge, no end thrust : U ; blade and nozzle angles.

Explain why, in multi-stage turbines, the first stage is often compounded for velocity and the remaining stages have single row wheels.

In a velocity-compounded impulse turbine the steam leaves the nozzles at 500 m/s and after passing through the fixed and moving blade rings it is discharged from the second moving blade ring in an axial direction. The axial velocity of flow in the first moving blade ring is twice that in the second moving blade ring. The discharge angle of the second moving blade ring is 30° (measured from the plane in which this blade ring rotates) and it is estimated that the relative velocity of steam falls by 10 per cent in passage through each blade ring, fixed and moving. The blade angles are such that the flow is shockless and that there is no end thrust on either moving ring.

Determine the blade speed, the necessary blade and nozzle angles and the work done per kg of steam.

Neglecting any increase of volume of the steam, find the ratio of the height required for each row of blades, taking the height of the nozzle as unity.

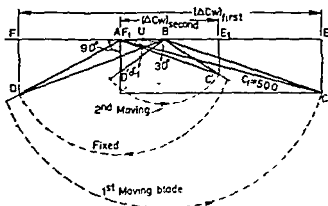


Fig. 13·20.

The following data are given in the question : $C_1=500$ m/s $\beta_2=90°$; C_a in first moving blade$=2C_a$ in second moving blade ; $\phi_2=30°$; blade velocity coefficient, $k=0·9$; $C_{1a}=C_{2a}$.

The velocity diagram is drawn in the reverse direction taking $AB=1$ unit.

$$AC=C_1=500 \text{ m/s}$$

$\therefore$ Scale of diagram · 1 unit$=\dfrac{500}{\text{length } AC}=\dfrac{500}{4·73}=105·7$

Hence, blade velocity, $U=105·7\times1=\underline{105·7 \text{ m/s}}$ Ans

From the diagram, nozzle angle, $\alpha_1=14·1°$
First moving ring : $\beta_1=17·9°$; $\beta_2=20°$
Fixed blade : $\alpha_4=28°$; $\alpha_3=15°$
Second moving ring : $\beta_1=26·7°$ Ans

First stage, $C_{2a}=C_{1a}=122$ m/s ; Second stage, $C_a=61$ m/s

$$\text{W.D per kg of steam}=\frac{AB(EF-E_1F_1)}{g}$$

$$=\frac{105·7(715+228)}{9·81}$$

$$=\underline{10,140 \text{ kgf m or } 23·75 \text{ kcal}}$$ Ans

For continuity of flow, the ratio $\frac{\text{area} \times \text{vel. of flow}}{\text{specific volume}}$ must be constant.

Assuming that the blades are parallel, the product blade height×velocity of flow=constant. Given height of nozzle as unity.

Referring the blade heights to exit conditions, as is customary,

$$\left.\begin{aligned} \text{Height of first moving blade} &= \frac{122 \times 1}{122} = 1 \\ \text{Height of fixed blade} &= \frac{122 \times 1}{122} = 1 \\ \text{Height of second moving blade} &= \frac{122 \times 1}{61} = 2 \end{aligned}\right\} \quad \text{Ans.}$$

(B) REACTION TURBINES

13·7. Reaction turbine : stage η ; blade height.

One stage comprising a pair of blade rings of a 50 per cent reaction axial flow turbine has inlet and outlet angles of 80° and 20°. The mean diameter of the blades is 1·8 m and the turbine runs at 1200 r.p.m.

Steam is admitted to the stage at 12 kgf/cm² pressure and 200°C temperature and undergoes an adiabatic heat drop of 4·2 kcal/kg. Five per cent of the steam supplied is lost through leakage. If the horse power developed in the stage is 620, determine :—

(a) the stage efficiency ;
(b) the blade height.

Fig. 13·21.

Given : $\alpha = \phi = 20°$; $\theta = \beta = 80°$

Mean blade speed, $U = \frac{\pi \times 1 \cdot 8 \times 1200}{60} = 113$ m/s

The velocity diagram can now be plotted to scale, as in Fig. above. As the blades are equiangular and degree of reaction is 50 per cent the diagram is symmetrical. Alternatively, the calculations may be made as follows :

In ΔABC, $\dfrac{AC}{\sin 100} = \dfrac{113}{\sin 60}$ $\quad \therefore \quad AC = C_1 = 128{\cdot}6$

$EF = 2AC \cos 20° - AB = 2 \times 128{\cdot}6 \cos 20 - 113 = 128{\cdot}6$

W.D. per kg of steam flowing over the blades

$$= \frac{113 \times 128{\cdot}6}{9{\cdot}81} = 1483 \text{ kgf m}$$

As there is 5 per cent leakage, W.D. per kg of steam entering the turbine $= 1483 \times 0{\cdot}95 = 1409$ kgf m

(a) Stage efficiency $= \dfrac{\text{W.D.}}{\text{Heat drop}}$

$$= \frac{1409}{4{\cdot}2 \times 427} = 78{\cdot}5 \text{ per cent} \qquad \textbf{Ans.}$$

(b) Let m be the actual steam flow over the blades

$\text{hp} = 620 = \dfrac{m \times 1409}{75}$ $\quad \therefore \quad m = 33{\cdot}03$ kg/s

Since the heat drop per stage is very small specific volume at entrance can be assumed to be equal to that at exit.

From Mollier chart, $\quad v = 0{\cdot}172\ m^3$

Velocity of flow, $C_{1a} = 128{\cdot}6 \sin 20 = 44$ m/s

Since, area $\times$ velocity of flow $=$ steam flow over blades $\times\, v$

$$\pi \times 1{\cdot}8 \times h \times 44 = 33{\cdot}03 \times 0{\cdot}172$$

$\therefore$ Height of blade, $h = 0{\cdot}02285$ or $2{\cdot}285$ cm **Ans.**

13.8. 8-stage reaction turbine : hp ; pressure at the end of expansion ; blade height.

The first expansion of reaction turbine is to be designed for a flow of 5 kg per second when supplied with dry saturated steam at 15 kgf/cm². It is to have eight pairs on a mean diameter 50 cm. The speed is 2600 rpm and the average value of blade speed/steam speed $= 0{\cdot}8$. The tip leakage of steam at all rows is 8 per cent of total and the efficiency of the working steam is 0·85. The blading outlet angle is 20°

for both fixed and moving blades. Determine (a) the power, (b) the pressure at the end of the expansion, and (c) the average blade height.

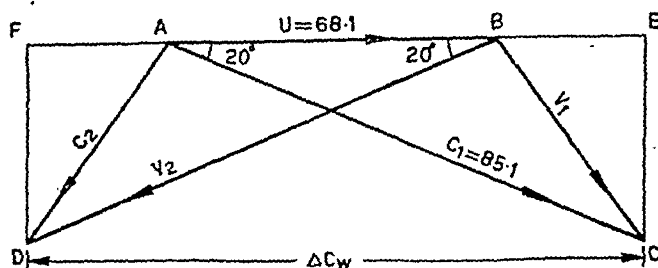


Fig. 13·22.

(*a*) Given : $\alpha_1 = \beta_2 = 20°$

Mean blade speed, $U = \dfrac{\pi \times 0{\cdot}5 \times 2600}{60} = 68{\cdot}1$ m/s

$\therefore$ Steam speed $= \dfrac{68{\cdot}1}{0{\cdot}8} = 85{\cdot}1$ m/s

Assuming that the degree of reaction is 50 per cent *i.e.* inlet and outlet diagrams are symmetrical, the diagram can be plotted to scale as in Fig. above or the necessary dimensions can be calculated,

$C_{1a} = 85{\cdot}1 \sin 20 = 29{\cdot}1$ m/s

$EF = 2\ AC \cos 20 - AB = 2 \times 85{\cdot}1 \cos 20 - 68{\cdot}1 = 91{\cdot}8$

$\underline{\text{hp}} = 8 \times \dfrac{5 \times 0{\cdot}92 \times 68\ 1 \times 91{\cdot}8}{9\ 81 \times 75} = \underline{312{\cdot}6}$ Ans.

(*b*) Theoretical heat drop

$\Delta h_T = \dfrac{312{\cdot}6 \times 75}{427 \times 5 \times 0{\cdot}85} = 12{\cdot}92$ kcal/kg

At 15 kgf/cm² dry saturated, $h_g = 666{\cdot}7$ kcal/kg, $s_1 = 1{\cdot}541$

$h_2 = 666{\cdot}7 - 12{\cdot}92 = 653{\cdot}8$ kcal/kg

As the heat drop is small it is difficult to estimate accurately the pressure at the end of expansion from Mollier chart. From the chart the pressure drops approximately to 12 kgf/cm².

We know entropy should be 1·541 and $h = 653{\cdot}8$ kcal/kg

From steam tables

at 12 kgf/cm², $h = 655{\cdot}8$; at 11·5 kgf/cm², $h = 653{\cdot}8$

$\therefore$ $\underline{\text{Final pressure} = 11\ 5\ \text{kgf/cm}^2}$ Ans

(c) Assuming the specific volume of the steam to remain constant in the stage

$$\pi \times \frac{50}{100} \times \frac{h}{100} \times 29 \cdot 1 = 5 \times 0 \cdot 92 \times 0 \cdot 1342$$

$\therefore$ Blade height, $h = 1 \cdot 35$ cm Ans.

13·9. Reaction turbine : θ ; W.D. ; increase in η for best speed.

Deduce an expression for work done per stage (one ring of fixed blades plus one ring of moving blades) of a reaction machine and determine the conditions for maximum efficiency.

At a stage of a reaction machine the mean ring diameter is 140 cm. The speed ratio is 0·7. The speed of rotation is 3000 rpm. Determine the required entrance angle for the blading if the exit angle is 20°. Determine the work done per kg of steam flowing per sec.

Calculate the percentage increase in diagram efficiency if the blades are designed for and run at the best theoretical speed, the exit angle being kept 20°.

Fig. 13·23.

For theory—See text.

$$\text{Blade speed, } U = \frac{\pi \times 1 \cdot 4 \times 3{,}000}{60} = 220 \text{ m/s}$$

$$\therefore \text{ Velocity of steam, } C_1 = \frac{220}{0 \cdot 7} = 314 \cdot 3 \text{ m/s}$$

$$V_1 = \sqrt{C_1^2 + U^2 - 2C_1 U \cos \alpha_1}$$

$$= \sqrt{314 \cdot 3^2 + 220^2 - 2 \times 314 \cdot 3 \times 220 \times \cos 20}$$

$$= 131 \cdot 4 \text{ m/s}$$

$$\frac{V_1}{\sin \alpha_1} = \frac{C_1}{\sin \beta_1}, \quad \sin \beta_1 = \frac{314 \cdot 3 \times \sin 20}{131 \cdot 4} \quad \therefore \beta_1 = 55^\circ \quad \text{Ans.}$$

$$\text{W.D. per kg of steam} = \frac{AB \times EF}{g} = \frac{U(2C_1 \cos \alpha_1 - U)}{g}$$

$$=\frac{220(2\times314\cdot3\times\cos 20-220)}{9\cdot81}$$

$=\underline{8310 \text{ kgf m}}$ Ans.

$$\text{Diagram } \eta=\frac{\text{W.D.}}{\text{Energy supplied}}=\frac{\text{W.D.}}{[C_1^2+V_2^2-V_1^2]/2g}$$

$$=\frac{8310}{[314\cdot3^2+314\cdot3^2-131\cdot4^2]/2g} \qquad [C_1=V_2]$$

$=0\cdot905$ or $90\cdot5$ per cent

From text, best theoretical blade speed $=C_1 \cos \alpha_1$

$=314\cdot3\times\cos 20=295\cdot3$ m/s

$$V_1=\sqrt{314\cdot3^2+295\cdot3^2-2\times314\cdot3\times295\cdot3\times\cos 20}$$

$=107\cdot2$ m/s

$$\text{Diagram } \eta=\frac{U(2C_1\cos\alpha_1-U)}{g[C_1^2+V_2^2-V_1^2]/2g}$$

$$=\frac{295\cdot3\times(2\times314\cdot3\times\cos 20-295\cdot3)}{\frac{1}{2}[314\cdot3^2+314\cdot3^2-107\cdot2^2]}=93\cdot8 \text{ per cent}$$

$$\underline{\text{Percentage increase in } \eta}=\frac{93\cdot8-90\cdot5}{90\cdot5}=\underline{3\cdot65\%} \qquad \text{Ans.}$$

$$\left[\text{Check : } \eta \text{ for best speed}=\frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}=\frac{2\cos^2 20}{1+\cos^2 20}=93\cdot8\%\right]$$

13·10. Reaction turbine : degree of reaction ; Δh_T ; η stage.

In a reaction turbine the mean blade speed is 150 m/s and the ratio of blade speed to steam speed is 0·625. The outlet angles of fixed and moving blades are 20° and 30° respectively. Calculate

(a) the degree of reaction, (b) the adiabatic heat drop in a pair of blade rings, (c) the gross stage efficiency.

The specific volume of steam at fixed blade outlet is 0·567 m³ and at moving blade outlet 0·6 m³. Assume the efficiency of the blades when considered as nozzles 0·90 and $K^2=0\cdot86$, where K is the blade velocity coefficient.

(a) Steam speed $=\dfrac{150}{0\cdot625}=240$ m/s

$C_{1a}=240 \sin 20=82\cdot1$ m/s

Since mass flow and area are constant,

$$\frac{C_{1a}}{v_{inlet}}=\frac{C_{2a}}{v_{outlet}}; \quad \frac{82\cdot1}{0\cdot567}=\frac{C_{2a}}{0\cdot6}, \quad \therefore \quad C_{2a}=86\cdot7 \text{ m/s}$$

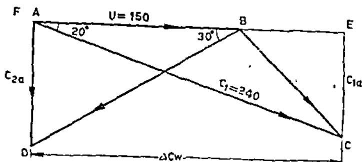


Fig 13·24.

$AE=240 \cos 20=225{\cdot}5$ $\quad\therefore\quad BE=225{\cdot}5-150=75{\cdot}5$

$$V_1^2=75{\cdot}5^2+82{\cdot}1^2=12440\ ;\ V_2=\frac{C_{2a}}{\sin 30}=173{\cdot}4$$

$FB=173{\cdot}4 \cos 30=150{\cdot}2$ $\quad\therefore\quad AF=150{\cdot}2-150=0{\cdot}2$

$$C_2^2=86{\cdot}7^2+0{\cdot}2^2=7517$$

$$\text{Heat drop in fixed blades}=\frac{C_1^2-K^2C_2^2}{2g\times J\times \eta_n}$$

$$=\frac{240^2-0{\cdot}86\times 7517}{2\times 9{\cdot}81\times 427\times 0{\cdot}90}=6{\cdot}781 \text{ kcal}$$

$$\text{Heat drop in moving blades}=\frac{V_2^2-K^2V_1^2}{2g\times J\times \eta_n}$$

$$=\frac{173{\cdot}4^2-0{\cdot}86\times 12440}{2\times 9{\cdot}81\times 427\times 0{\cdot}90}$$

$$=2{\cdot}568 \text{ kcal}$$

$\therefore$ Degree of reaction $=\dfrac{2{\cdot}568}{6{\cdot}781+2{\cdot}568}=27{\cdot}5\%$ Ans.

(b) Addition heat drop $=6{\cdot}781+2{\cdot}568=9{\cdot}349$ kcal/kg Ans.

(c) $\text{W.D.}=\dfrac{AB\times EF}{g}=\dfrac{150(150{\cdot}2+75{\cdot}5)}{9{\cdot}81}=3450$ kgf m

$\therefore$ Stage efficiency $=\dfrac{3450}{9{\cdot}349\times 427}=86{\cdot}5\%$ Ans.

13·11. Radial double motion reaction turbine : blade angles ; steam/sec ; stage η.

The following data relates to a radial double motion reaction turbine : hp, 1,200 : steam supplied at 20 kgf/cm², dry saturated ; rpm of each rotor, 3000 ; mean diameter, 75 cm ; steam speed at inlet 600 m/s ; radial steam velocity twice that of each blade. Find :—

(a) the blade angles ; (b) the steam consumption per sec ; and (c) the stage efficiency if the exit pressure is 5 kgf/cm².

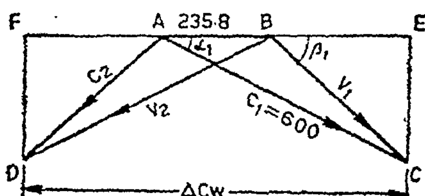


Fig. 13·25.

In radial flow double motion turbines there are two rotors rotating in opposite direction. There are no fixed blades in this turbine. Both blades are in motion, but the problem may be solved by considering one at rest and the other as moving with its velocity relative to the first. Thus the velocity diagram may be drawn taking U as twice the actual blade speed.

(a) Mean blade speed$=\dfrac{\pi\times 0{\cdot}75\times 3000}{60}=117{\cdot}9$ m/s

Relative velocity U $(AB)=2\times 117{\cdot}9=235{\cdot}8$ m/s

The diagram is plotted from the given data and blade angles can be measured. Blade angles can also be calculated.

$\sin\alpha=\dfrac{235.8}{600}$ ∴ Blade exit angle, $\alpha_1=23{\cdot}1°$ Ans.

$\tan\beta_1=\dfrac{235{\cdot}8}{600\cos 23{\cdot}1-235{\cdot}8}$ ∴ Blade inlet angle, $\beta_1=36{\cdot}5°$ Ans.

(b) $EF=2\times 600\cos 23{\cdot}1-235{\cdot}8=866{\cdot}2$

$$\text{hp}=\frac{m(AB\times EF)}{g\times 75}$$

∴ Steam consumption

$$m=\frac{1200\times 9{\cdot}81\times 75}{235{\cdot}8\times 866{\cdot}2}=4{\cdot}34 \text{ kg/s}$$ Ans.

(c) Useful heat drop$=\dfrac{235{\cdot}8\times 866{\cdot}2}{9{\cdot}81\times 427}=48{\cdot}7$ kcal/kg

From Mollier chart, $\Delta h_T=61{\cdot}5$ kcal/kg

∴ Stage $\eta=\dfrac{48{\cdot}7}{61{\cdot}5}=79{\cdot}2$ per cent Ans.

EXAMPLES 13

(A) IMPULSE TURBINES

13.1. Impulse turbine : blade dia ; residual energy ; hp

The rotational speed of an impulse steam turbine wheel is 3000 rev/min. The nozzle is inclined at 20° to the plane of the wheel and its efficiency is 0·89. The isentropic heat drop for the stage is 38 kcal/kg. If the ratio of blade speed to steam speed is 0·4, the blade velocity coefficient is 0·82 and the blading efficiency is 0·76, find (a) the mean blade ring diameter, (b) the residual energy of steam at outlet in kcal per kg of steam, and (c) the horse power developed by the wheel when the steam flow is 15 kg/s.

[C_1=532 m/s , U=212·8 m/s ; D=1·356 m , EF=506 m/s ; C_2=173·3 m/s ; residual energy=12·3 kcal/kg , hp=2195]

13.2. Impulse turbine, discharge axial : θ ; ϕ ; axial thrust.

An impulse turbine has a mean blade ring diameter of 60 cm and runs at 3000 rev/min. The ratio of mean blade speed to jet velocity is 0·45 and the nozzle angle relative to the direction of blade motion is 20°. The relative velocity at outlet from the blading is 88 per cent of that at entry. Determine :—

(a) the blade inlet angle ; (b) the blade outlet angle if the steam is to discharge from the blades in an axial direction ; and (c) the axial thrust per kg of steam flowing per second.

[U=94·3 m/s ; C_1=209·6 m/s , θ_1=35° , θ_2=30·7° ; axial thrust=1·59 kg]

13.3. Impulse turbine, max η ; blade angles ; velocity coefficient ; loss in friction given blading η and axial thrust.

Deduce an expression for work done per stage of an impulse machine and determine the conditions for maximum efficiency.

In a simple impulse turbine steam leaves the nozzle with a velocity of 1000 m/s inclined at an angle of 20° to the plane of rotation. The blade velocity is 60 per cent of the velocity for maximum efficiency. If the blading efficiency is 70 per cent and axial thrust 4 kg per kg

steam, calculate (a) the blade angles. (b) the blade velocity coefficient, (c) the heat lost in friction in kcal/kg.

[For max. η, $\rho=\cos\alpha_1/2$; $U=282$ m/s ; $C_{2a}=302{\cdot}8$ m/s ; $EF=1241$ m/s ; $\beta_1=27{\cdot}4°$; $\beta_2=27{\cdot}4°$; $K=88{\cdot}5\%$; loss in friction $=14{\cdot}3$ kcal/kg]

13·4. Impulse turbine : W.D. ; losses : heat balance sheet.

The Rankine heat drop, at a stage of an impulse steam turbine is 32 kcal/kg and the ratio of blade speed to jet speed is 0·33. The inclination of the steam jet to the plane of the wheel is 16° and the blade outlet angle is 26°. The nozzle efficiency is 0·9; the blade velocity coefficient is 0·8 and 3 per cent of the work done on the blades is lost in disc and vane friction. Draw the velocity diagrams and find (a) the nozzle loss, (b) the loss in the blade channels, (c) the disc and vane friction loss, (d) the carry-over or residual jet energy loss and (e) the net work available. Express the quantities as percentages of the Rankine heat drop in the stage.

[$C_1=491$ m/s ; $U=162$ m/s ; heat balance in kcal/kg : useful work, 20·77 kcal (64·9%) ; nozzle friction, 3·2 kcal (10%) ; blade friction, 4·92 kcal (15·4%) ; disc friction, 0·64 kcal (2%) ; residual, 2·47 kcal (7·7%)].

13·5. Velocity compounding : blade angles ; C_3 ; W. D. ; blade heights.

Give reasons why velocity-compounded impulse stages are usually situated at the high-pressure inlet end of steam turbines.

In the first pressure stage of a turbine of the Curtis type, there are two velocity stages. The steam velocity at the nozzle is 750 m/s, whilst the mean peripheral speed of the moving blades is 150 m/s. In its passage through the fixed blades, the steam may be assumed to have its velocity unaltered, but in passing through the moving blades, the relative steam velocity may be assumed to be diminished by 20 per cent in each case. The steam jet angle is 20°, the exit angles of the blades being (i) first moving, 22°, (ii) fixed, 24°, (iii) second moving, 35°. Find the inlet angles, the velocity in magnitude and direction of the exhausting steam, and the work done per kg of steam.

Calculate how the blade heights vary (in terms of the nozzle height) neglecting the variations of the quality of the steam due to friction.

[$\beta_1=24{\cdot}8°$; $\beta_3=39{\cdot}7°$; $\alpha_2=31{\cdot}1°$; $\alpha_4=90°$; $W.D.$/kg$=20\,350$ kgf m ; blade heights 1 : 1·4 : 1·778 : 2·541]

13·6. Velocity compounding, axial discharge : U ; stage η.

The following particulars relate to a stage of an impulse turbine, compounded for velocity, in which the steam supplied to the nozzle box is at a pressure of 10 kgf/cm² and superheated to 220°C :—

Pressure of steam in wheel chamber, [illegible] kgf/cm² ; discharge angle for two rings of moving blades, 30° ; discharge angle for nozzle and single row of fixed blades, 20°.

Find (a) the blade speed for final discharge to be axial assuming that friction causes reduction of 10 per cent in relative velocity for fixed and moving blades, (b) the stage efficiency.

Assume nozzle efficiency as [illegible] per cent.

[$\Delta h = 38·3$ kcal, $C_1 = 566$ m/s ; let $C = 100$, then $C_1 = 512·4$
$\therefore$ scale $= 1·105$ ∴ $U = 110·5$ m/s ; $W.D. = 11\ 120$ kgf m ;
stage $\eta = 60·5\%$]

(B) REACTION TURBINES

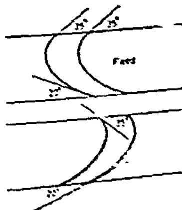
13·7. Reaction turbine : blade height ; drum diameter.

The measured consumption of a Parson's reaction turbine is 16 000 kg/hr. The speed of the turbine is [illegible] rpm. At a certain pair in the turbine the pressure is found to be [illegible] kgf/cm² and steam dryness fraction 0·95. The hp developed by the turbine is [illegible]. The discharging blade tip angle is 20° for both fixed and moving blades and the axial velocity of flow is 0·75 of the blade velocity. The tip leakage steam is found to be 10 per cent. Neglecting blade thickness, determine drum diameter and blade height of the pair considered.

[$EF = 3\ 121\ V$, $U = 18$[illegible] m/s ∴ $D = 85·5$ cm ; $h = 9·05$ cm]

13·8 Reaction turbine ; no of stages ; height of blades.

The figure shows, diagrammatically, the high pressure blading of a Parson's turbine. The blade speed is [illegible] m/s. The turbine is supplied



steam, calculate (a) the blade angles, (b) the blade velocity coefficient, (c) the heat lost in friction in kcal/kg.

[For max. η, $\rho=\cos\alpha_1/2$; $U=282$ m/s ; $C_{2a}=302{\cdot}8$ m/s ; $EF=1241$ m/s ; $\beta_1=27{\cdot}4°$; $\beta_2=27{\cdot}4°$; $K=88{\cdot}5\%$; loss in friction $=14{\cdot}3$ kcal/kg]

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The Rankine heat drop, at a stage of an impulse steam turbine is 32 kcal/kg and the ratio of blade speed to jet speed is 0·33. The inclination of the steam jet to the plane of the wheel is 16° and the blade outlet angle is 26°. The nozzle efficiency is 0·9; the blade velocity coefficient is 0·8 and 3 per cent of the work done on the blades is lost in disc and vane friction. Draw the velocity diagrams and find (a) the nozzle loss, (b) the loss in the blade channels, (c) the disc and vane friction loss, (d) the carry-over or residual jet energy loss and (e) the net work available. Express the quantities as percentages of the Rankine heat drop in the stage.

[$C_1=491$ m/s ; $U=162$ m/s ; heat balance in kcal/kg : useful work, 20·77 kcal (64·9%) ; nozzle friction, 3·2 kcal (10%) ; blade friction, 4·92 kcal (15·4%) ; disc friction, 0·64 kcal (2%) ; residual, 2·47 kcal (7·7%)].

13·5. Velocity compounding : blade angles ; C_2 ; W. D. ; blade heights.

Give reasons why velocity-compounded impulse stages are usually situated at the high-pressure inlet end of steam turbines.

In the first pressure stage of a turbine of the Curtis type, there are two velocity stages. The steam velocity at the nozzle is 750 m/s, whilst the mean peripheral speed of the moving blades is 150 m/s. In its passage through the fixed blades, the steam may be assumed to have its velocity unaltered, but in passing through the moving blades, the relative steam velocity may be assumed to be diminished by 20 per cent in each case. The steam jet angle is 20°, the exit angles of the blades being (i) first moving, 22°, (ii) fixed, 24°, (iii) second moving, 35°. Find the inlet angles, the velocity in magnitude and direction of the exhausting steam, and the work done per kg of steam.

Calculate how the blade heights vary (in terms of the nozzle height) neglecting the variations of the quality of the steam due to friction.

[$\beta_1=24{\cdot}8°$; $\beta_3=39{\cdot}7°$; $\alpha_2=31{\cdot}1°$; $\alpha_4=90°$; W.D./kg $=20\,350$ kgf.m ; blade heights 1 : 1·4 : 1·778 ; 2·54]

13 6. Velocity compounding, axial discharge : U ; stage η_s.

The following particulars relate to a stage of an impulse turbine, compounded for velocity, in which the steam supplied to the nozzle box is at a pressure of 10 kgf/cm² and superheated to 220°C :—

Pressure of steam in wheel chamber, 5 kgf/cm² ; discharge angle for two rings of moving blades, 30° ; discharge angle for nozzle and single row of fixed blades, 20°.

Find (a) the blade speed for final discharge to be axial assuming that friction causes reduction of 16 per cent in relative velocity for fixed and moving blades, (b) the stage efficiency.

Assume nozzle efficiency as 88 per cent.

[Δh=38·3 kcal ; C_1=566 m/s ; let U=100, then C_1=512·4 ∴ scale=1·105 ∴ $\underline{U=110\text{·}5 \text{ m/s}}$; $W.D.$=11 120 kgf m ; stage $\underline{\eta_s=60\text{·}5\%}$]

(B) Recation Turbines

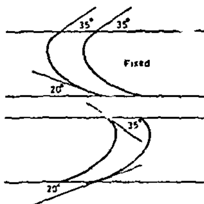
13 7. Reaction turbine : blade height ; drum diameter.

The measured consumption of a Parson's reaction turbine is 16 000 kg/hr. The speed of the turbine is 426 rpm. At a certain pair in the turbine the pressure is found to be 2 kgf/cm² and steam dryness fraction 0·95. The hp developed by the turbine is 6 0. The discharging blade tip angle is 20° for both fixed and moving blades and the axial velocity of flow is 0·75 of the blade velocity. The tip leakage steam is found to be 10 per cent. Neglecting blade thickness, determine drum diameter and blade height of the pair considered.

[EF=3·121 U ; U=18·8 m/s ∴ $\underline{D=85\text{·}5 \text{ cm}}$; $\underline{h=9\text{·}05 \text{ cm}}$]

13 8 Reaction turbine ; no of stages ; height of blades

The figure shows, diagrammatically, the high pressure blading of a Parson's turbine. The blade speed is 70 m/s. The turbine is supplied



with steam at 16 kgf/cm², dry saturated and the exit pressure is 2 kgf/cm². Calculate the number of stages, neglecting all losses. Assume blade height are graded so as to give constant steam velocity.

If the efficiency of the turbine due to friction and other losses is 60 per cent of that of the Rankine engine, calculate the height of the blades at entry for a turbine giving 5 000 hp at 750 rpm.

[W.D./stage=3·7 kcal, x_{exit}=0·875 ; Rankine heat drop=86 kcal ; ∴ no. of stages=24 ; D=1·782 m ; steam/sec=17 kg ; h=0·721 cm]

13·9. Reaction turbine : max η : area of blade annulus; heat drop ; pressure drop.

What is the essential difference between impulse and reaction turbines so far as the flow of steam is concerned?

In a stage of Parson's turbine the blade speed is 70 m/s and the ratio of blade speed/steam speed is 0·48 of that required for maximum efficiency. The exit angle of both fixed and moving blades is 20°. The flow of steam is 16200 kg/hr at 1·4 kgf/cm², dry and saturated. Calculate (a) the required area of blade annulus, (b) the heat drop in kcal required by the pair if the steam expands with an efficiency ratio of 0·8. and (c) the pressure drop in the pair.

[For maximum η, ρ=cos α_1 ; C_1=155·2 m/s ; area=0·1 m² ; h_T=4·62 kcal ; pressure drop=0·146 kgf/cm²]

13·10. Reaction turbine : hp ; rpm ; degree of reaction ; heat drop.

In a stage of a reaction turbine the steam leaves the fixed blades at a pressure of 3 kgf/cm², with a dryness fraction of 0·94 and a velocity of 143 m/s. The ratio of axial velocity of flow to blade velocity is 0·70 at entry to and 0·75 at exit from the moving blades. The discharge angle of both fixed and moving blades are equal. If the height of the blade is 1·8 cm find for a flow of 2·5 kg of steam per sec, determine

(a) the hp developed, (b) the speed of the turbine in rpm, (c) the degree of reaction, and (d) the heat drop per kg of the steam flow to the turbine with an efficiency ratio of 0·85.

Assume blade velocity to be 70 m/s.

Neglect the change of volume in passage over the moving blade and effect of blade thickness but assume a tip leakage of 9 per cent of the total steam.

[EF=208·6 m/s ; hp=95·2 ; D=44·5 cm ; N=3,000 rpm : Δh_U/kg of steam flow over the blades, fixed=1·454 kcal, moving 2·03 kcal, degree of reaction=58·2% ; Δh_T/kg of total flow=3·73 kcal]

13 11. Radial double motion reaction turbine : θ ; hp ; Δh_T.

In a radial reaction turbine with double motion the blade exit angle at a certain pair is 20°, the mean radius 45 cm, the speed of each rotor is 3000 rpm, radial steam velocity is twice that of each blade, steam flow 5 kg/s. Find the blade inlet angle, the horse powre developed by the pair, and the adiabatic heat drop in kcal, if the stage efficiency is 75 per cent.

[U=141·4 m/s ; as double flow, take U=141·4×2=282·8 ; EF=1271 m/s ; β_1=29 6° ; hp=2444 ; Δh_T=114·5 kcal/kg]

14

Steam Turbine Performance

14·1. Reheat Factor. In steam turbines the blade friction, shock, leakage, etc., reduce the effective heat drop. This energy wasted in friction, etc., reheats the steam and improves its quality

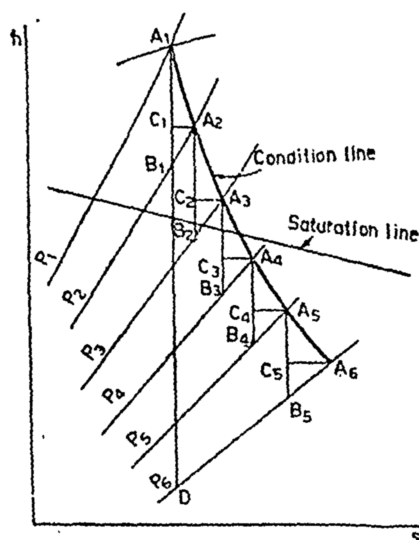


Fig. 14·1. Expansion in a multi-stage turbine.

as it emerges from the stage. Referring to Fig. 14·1, $A_1B_1(\Delta h_T)$ is the isentropic heat drop for the first stage, out of which $A_1C_1(\Delta h_U)$ is the useful heat drop. As defined in previous chapter,

$$\text{Stage efficiency or efficiency ratio, } \eta_s = \frac{AC}{AB} = \frac{\Delta h_U}{\Delta h_T} \qquad (14\cdot1)$$

The condition of the steam at exit from the first stage of a multi-stage pressure compounded or reaction turbine, A_2, is obtained at the pressure of B_1 and the enthalpy of C_1. The condition of steam at exit

from other stages is found in the similar way and the line joining these points is known as *condition line* which represents the probable path of steam expansion. The condition line is useful in finding the superheat or dryness fraction and the specific volume of steam at any pressure in the expansion.

As the constant pressure lines in Mollier chart converge near the origin and diverge towards the right, the sum of the individual heat drops, known as the cumulative heat drop, in the different stages (ΣAB) is greater than the direct or adiabatic drop (A_1D) in a single stage. The ratio $\frac{\Sigma AB}{A_1D}$ is known as the *reheat factor.*

$$\text{Reheat factor, R.F.} = \frac{\text{cumulative heat drop}}{\text{adiabatic heat drop}} = \frac{\Sigma AB}{A_1D} \qquad (14\cdot2)$$

The reheat factor depends on turbine stage efficiency, initial pressure and superheat and exit pressure It is greater the larger the number of stages and lower the stage efficiency. The value varies from 1 02 for turbines with a few stages to 1·06 in large turbines with many stages.

14 2. Reheat Factor on T-s Diagram. Fig. 14 2 represents multi-stage expansion on T-s diagram. The adiabatic heat drop

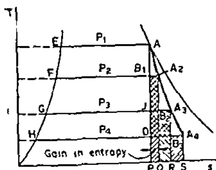


Fig. 14·2 Expansion of steam on T-s diagram.

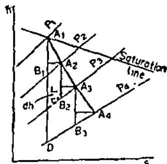


Fig 14 3 Expansion of steam on h-s diagram.

in the first stage is represented by the area EA_1B_1F, corresponding to length A_1B_1 in h s diagram, Fig 14·3 The energy dissipated in friction and returned to the steam as heat, is represented by the area B_1A_1QP, increase in entropy due to this reheating being PQ. Out of the area B_1A_2QP the heat available for doing work in the next stage is given by cross hatched area. The increase in the adiabatic heat drop in second stage due to friction losses in the first stage is given b

area $B_1A_2B_2J$, corresponding to dh in h-s diagram by drawing a parallel line B_1A_2.

Similar reheating of steam takes place in subsequent stages but the availability of reheat goes on decreasing. For this reason it is desirable to have an inefficient stage *i.e.* a two-row velocity-compounded wheel, at the high pressure end of the turbine rather than at the low pressure end.

14·3. Efficiencies. Due to reheat factor the efficiency of the machine as a whole is greater than its individual stages. The efficiency of the complete machine is known as the internal efficiency of the turbine.

$$\text{Internal turbine efficiency} = \frac{\text{total useful heat drop}}{\text{adiabatic heat drop}}$$

or
$$\eta_i = \frac{\Sigma AC}{A_1D} = \frac{\Sigma \Delta h_U}{A_1D} \qquad (14\cdot3)$$

From equation (14·2), $A_1D = \dfrac{\Sigma AB}{RF}$

Hence, assuming stage efficiency constant for all stages,

Internal turbine efficiency, $\eta_i = \text{stage } \eta \times R.F.$ (14·4)

Note. If the stage efficiency is not constant for all the stages average value may be used in the above equation.

It may be recalled that

(*i*) Rankine $\eta = \dfrac{\text{adiabatic heat drop}}{\text{heat supplied}} = \dfrac{h_{A1} - h_D}{h_{A1} - h_{fG}} = \dfrac{A_1D}{H_{A1} - h_{fD}}$ (14·5)

(*ii*) Overall thermal $\eta = \dfrac{\text{useful heat drop}}{\text{heat supplied}} = \dfrac{\Sigma AC}{H_{A1} - h_{fD}}$ (14·6)

(*iii*) By the usual definition of relative efficiency,

$$\text{Relative } \eta = \frac{\text{overall thermal } \eta}{\text{Rankine } \eta}$$

$$= \frac{\Sigma AC}{H_{A1} - h_{fD}} \times \frac{H_{A1} - h_{fD}}{A_1D} = \frac{\Sigma AC}{A_1D} \qquad [14\cdot7(a)]$$

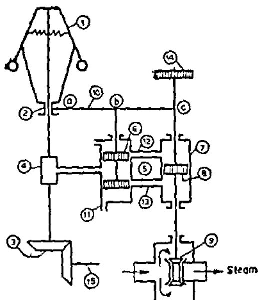
$$= \frac{\text{actual W.D. per kg}}{\text{Rankine W.D. per kg}} \qquad [14\cdot7\,(b)]$$

It may be noted that the relative efficiency is the same as internal efficiency. It is also sometimes known as the adiabatic efficiency or simply efficiency of the turbine.

14.4. Governing of Turbines. There are four methods of governing used in steam turbines.

1. Throttle governing.
2. Nozzle control governing.
3. By pass governing.
4. Combination of throttle and nozzle governing or throttle and by pass governing.

1. *Throttle Governing* In throttle governing at low speed the steam is throttled to reduce the mass flow. The throttling is done by



Index

1. Centrifugal speed governor.
2. Sleave.
3. Gearing.
4. Oil pump (3-4 kgf/cm²)
5. Central chamber.
6. Regulating valve.
7. Piston type servomotor.
8. Servomotor piston.
9. Throttle valve.
10. Differential lever.
11. Oil return passage.
12. Pipe to *close* valve
13. Pipe to *open* valve.
14. Hand wheel.
15. Turbine shaft.

Fig. 14·4. Throttle governing (indirect regulation by servomotor).

a double beat balanced valve operated by a servomotor as sho... Fig. 14·4. The servomotor is controlled by a centrifugal g... which is driven by a reduction gear fitted on the turbine shaft,

oil relay may be interconnected with the lubrication system to stop the turbine in case of abnormally low lubricating pressure). In case of throttle governing the steam consumption is given by Willan's line which is a straight line.

Throttle governing is mechanically simple but thermodynamically inefficient and is therefore generally restricted to small machines.

2. *Nozzle Control Governing.* In this method the nozzles are grouped together in 3 to 5 or more groups and each group of nozzles is supplied steam controlled by valves. Each valve may have its own relay cylinder or pilot valve or the valves may be opened by a single relay governor in a particular order by the help of a camshaft.

The nozzle control governing is restricted to the first stage of the turbine, the nozzle area in the other stages remaining constant. One arrangement of this type of governing is shown in Fig. 14·5.

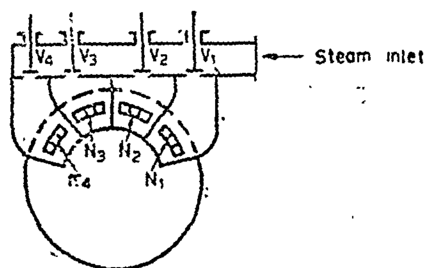


Fig. 14·5. Nozzle control governing.

Nozzle control governing is more efficient than throttle governing and is so preferred for large machines.

3. *By-pass Governing.* The high pressure impulse turbines generally comprise a number of stages of comparatively small mean diameter of wheel. These turbines are generally designed for maximum efficiency at an economic load which is about 80 per cent of the maximum continuous rating. Due to the small heat drop in first stage nozzle control governing cannot be efficiently used. Secondly, it is desirable to have full admission into high pressure stages at the rated economic load to eliminate the partial admission losses. Hence it is not possible to admit the extra steam required to generate the full

power through additional nozzles in the first stage. In such cases by-pass governing is employed, *see* Fig. 14·6. In this arrangement, for

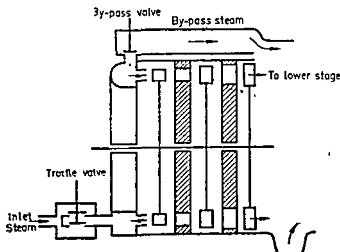


Fig. 14·6 By-pass governing

high loads a by-pass line is provided for the steam from the first stage nozzle box into a latter stage. The by-pass of steam is automatically regulated by lift of the valve. The by-pass valve is under the control of the speed governor for all loads within its range.

14·5. Emergency Governors Turbines are generally provided with emergency governors to protect it against overspeeding.

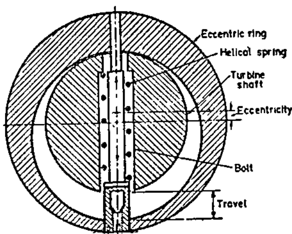


Fig 14·7. Eccentric bolt type emergency governor

These governors shut off the steam supply in the event the turbine speed exceeds 9 to 12 per cent the normal rated speed. Two types of emergency governors are commonly used. These are (*i*) eccentric bolt type (Fig. 14·7) and (*ii*) eccentric ring type (Fig. 14·8).

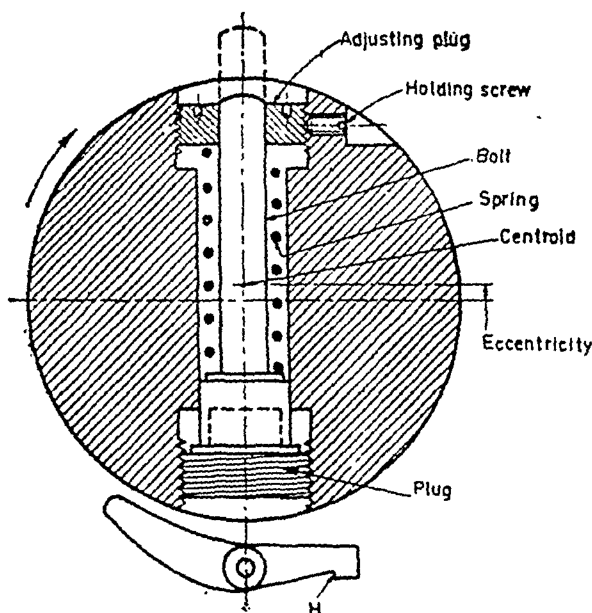


Fig. 14·8. Eccentric ring type emergency governor.

The operation of these may be either direct or through an oil relay (servomotor). The eccentric bolt mechanism is screwed into the high pressure end of the turbine spindle whereas the eccentric ring type mechanism is screwed on the end of the turbine spindle.

Emergency governors must be tested atleast once in a week.

14·6. Back Pressure Turbines. In applications where combined power and heat in steam for process work is required back pressure and pass-out turbines are used. In back pressure turbine the steam leaves the turbine at higher pressure than in normal turbine and is generally superheated. Superheated steam is not suitable for heating because the rate of heat transfer is lower than that of saturated steam and also because control of temperature is not very convenient. The exhaust is therefore passed in a desuperheater making the steam

saturated. The saturated steam is then passed through a heater where it is fully condensed. The diagrammatic arrangement of a back pressure installation is shown in Fig 14·9.

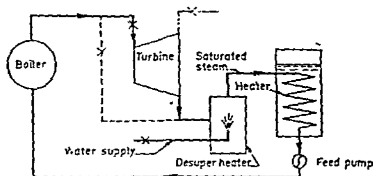


Fig. 14·9. Back pressure turbine.

14·7. Pass-out or Extraction Turbines. In combined power and process steam plant if the steam available from back-pressure turbine is more than required a pass-out or extraction turbine may be used. In such a turbine a part of the steam is continuously extracted at the pressure required for process work and the remainder

Fig. 14·10. Pass-out or extraction turbine.

steam passes into low pressure section of turbine through a pressure control valve (*see* Fig. 14·10). The low pressure section of turbine is provided with a control mechanism to keep the speed of the turbine and the pressure of the steam extracted constant irrespective of variation in power of steam for process and heating loads. The efficiency of a pass out turbine is low because it is usually small in size and has to operate under wide variations of load. The possible conditions of operation in pass-out turbine are full extraction, no extraction or partial extraction. In partial extraction either nozzle control or throttle control governing may be used.

14·8. Mixed-pressure Turbines. Mixed-pressure turbines are those which utilize high pressure steam from a boiler and also admit low pressure steam from the exhaust of a non-condensing engine or auxiliaries of the plant. Such turbines are used where an intermittent supply of exhaust steam is available but which is not sufficient to produce the power required. A mixed-pressure turbine consists virtually of two turbines in one cylinder (*see* Fig. 14·11). In these turbines governor control is provided to utilize the whole of the exhaust steam available, without allowing exhaust steam pressure

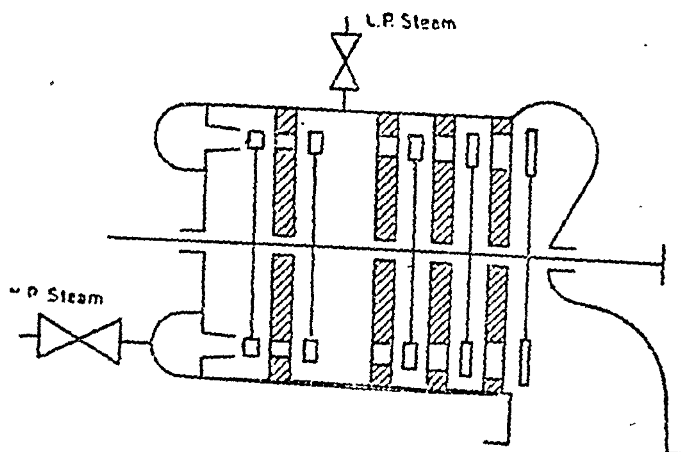


Fig. 14·11. Mixed pressure turbine.

to fall and to supply high pressure steam as and when required to keep the speed constant irrespective of load. Depending upon the

supply of high pressure and low pressure steam the turbine may work either by high pressure steam alone, by low pressure steam alone or partly by high pressure and partly by low pressure steam.

IMPORTANT POINTS

1. Sometimes the term overall efficiency (though not overall thermal efficiency) is used in the sense of internal efficiency. To know exactly which efficiency is meant it should be remembered that the value of internal efficiency lies between 60 to 80 per cent whereas that of overall efficiency (or overall thermal efficiency) lies between 20 to 35 per cent.

2. Whenever the exit velocity of steam is given the actual state of steam leaving is found by subtracting the K. E from the enthalpy.

3. Equal work in stages implies Δh_U is equal; Δh_T may be different due to different stage efficiencies.

4. The cumulative heat drop in equation (14 2) is the theoretical cumulative heat drop scaled from Mollier chart plus the losses, if any, in receivers, etc.

ILLUSTRATIVE EXAMPLES

14 1 Turbo-alternator plant · steam consumption; overall η, dryness fraction at exit, given steam velocity.

Steam is supplied to a 10,000 kw turbo-alternator at 50 kgf/cm² and 400°C. The power required by auxiliaries is 9 per cent of the alternator output. The exhaust pressure is 0 1 kgf/cm² and the condensate is cooled to 40°C Assuming the boiler efficiency as 0 82, relative efficiency of turbine 0·8 and alternator efficiency 0 95, calculate :

(a) the consumption of steam per min ,

(b) the overall efficiency of the plant ;

(c) the dryness of the steam at exit from the turbine, if its velocity is 200 m/s.

(a) From Mollier chart, $\Delta h_T = AB = 260$ kcal/kg

$\therefore \quad \Delta h_U = AC = 0{\cdot}8 \times 260 = 208$ kcal/kg

$$\text{Input to alternator} = \frac{10{,}000}{0{\cdot}95} \times \frac{856{\cdot}7}{3600} = 2510 \text{ kcal/s}$$

Let m kg be the steam consumption per min

$$\therefore \quad m = \frac{\text{Input to alternator}}{\Delta h_U} = \frac{2{,}510 \times 60}{208} = \underline{725 \text{ kg/min}}$$

steam passes into low pressure section of turbine through a pressure control valve (*see* Fig. 14·10). The low pressure section of turbine is provided with a control mechanism to keep the speed of the turbine and the pressure of the steam extracted constant irrespective of variation in power of steam for process and heating loads. The efficiency of a pass out turbine is low because it is usually small in size and has to operate under wide variations of load. The possible conditions of operation in pass-out turbine are full extraction, no extraction or partial extraction. In partial extraction either nozzle control or throttle control governing may be used.

14·8. Mixed-pressure Turbines. Mixed-pressure turbines are those which utilize high pressure steam from a boiler and also admit low pressure steam from the exhaust of a non-condensing engine or auxiliaries of the plant. Such turbines are used where an intermittent supply of exhaust steam is available but which is not sufficient to produce the power required. A mixed-pressure turbine consists virtually of two turbines in one cylinder (*see* Fig. 14·11). In these turbines governor control is provided to utilize the whole of the exhaust steam available, without allowing exhaust steam pressure

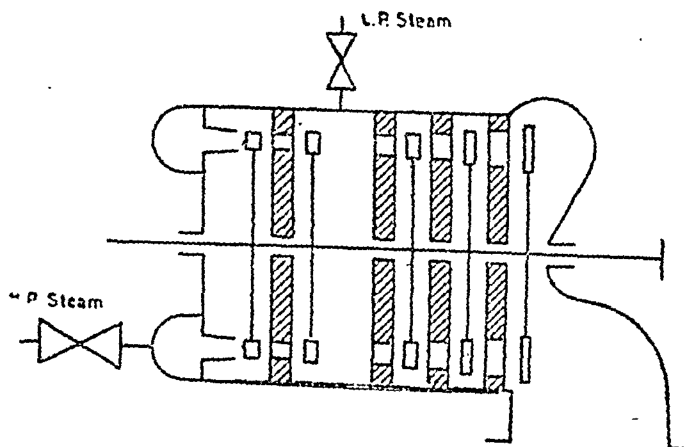


Fig. 14·11. Mixed pressure turbine.

to fall and to supply high pressure steam as and when required to keep the speed constant irrespective of load. Depending upon the

supply of high pressure and low pressure steam the turbine may work either by high pressure steam alone, by low pressure steam alone or partly by high pressure and partly by low pressure steam.

IMPORTANT POINTS

1. *Sometimes the term overall efficiency (though not overall* thermal efficiency) is used in the sense of internal efficiency. To know exactly which efficiency is meant it should be remembered that the value of internal efficiency lies between 60 to 80 per cent whereas that of overall efficiency (or overall thermal efficiency) lies between 20 to 35 per cent.

2. Whenever the exit velocity of steam is given the actual state *of steam leaving is found by subtracting the K. E from the enthalpy.*

3. Equal work in stages implies Δh_U is equal, Δh_T may be different due to different stage efficiencies.

4. The cumulative heat drop in equation (14 2) is the theoretical cumulative heat drop scaled from Mollier chart plus the losses, if any, in receivers, etc.

ILLUSTRATIVE EXAMPLES

14 1. Turbo-alternator plant : steam consumption ; overall η ; dryness fraction at exit, given steam velocity.

Steam is supplied to a 10,000 kw turbo-alternator at 50 kgf/cm² and 400°C. The power required by auxiliaries is 9 per cent of the alternator output. The exhaust pressure is 0 1 kgf/cm² and the condensate is cooled to 40°C. Assuming the boiler efficiency as 0 82, relative efficiency of turbine 0 8 and alternator efficiency 0·95, calculate :

(a) the consumption of steam per min ;

(b) the overall efficiency of the plant ,

(c) the dryness of the steam at exit from the turbine, if its velocity is 200 m/s.

(a) From Mollier chart, $\Delta h_T = AB = 260$ kcal/kg

$\therefore \quad \Delta h_U = AC = 0{\cdot}8 \times 260 = 208$ kcal/kg

Input to alternator $= \dfrac{10{,}000}{0{\cdot}95} \times \dfrac{856{\cdot}7}{3600} = 2510$ kcal/s

Let m kg be the steam consumption per min

$$\therefore \quad m = \frac{\text{Input to alternator}}{\Delta h_U} = \frac{2{,}510 \times 60}{208} = \underline{725 \text{ kg/min}} \qquad \text{Ans.}$$

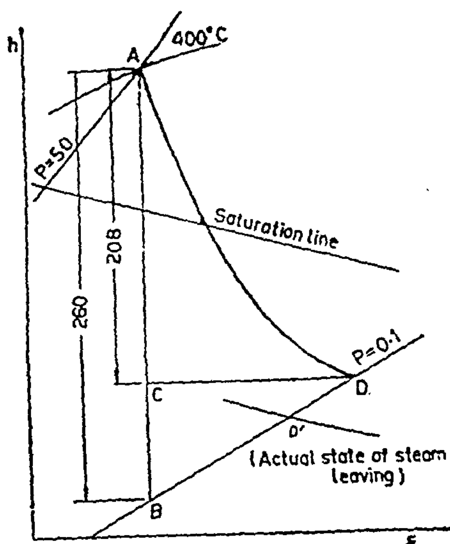


Fig. 14·12.

(b) Heat supplied to boiler $=725[762{\cdot}9-40]\times\dfrac{1}{0{\cdot}82}$

$=6\ 40\ 000$ kcal/min

Useful output $=(10\ 000\times0{\cdot}91)\times856{\cdot}7=1\ 30\ 500$ kcal/min

$\therefore$ Overall efficiency $=\dfrac{1\ 30\ 500}{6\ 40\ 000}=\underline{20{\cdot}4\%}$ Ans.

(c) The total energy at D includes the K.E.

$$\text{K.E.}=\frac{200^2}{2\times9{\cdot}81\times427}=4{\cdot}76 \text{ kcal/kg}$$

Neglecting the velocity of approach, this K.E. is produced at the expense of a reduction of enthalpy $h_D'=h_D-\text{K.E.}$

From Steam Tables,

Enthalpy at $D'=762{\cdot}9-208-4{\cdot}76=550{\cdot}1$ kcal/kg

$\therefore$ df at exit, $x_D'=\dfrac{550{\cdot}1-45{\cdot}4}{571{\cdot}6}=\underline{0{\cdot}883}$ Ans.

14·2. Pressure-compounded turbine : W.D. ; ηs : number of stages for equal work, R.F. given.

Explain what is meant by reheat factor in a steam turbine.

An impulse turbine has a number of pressure stages each having a row of nozzles and a single ring of blades. The nozzle angle in the first stage is 20 deg and the blade exit angle 30 deg, with reference to the plane of rotation ; the blade speed is 120 m/s, and the velocity of steam leaving the nozzles 300 m/s.

(a) Taking a velocity coefficient for the blade row of 0·8 and a nozzle efficiency of 85 per cent, determine the work done in the stage per kg of steam and the stage efficiency.

(b) If the steam supply to the first stage is at 20 kgf/cm² and 260°C and the condenser pressure is 0 07 kgf/cm², estimate the number of stages required assuming that the stage efficiency and the work done are the same for all stages and that the reheat factor is 1·06.

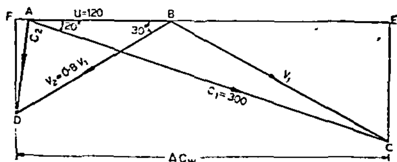


Fig. 14·13.

Given : $U=120$ m/s ; $C_1=300$ m/s , $\alpha_1=20°$; $\beta_2=30°$; $K=V_2/V_1=0\ 8$. The velocity diagram is drawn from given data.

From velocity diagram, $EF=295$ m/s

(a) W.D. in the stage $=\dfrac{EF\times AB}{g\times J}$

$$=\frac{295\times 120}{9\ 81\times 427}=\underline{3\ 610\ \text{kgf m/kg}} \qquad \text{Ans.}$$

(b) $$\Delta h_T=\frac{C_1^2}{2gJ\times\eta_1}=\frac{300\times 300}{2\times 9\ 81\times 427\times 0{\cdot}85}=5\ 400\ \text{kgf m/kg}$$

$\therefore$ Stage $\eta=\dfrac{\text{W.D.}}{\Delta h_T}=\dfrac{3\ 610}{5\ 400}=\underline{66{\cdot}7\%}$ Ans.

From Mollier chart, adiabatic heat drop $=207{\cdot}5$ kcal

$\therefore$ Cumulative heat drop $=207{\cdot}5\times 1{\cdot}06=220$ kcal. [R.F. $=1{\cdot}06$

As the stage η and W.D. are same Δh_T is same in each stage.

$\therefore$ No. of stage $=\dfrac{220\times 427}{5\ 400}=\underline{17\ 4 \text{ or } 18 \text{ (say)}}$ Ans.

14·3. Three stage turbine : Rankine η ; dryness fraction , η ratio ; Reheat factor given intermediate pressures.

Develop the relation between stage efficiency, internal efficiency and reheat factor for a multi-stage turbine.

Steam enters a three-stage turbine at 30 kgf/cm² and 320°C and leaves finally at 0·07 kgf/cm². The steam leaves the first stage at 7 kgf/cm² and the second stage 1 kgf/cm². The stage efficiency of all three stages are 75 per cent.

Show the expansion on a sketch of a total heat-entropy chart and determine :—

(*a*) *the Rankine efficiency ;*

(*b*) *the steam condition at the end of each stage ;*

(*c*) *the work done in kcal/kg of steam in each stage,*

(*d*) *the efficiency ratio ;*

(*e*) *the reheat factor.*

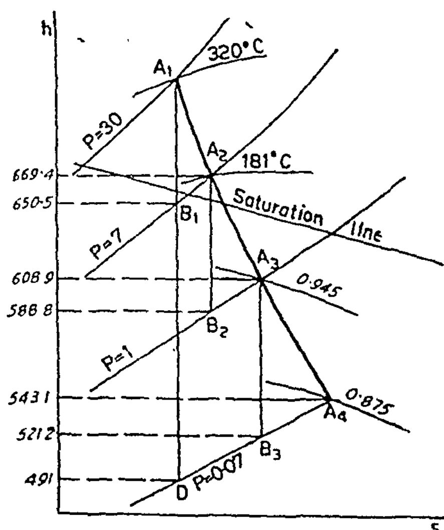


Fig. 14·14.

(*a*) From Mollier chart

Rankine drop, $h_{A1}-h_D=235$ kcal

Rankine $\eta=\frac{h_{A1}-h_D}{h_{A1}-h_{fd}}=\frac{235}{726-38\cdot7}=\underline{34\cdot2\%}$ Ans.

(*b*) and (*c*). Knowing intermediate pressures and stage η the diagram is plotted on Mollier chart and the following readings are obtained :

Pressure P kgf/cm²	Temp. or df	Δh_T of stage, kcal	Δh_U of stage, kcal	
30	320°C			
7	181°C	75·6	56·7	Ans.
1	0·945	80 6	60 5	
0·07	0·875	87·7	65 8	
		$\Sigma\Delta h_T$=243 9	$\Sigma\Delta h_U$=183	

The condition of steam and W.D. is obtained in the above table.

(d) Efficiency ratio $=\frac{\Sigma\Delta h_U}{A_D}=\frac{183}{235}=77{\cdot}9\%$ Ans.

(e) Reheat factor $=\frac{\Sigma\Delta h_T}{AD}=\frac{243{\cdot}9}{235}=1{\cdot}037$ Ans.

14 4. 5-stage turbine : reheat factor ; steam condition at exit from stages, given equal work and η in stages.

A five stage turbine receives steam at 15 kgf/cm² and 260°C and exhausts at 0·07 kgf/cm². Each stage has an efficiency ratio of 75 per cent and all do equal work. Find the reheat factor and determine the steam conditions at exit from each stage.

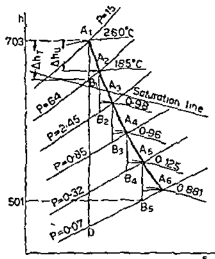


Fig. 14 15.

From Mollier chart, Rankine heat drop = 202 kcal

As all stages do equal work, Δh_U for each stage is same, as stage efficiencies are equal Δh_T is also same for each stage.

Now $\Sigma \Delta h_T = \text{R.F.} \times 202$

The problem can only be solved by trial and error from Mollier chart. Assuming a usual value of R.F., say 1·05

$$\Sigma \Delta h_T = 1{\cdot}05 \times 202 = 212 \text{ kcal}$$

$\therefore$ Δh_T per stage $= \frac{212}{5} = 42{\cdot}4$ kcal

Plotting this on Mollier chart we find that the last point goes slightly below 0·07 kgf/cm² pressure line which means that the value of R.F. assumed is slightly on higher side. By trial we see that 210 kcal satisfies the condition more correctly.

$\therefore$ $\Sigma \Delta h_T = 210$ kcal

$\therefore$ $\underline{\text{R.F.}} = \frac{210}{202} = \underline{1{\cdot}04}$ Ans.

From Mollier chart, the steam condition at exit from each stage is

Pr. in kgf/cm²	6·4	2·45	0·85	0·32	0·07	
Condition of steam (Temp. or df)	185°C	0·983	0·96	0·913	0·881	Ans.

14·5. Three-stage turbine : Δh_T for given ratio of work.

In an impulse turbine with three pressure stages, the initial steam conditions are, pressure, 20 kgf/cm² and temperature 300°C. The condenser vacuum is 66 cm (barometer, 73·36 cm). Calculate the heat drop required in each stage for developing power in the stages in the ratio H.P. ; I.P. ; L.P. : : 1 : 1 ; 2. Take stage efficiencies of H.P., I.P. and L.P. as 0·78, 0·75 and 0·71 respectively. Assume a reheat factor of 1·06 and a loss of available heat in receiver pipe, etc., of 5·5 kcal per kg.

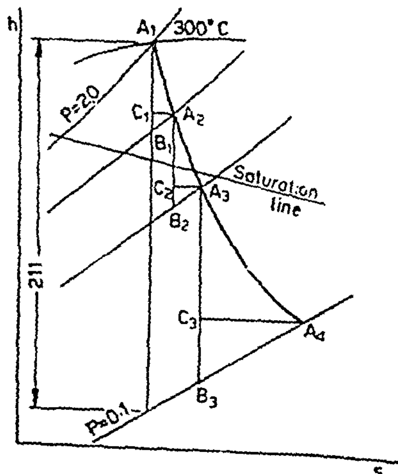


Fig. 14·16.

Absolute pressure in condenser $= \dfrac{(73\cdot36-66)\times1}{735\cdot6} = 0\cdot1$ kgf/cm²

From Mollier chart, Adiabatic heat drop = 211 kcal/kg

Reheat factor $= 1\cdot06 = (\Sigma AB + 5\cdot5/211)$ $\quad \therefore \Sigma AB = 218\cdot2$ kcal

Given : $A_1C_1 = A_2C_2 = A_3C_3/2$

$\therefore \dfrac{A_1C_1}{0\cdot78} + \dfrac{A_1C_1}{0\cdot75} + \dfrac{2A_1C_1}{0\cdot71} = 218\cdot2 \qquad \therefore A_1C_1 = 40\cdot18$ kcal

Hence, in kcal/kg

	Δh_U	η_S	Δh_T	
H.P. stage	$A_1C_1 = 40\cdot18$	0·78	$A_1B_1 = 51\cdot51$	Ans.
I. P. stage	$A_2C_2 = 40\cdot18$	0·75	$A_2B_2 = 53\cdot57$	
L.P. stage	$A_3C_3 = 80\cdot36$	0·71	$A_3B_3 = 113\cdot12$	
	$\Sigma AC = 160\cdot72$		$\Sigma AB = 218\cdot2$	

Note. Cumulative heat drop is reduced by the amount of heat lost in receiver, etc.

14 6. Five stage turbine : steam condition at entry to stages ; η_S ; R.F. ; overall η_t.

Steam is supplied to a five-stage turbine at 35 kgf/cm² and 450°C and exhausts at 0·07 kgf/cm², 0 89 dry. Calculate for equal work between the stages, (a) the condition at entry to each stage ; (b) the stage efficiencies ; (c) the reheat factor ; and (d) the overall turbine efficiency ; Assume the condition line to be straight.

(a) Knowing initial and final condition, condition line $A_1 A_6$ is drawn on mollier chart and useful W.D., ΣAC_1, is measured equal to 246 kcal.

Since the work is shared equally between the stages, useful work per stage, $\Delta h_T = \dfrac{246}{5} = 49\cdot2$ kcal.

The diagram is now completed and the following results are obtained :—

Pressure kgf/cm²	35	14·4	5·3	1·6	0·355	0 07	Ans.
Temperature or df	450	342	235	123	0·948	0·89	

$A_1B_1 = 63$ kcal ; $A_2B_2 = 58\cdot8$ kcal ; $A_3B_3 = 57\cdot6$ kcal ;
$A_4B_4 = 57\cdot4$ kcal ; $A_5B_5 = 56\cdot2$ kcal

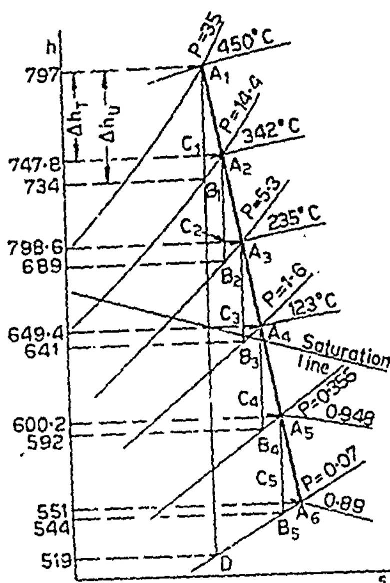


Fig. 14·17.

Rankine heat drop, $AD=278$ kcal

(b) $\text{Stage } \eta = \dfrac{\text{useful heat drop}}{\text{Rankine heat drop}}$

∴ The efficiencies of the stage are

$$\frac{49\cdot2}{63}, \frac{49\cdot2}{58\cdot8}, \frac{49\cdot2}{57\cdot6}, \frac{49\cdot2}{57\cdot4}, \frac{49\cdot2}{56\cdot2}$$

78·2%, 83·7%, 85·4%, 85·9%, 87·5% Ans.

(c) Reheat factor $= \dfrac{\Sigma AB}{AD} = \dfrac{63+58\cdot8+57\cdot6+57\cdot4+56\cdot2}{278}$

$=1\cdot052$ Ans.

(d) Overall $\eta = \dfrac{\text{useful heat drop}}{\text{adiabatic heat drop}} = \dfrac{246}{278} = 88\cdot5\%$ Ans.

Note. Here the term overall efficiency has been used in the sense of internal efficiency.

14·7. Back-pressure turbine ; throttle governing.

Steam is supplied to a back-pressure turbine at 15 kgf/cm² and 250°C. It is expanded to 1·4 kgf/cm² with an internal efficiency ratio of 0·70. Calculate the total amount of heat available if the brake power is 980 kW, and the mechanical losses amounts to 20 kW.

Calculate the brake power and heat available when the quantity of steam is reduced to one half of that at 980 kW brake power. Assume

that the turbine is governed by throttling, and the internal efficiency ratio, exhaust pressure, mechanical losses remain same as before.

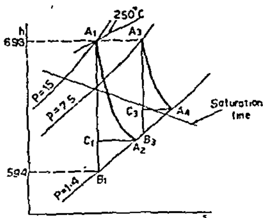


Fig. 14·18

From steam table at 15 kgf/cm² and 250°C

$h_1 = 698$ kcal

From Mollier chart, $h_{A1} - h_{B1} = 104$ kcal

Useful work per kg of steam $= 104 \times 0.7 = 72{\cdot}8$ kcal

Internal power developed by the turbine

$=$ coupling power $+$ mechanical losses

$= 980 + 20 = 1000$ kW

$\therefore$ Steam flow through turbine $= \dfrac{WD}{\text{work/kg of steam}}$

$= \dfrac{856.7 \times 1000}{72{\cdot}8} = \underline{11780 \text{ kg/hr}}$ Ans.

Heat available $=$ mass flow $\times (h_{A1} - h_{fA2})$

$= 11780[(698 - 72{\cdot}8) - 108.9] = \underline{6.077 \times 10^6 \text{ kcal/hr}}$ Ans.

After throttle governing.

Steam consumption $= \dfrac{11780}{2} = 5890$ kg/hr

Now from the proportionality of steam consumption and nozzle box pressure, the nozzle box pressure

$$P_{A3} = \frac{15}{2} = 7.5 \text{ kgf/cm}^2$$

From Mollier chart, $h_{A3} - h_{B3} = 67$ kcal

Useful work per kg of steam $= 67 \times 0{\cdot}7 = 46{\cdot}9$ kcal

$$\text{Power developed} = \frac{5890 \times 46{\cdot}9}{856{\cdot}7} = 322 \text{ kW}$$

Coupling power = power developed − mechanical losses

$= 322 - 20 = 302$ kW **Ans.**

Heat available $= 5890[(698 - 46{\cdot}9) - 108{\cdot}9]$

$= 3{\cdot}194 \times 10^6$ kcal/hr **Ans.**

14.8. Pass-out turbine : steam/hr ; process heat.

The following results were obtained during a test of pass-out steam turbine :—

Steam pressure at turbine inlet	*...30 kgf/cm²*
Steam temperature at turbine inlet	*...400°C*
Steam pressure before first stage nozzles	*...24 kgf/cm²*
Pressure at which steam is extracted for heating	*...5 kgf/cm²*
Pressure before L.P. first stage nozzles	*...3·5 kgf/cm²*
Exhaust pressure	*...0·07 kgf/cm²*
Temperature of condensed process steam	*...100°C*
Electrical power developed	*...7800 kW*
Discharge from surface condenser per hour	*...30 000 kg*

Calculate the steam consumption of the turbine and the total amount of heat available, if the efficiency ratios of the H.P. and L.P. groups of stages are each 80 per cent, the alternator efficiency is 96 per cent, and the turbine mechanical losses amount to 60 kW.

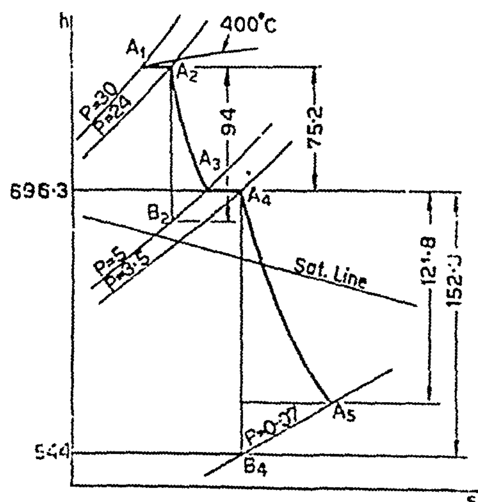


Fig. 14·19.

Plot the cycle on Mollier chart. There is initial throttling at constant total heat from 3 kgf/cm² to 24 kgf/cm². Again there is throttl. ing between H.P. and L.P. stage from 4 kgf/cm² to 3 5 kgf/cm².

From Mollier chart, $h_{A_2}-h_{B_2}=94$ kcal [In H.P. stage

∴ Useful heat drop, $h_{A_2}-h_{A_3}=0\ 8\times 94=75{\cdot}2$ kcal

Form Mollier chart, $h_{A_4}-h_{B_4}=152{\cdot}3$ kcal [In L.P. stage

∴ Useful heat drop, $h_{A_4}-h_{A_5}=0\ 8\times 152{\cdot}3=121{\cdot}8$ kcal.

Let the steam consumption be m kg per hour. The total mass m is flowing through H.P. stage, but steam flowing through L.P. stage is less by the amount extracted for heating.

$$\therefore \quad \frac{7\ 860}{0\ 96}=\frac{m\times 75{\cdot}2+30\ 000\times 121{\cdot}8}{856{\cdot}7}$$

$m=44\ 800$ kg/hr Ans.

In the heater the steam enters with enthalpy 69 3 kcal and leaves with 100 kcal.

∴ Amount of heat available.

$=(44\ 800-30\ 000)\times[696{\cdot}3-100]$

$=8\ 240\ 000$ kcal/hr Ans.

14 9. Mixed stage-pressure turbine: dryness fraction before entrance of L.P. stage ; L.P. stages η_s ; total power.

The low pressure unit of a turbine receives 20 kg of steam per second at 2·5 kgf/cm² and 0 95 dry. The steam expands to 1 kgf/cm² in the first two stages of the unit with an efficiency ratio of 0 7. The exhaust steam from the auxiliaries amounting to 5 kg per second at 1·5 kgf/cm² and 0 93 dry, is passed into the other stages through a throttle valve, and the mixed steam then expands in the remaining stages of the turbine leaving finally at a pressure of 0·07 kgf/cm² with a dryness fraction of 0 87 and a speed of 200 m/s.

Determine (a) the steam condition at the entrance of the latter stages, (b) the efficiency ratio of the latter stages, and (c) the total power of the L.P. unit.

The diagram may be plotted on Mollier chart for first two stages.

(a) From Mollier chart, $h_{A_1}-h_{B_1}=36{\cdot}5$ kcal

Useful heat drop $=0{\cdot}7\times 36{\cdot}5=25{\cdot}6$ kcal

$h_{A_2}=h_{A_1}-25{\cdot}6=623{\cdot}6-25{\cdot}6=598$ kcal

From Steam Tables, heat per kg of steam from auxiliaries at 1·5 kgf/cm² and 0·93 dry, $h_{A3}=111+0{\cdot}93\times532{\cdot}1=605{\cdot}8$ kcal

This is throttled to 1 kgf/cm², $h_{A4}=h_{A3}=605{\cdot}8$ kcal

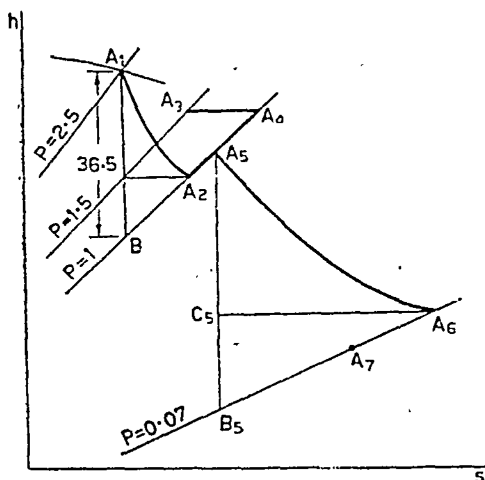


Fig. 14·20.

The enthalpy per kg of this steam is greater than enthalpy per kg of steam from turbine stage. Point A_5 is the final condition, after mixing 5 kg of steam from auxiliaries and 20 kg of steam from turbine stage.

Enthalpy before mixing = Enthalpy after mixing

$$5\times605{\cdot}8+20\times598=25\times h_{A5}$$

∴ Enthalpy per kg at A_5, $h_{A5}=600$ kcal/kg

Steam condition at entrance to latter stages, at 1 kgf/cm²

$$x_A=\frac{600-99{\cdot}2}{539{\cdot}5}=0{\cdot}927 \qquad \text{Ans.}$$

The diagram may be plotted on Mollier chart for latter stages.

From Mollier chart, $h_{A5}-h_{B5}=87$ kcal

$$h_{A7}=38{\cdot}7+0{\cdot}87\times575{\cdot}4=539{\cdot}3 \text{ kcal}$$

$$\text{K.E. at outlet}=\frac{200^2}{2\times9{\cdot}81\times427}=4{\cdot}77 \text{ kcal}$$

$$h_{A6}=h_{A7}+\text{K.E.}=539{\cdot}3+4{\cdot}77=544{\cdot}1 \text{ kcal}$$

Useful heat drop $=h_{A5}-h_{A6}=600-544{\cdot}1=55{\cdot}9$ kcal

$$\therefore \quad \eta \text{ ratio or stage } \eta=\frac{h_{A5}-h_{A6}}{h_{A5}-h_{B5}}=\frac{55{\cdot}9}{87}=64{\cdot}3\% \qquad \text{Ans.}$$

(c) Total power$=\dfrac{(20\times36\cdot5+25\times55\cdot9)\times427}{75}=$ 12 100 hp

Ans.

14 10. Mixed-pressure turbine ; Willan's line ; H.P. steam from given power and L.P. steam.

A mixed pressure turbine of output 3 000 hp receives high pressure steam at 15 kgf/cm² and 260°C and low pressure steam at 1·5 kgf/cm² dry saturated. The exhaust pressure is 0 035 kgf/cm². The efficiency ratios of both H.P. and L.P. stages is 0·76. Willan's line for both supplies are straight and the consumption at no load is 11·4 per cent of that on full load.

If 7200 kg per hour of L.P. steam is available, find the amount of H.P. steam required for developing 2200 horse-power.

H P. Turbine alone

From Mollier chart $\Delta h_T=AB=221$ kcal/kg

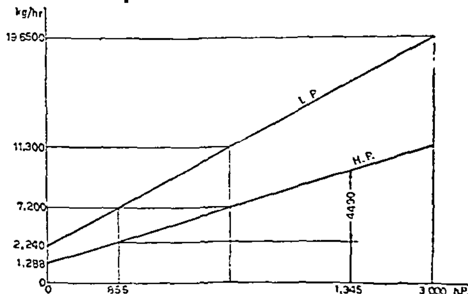


Fig. 14 21.

$\therefore\ \Delta h_U=0\cdot76\times221=168$ kcal/kg

$\therefore$ Steam consumption on full load$=\dfrac{3\ 000\times75\times3\ 600}{427\times168}$

$=11\ 300$ kcal/hr

Steam consumption on no load$=0\cdot114\times11\ 300=1\ 288$ kg/hr

L.P. Turbine alone

From Mollier chart, $\Delta h_T = 127$ kcal/kg

$\therefore \quad \Delta h_U = 0{\cdot}76 \times 127 = 96{\cdot}5$ kcal/kg

$\therefore$ Steam consumption on full load $= \dfrac{3\ 000 \times 75 \times 3\ 600}{427 \times 96{\cdot}5}$

$= 19\ 650$ kg/hr

Steam consumption on no load $= 0{\cdot}114 \times 19\ 650 = 2\ 240$ kg/hr

The Willan's line may now be plotted for both H.P. and L.P. supplies. From L.P. Willan's line, power developed by 7 200 kg per hour of L.P. steam is 855 hp

$\therefore$ Power to be developed by H.P. steam $= 2\ 200 - 855 = 1\ 345$ hp

As the two turbines are in one cylinder and on the same shaft the friction losses, etc., are overcome by L.P. steam.

$\therefore$ From H.P. Willan's line ;

<u>H.P. steam required</u> $=$ <u>4 490 kg/hr</u> Ans.

The problem can also be solved by calculations as indicated below

L.P. Turbine : Let the equation for Willan's line be $y = mx + c$

Putting the values for no load ; $2\ 240 = m \times o + c$ $\quad \therefore \ c = 2\ 240$

For full load, $19\ 650 = m \times 3\ 000 + 2\ 240$ $\quad \therefore \ m = 5{\cdot}803$

When steam consumption is, 7200 kg

$7\ 200 = 5803x + 2\ 240 \quad \therefore \ x = 855$

Power to be supplied by H.P. turbine $= 2\ 200 - 355 = 1\ 845$ hp

As the friction losses have been included by L.P. steam the steam required for no load should not be considered.

$\therefore$ <u>H.P. steam required</u> $= \dfrac{11\ 300 - 1\ 288}{3{,}000} \times 1\ 345$

$=$ <u>4 490 kg/hr</u> Ans.

14·11. Efficiency of generation of steam by turbo-compressor and boiler.

In a factory equal power is generated in two turbines one working on 8 kgf/cm², another on 40 kgf/cm². There is a single boiler generating steam at 8 kgf/cm² dry saturated. Part of this steam is supplied to L.P. turbine and part to a turbo-compressor coupled to L.P. turbine which raises the steam pressure to 40 kgf/cm². The steam from turbo-compressor is supplied to H.P. turbine. The back pressure of both

Let 1 kg of steam be used in H.P. turbine (which produces 228·7 kcal of useful work). Steam used in H.P. turbine is supplied by turbo-compressor.

∴ Steam supply to turbo-compressor = 1 kg

Steam supply to L.P. turbine

(i) For producing external power equal to H.P. turbine

$$=\frac{228\cdot7}{142}=1\cdot61 \text{ kg}$$

(ii) For driving turbo-compressor $=\dfrac{111\cdot9}{142}=0\cdot787$ kg

Total supply to L.P. Turbine = 2·397 kg

∴ Total steam supply from boiler = 3·397 kg

Useful heat required in boiler for producing 3·397 kg of dry saturated steam at 8 kgf/cm² = 3·397 (661·2 − 38·7) = 2115 kcal

If H.P. steam is separately generated in a H.P. boiler at the same superheat as in the turbo-compressor, useful heat required in H.P. boiler for generating 1 kg of steam at 40 kgf/cm²

$$=773-38\cdot7=734\cdot3 \text{ kcal}$$

Steam required in L.P. turbine (*i.e.* generated in L.P. boiler) is for producing power equal to hp turbine and useful heat required in L.P. boiler for generating 1·61 kg of steam at 8 kgf/cm²

$$=1\cdot61(661\cdot2-38\cdot7)=1\ 002 \text{ kcal}$$

∴ Total heat required = 734·3 + 1002 = 1736·3 kcal

$$\therefore \underline{\text{Relative } \eta=\frac{1{,}736\cdot3}{2{,}115}=82\cdot1\%} \qquad \text{Ans.}$$

EXAMPLES 14

14·1. Turbo-generator plant : steam consumption : dryness fraction at exit ; heat balance.

A turbo-generator is supplied with steam at 50 kgf/cm² and temperature 450°C and exhausts to a vacuum of 69 cm of mercury when the barometer reads 76 cm. The turbine develops 54 000 kW, of which 4 000 kW is expended in driving the auxiliaries. The condensate temperature is 40°C. Assuming the boiler efficiency 84 per cent, turbine efficiency 83 per cent and generator efficiency 96 per cent, determine :

(a) *the consumption of steam in kg per hour ;*

(b) *the dryness fraction of steam leaving the turbine if its speed is 200 m /second. Draw up a heat balance sheet on the basis of heat supplied to the boiler per kg of steam.*

[Δh_T=278·5 kcal ∴ Δh_U=231 kcal m=2 09 000 kg/hr K.E.=4·77 kcal ; d.f.=0 895 ; heat balance ; heat suppplied/kg of steam=895 kcal ; loss in boiler=143 2 kcal (16%) net output =205·53 kcal (22·95%), auxiliaries=16 42 kcal (1·838%), generator =9 05 kcal (1·012%), condenser 521 kcal (58·3%)].

14 2. Reaction turbine : power developed, given R.F. ; height of blades.

A low pressure steam turbine is supplied with steam at 3·5 kgf/cm² 0 93 dry and exhausts at 0·13 kgf/cm². Steam consumption is 15 kg/s and speed 480 rev/min. Calculate the power developed, assuming th a 20 per cent of the adiabatic heat drop is lost in friction in each stage. Reheat factor 1·05.

Taking the blade velocity as 0·7 times the relative velocity of the discharge steam, the blade height 1/12 of the mean diameter of the row of blades, and the blade angles at discharge as 20°, find the height of blades at a point in the expansion, where the pressure is 1·2 kgf/cm². Neglect the effects of friction and reheating.

[Δh_T=112 kcal ; Δh_U=94·2 kcal ; hp=8 060 ; symmetrical diag. ; $mv_s=\pi dhC_a$ ∴ d=56 cm ; h=4·67 cm]

14 3 Compound turbine ; internal Δh ; overall η : R.F.

The following data refer to a compound steam turbine. Inlet pressure and temperature of the H.P. section 15 kgf/cm² and 300°C, pressure at entrance to the L.P. section 1·4 kgf/cm², average stage efficiency 0 77 and exhaust pressure 0 04 kgf/cm². Find, with the aid of the steam tables (a) the internal heat drop in kcal per kg (b) the overall efficiency ratio of the turbine if the external losses amount to 4 per cent of the isentropic heat drop between the turbine inlet and the condenser inlet, and (c) the reheat factor.

[$h_{A_1}-h_{B_1}$=115·2 kcal ; $h_{A_2}-h_{B_2}$=111·7 kcal ; internal heat drop=226·9 kcal ; isentropic heat drop=221·4 kcal ; net W.D.=165·84 kcal ; overall η=25% ; R.F.=1·025].

14·4. Three-stage turbine, throttling at inlet, stage pressure given : d.f. at exit ; R.F. : nozzle throat area.

Steam dry and saturated at absolute temperature T_1 expands in a turbine to an absolute temperature T_2 with a stage efficiency of η_s. Assuming infinite number of stages and that the condition line to be straight, show that the reheat factor is given by

$$R = \frac{T_1 + T_2}{2T_2 + \eta_s(T_1 - T_2)}\ .$$

The steam supplied to a three-stage impulse machine is at 10 kgf/cm² and 250°C. There is a drop of pressure across the throttle valve of 2 kgf/cm² and each stage has an efficiency ratio of 0·66. The pressure in the three wheel casings are 2, 0·45 and 0·07 kgf/cm² respectively. Find,

(a) the dryness of the exhaust steam ; (b) the reheat factor based on conditions after the throttle valve, (c) the required area of the throats of the third-stage nozzle to pass 1 kg/s of steam.

[$\Sigma\Delta h_T = 65{\cdot}4 + (59{\cdot}2 + 63{\cdot}1)$ kcal ; Rankine drop $= 181{\cdot}4$ kcal ; $x_e = 0{\cdot}94$; R.F. $= 1{\cdot}032$; $P_T = 0{\cdot}261$ kgf/cm² ; $\Delta h_U = 12{\cdot}6$ kcal $\therefore$ $C = 325{\cdot}5$ m/s ; $v = 5{\cdot}9$ m³ ; $A = 181{\cdot}2$ cm²]

14·5. Three-stage turbine, equal ideal expansion : R.F. ; internal efficiency ratio ; hp.

A three-stage steam is supplied with steam at a pressure of 24 kgf/cm² and temperature 380°C. The exhaust pressure is 0·07 kgf/cm² and the stage pressures are such as to divide the ideal expasion line between the initial and final pressures into three equal parts. If the mean stage efficiency be 75 per cent, find the reheat factor and the internal efficiency ratio of the turbine. Find also the power developed when the rate of steam flow is 6 kg per second.

[Rankine drop $= 246$ kcal ; $\Sigma AB = (82 + 87 + 88{\cdot}3)$ kcal R.F. $= 1{\cdot}045$; $\Sigma AC = (61{\cdot}5 + 65{\cdot}3 + 66{\cdot}2)$ kcal; $\eta = 78{\cdot}5\%$; h.p. $= 6{,}600$]

14·6. Two-stage turbine : stage pressures given, equal power in stages : R.F. ; steam flow.

Prove that the steam flow through a multi-stage turbine is

$$m = k\sqrt{(P_1/v_1 - P_n/v_n)}$$

where P_1 and P_n are the steam pressures before the first stage nozzles and at exhaust, respectively and v_1 and v_n are the corresponding specific volumes.

The following data refer to a two-stage impulse steam turbine. Initial pressure, 10 kgf/cm² : initial temperature, 200°C ; pressure in first and second stage chambers 1·5 and 0·16 kgf/cm² ; first-stage efficiency ratio, 0·68. If each stage develops the same actual power, find with the aid of the steam tables, (a) the reheat factor, and (b) the rate of steam flow in kg per second when the horse-power developed is 1000.

What is the second stage efficiency ratio and the dryness fraction of steam at exit if it leaves with a velocity of 150 m/sec. ?

[Rankine drop=156·5 kcal ; ΣAB=(80·4+85·8) kcal R.F.=1·061 ; m=1·605 kg/per ; second stage η=63·8% ; K.E.=2·68 kcal ; d.f.$_{exit}$=0·898].

14·7. Compound turbine, mixing of steam with steam : d.f. ; h.p. ; input to H.P. boiler.

A steam turbine is built in sections : A, the high pressure section and B, the low. Section A expands 5000 kg/hr of steam from 20 kgf/cm² and 350°C to 1·8 kgf/cm² with an efficiency ratio of 0·8. The exhaust of A enters a receiver into which are also passed 2500 kg/hr of steam from another source. This steam is at 1·8 kgf/cm², 0·8 dry. The mixed steam expands in section B to the final exhaust pressure of 0·07 kgf/cm² with an efficiency ratio of 0·8. Find (a) the quality of mixed steam entering section B ; (b) the horse-power output of the turbine neglecting mechanical losses ; (c) the heat to be supplied per hour in the high pressure boiler supplying turbine section A.

[Δh_{U_1}=98·6 kcal ; df after mixing=0·941 ; Δh_{U_2}=55·1 kcal ; hp=1435 ; heat in boiler=3 550 500 kcal/hr].

14·8. Four stage turbine ; initial pressure and quality of steam at each stage ; η_i ; R.F. by trial

The initial steam conditions for a velocity-compounded turbine with four pressure stages are 35 kgf/cm² and 420°C while the pressure at the exhaust flange is 0·07 kgf/cm². Find, by means of a Mollier chart, the probable values of each initial stage pressure and steam quality, and the internal efficiency of the turbine if the average efficiency is 0·7.

[By trial, R.F.=1·066 ; Rankine heat drop=269·5 kcal , Δh_U/stage=50·28 kcal ; stage pr. and temp. : 11·3 kgf/cm², 301°C ; 2·9 kgf/cm², 189°C ; 0·55 kgf/cm², 0·991 ; 0·07 kgf/cm², 0·935 , Σh_U=201·1 kcal ∴ η_i=74·6%].

14·9 Pass-out turbine : nozzle control governing ; steam/hr.

The following particulars apply to pass-out turbine producing 1000 kw and which has a heat load varying from zero to 5 million kcal/hr : initial steam pressure = 20 kgf/cm² ; initial steam temperature = 350°C ; pass-out pressure = 3 kgf/cm² ; exhaust pressure = 0·07 kgf/cm².

If the efficiency ratio of H.P. and L.P. turbine is 0·7 under all conditions of operation, estimate maximum and minimum hourly steam consumption. Assume that heating steam gives up heat by condensing only, and the turbine is provided with nozzle control governing, with no pressure drop at the control valves.

[Δh_{u1} = 70·7 kcal/kg Δh_{u2} = 98 kcal min steam = 5150 kg/hr ; mass for heating = 9 190 kg/hr ; max. steam = 10 400 kg/hr].

14·10. Pass-out turbine : steam supply and hp of high pressure stage.

Steam is supplied at 15 kgf/cm² and superheated to 250°C to the high pressure stage of an extraction turbine which develops 2000 hp. In the high pressure steam expands to 1·8 kgf/cm² when it is extracted at the rate of 10 800 kg per hr. Before being admitted to the low pressure section the balance steam is throttled to 0·7 kgf/cm². It is then expanded in low pressure stage to 0·15 kgf/cm².

If each section of turbine is assumed to convert into useful work 80 per cent of adiabatic drop, how much steam is being supplied to the high pressure stage and what power does this stage develop.

[$\Delta h U_1$ = 76·2 kcal ; $\Delta h U_2$ = 41·36 kcal, m = 14 540 kg/hr ; power of H.P. stage = 1 750 hp].

14·11. Mixed-pressure turbine ; total output ; L.P. steam required ; Willans line straight.

A mixed pressure turbine receives 4 300 kg of steam per hour, the pressure and temperature of the steam being 15 kgf/cm² and 250°C respectively. The L.P. portion receives 5000 kg dry saturated

steam per hour at 1·3 kgf/cm². The exhaust pressure is 0·03 kgf/cm². If the efficiency ratios of both high pressure and low pressure sections of the turbine are 0·7, whether H.P. or L.P. steam alone is supplied, calculate

(a) the output at the turbine coupling ;

(b) the amount of L.P. steam required when the turbine is developing 950 kW and 3600 kg per hour of H.P. steam are being used. Assume in both cases Willans lines are straight and the consumption on no load is 10 per cent of that on full load.

[H.P. section : $\Delta h_U = 146{\cdot}6$ kcal ; kW=736 , L.P. section; $\Delta h_U = 79$ kcal ; kW=61 ; total output=1197 kW , power by H.P. steam=601 kW ; no load losses provided by H.P. steam ; L.P steam for 346 kW =3 375 kg hr].

15

Improvements in Turbine Plants

15·1. Ideal Regenerative Cycle. The efficiency of Rankine cycle is less than Carnot cycle because in the former all the heat is not added at the highest temperature, as in the latter. Heat added during the heating of feed water from condition 3 to 4 is at lower temperature. If by some means the feed water temperature is raised to boiler temperature by *reversible interchange* of heat before it enters the boiler, the cycle will be as efficient as Carnot cycle.

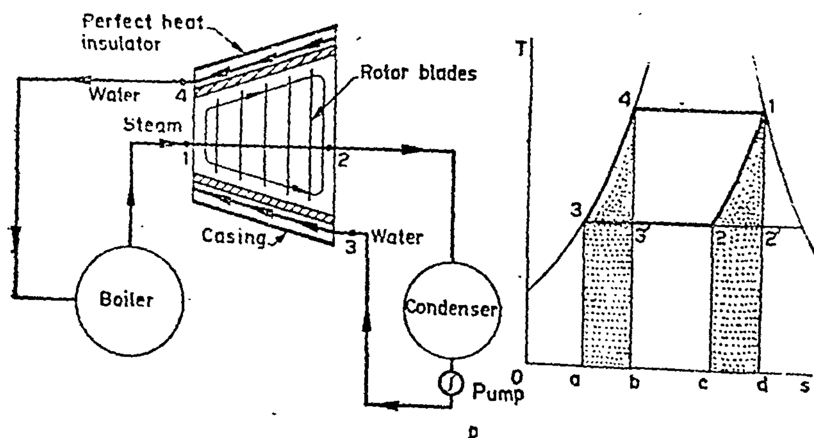


Fig. 15·1. Ideal regenerative cycle.

The ideal arrangement to achieve this is represented diagrametically and on *T-s* diagram in Fig. 15·1. The steam enters the turbine, dry saturated, at temperature T_1 and expands adiabatically to temperature T_2. The condensate from the condenser is pumped back through an annular space in the turbine casing and the feed water is heated by the steam in a reversible manner, the temperature of steam and feed water being same at any section. Such heating is known as *regenerative heating* as steam is used to heat itself. The water enters

the boiler in a saturated condition at 4. The heat gained by feed water during 3 4, (area 3 4 *b a*), is equal to the heat given by the steam during 1 2, (area 1 *d c* 2). It can be shown that the efficiency of this ideal regenerative cycle is equal to that of Carnot cycle.

Heat supplied externally=area under 4 1=4 1 *d b*

Heat rejected internally=area under 2 3

=area under 2′ 3′=area 2′ *d b* 3′

The above expressions are same as in Carnot cycle. The advantage of regeneration is explained by the fact that in Rankine cycle more latent heat is thrown in condenser than in regenerative cycle.

The ideal regenerative cycle cannot be followed in actual practice. Even if we could practically approach it, it would not be used because of the low dryness fraction of the steam in the latter stages of the turbine. Therefore, in actual practice advantage is taken of the principle of regeneration by *bleeding* a part of the steam flowing at certain stages of expansion for feed heating so that the dryness fraction of the remaining part is not greatly reduced.

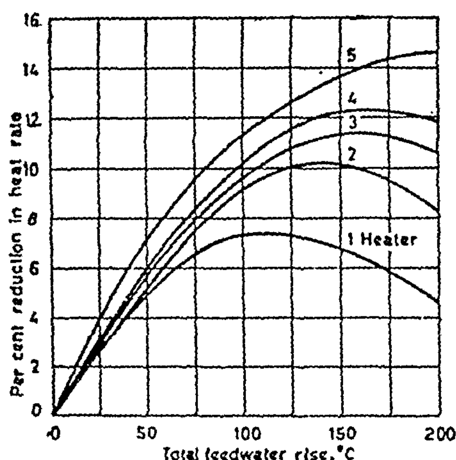
15·2. Improvement in Turbine Efficiency by Bleeding. Bleeding is the process of abstracting steam from any point in the turbine and using it for heating the feed water. This cycle is also known as regenerative cycle because part of the steam after its partial expansion is utilised to heat feed water, which is same working substance as steam.

Its main advantages are (*i*) it increases the thermodynamic efficiency as the heat of the bled steam is not lost in the condenser but is utilised in feed heating, which increases the average temperature at which the system receives heat from the external source, (*ii*) the temperature stresses in the boiler are reduced due to hotter feed and, (*iii*) the hotter feed prevents the condensation of sulphur-dioxide gases on economiser.

The disadvantages of bleeding are that the cost of plant increases and the work done per kg of steam is reduced which results in higher boiler capacity for given output ; however, the capacity of condenser required is reduced. Sometimes feed heating renders economiser unnecessary and hence air heaters may be required to remove the heat from exhaust gases.

Theoretically the efficiency increases with the increased number of feed heaters but the gain is in diminishing return as shown in *Fig. 15·2*.

Due to cost element the number of feed heaters is limited to 3 to 5 (maximum 8) and the temperature rise of the feed water per heater is about 10 to 15°C.



15·2. Saving in heat rate from regenerative heating.

15·3. Disposal of Bled Steam Condensate. Bled steam condensate may be disposed off in following ways :—

(1) *Direct Contact Heaters.* In this method there is mixing of bled steam with feed water, both of which are extracted and pumped into the next high pressure heaters as shown in *Fig. 15·3*. This system is rarely used due to difficult conditions caused by high temperature in which pump has to work.

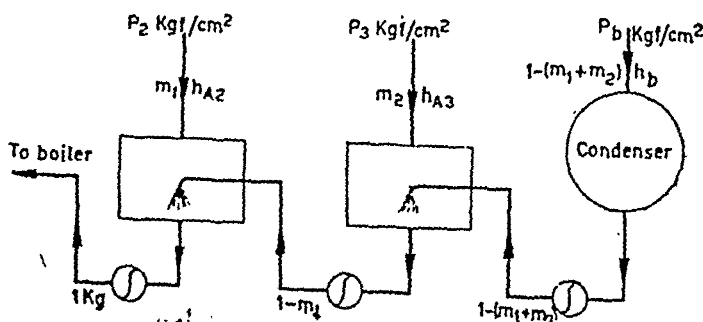


Fig. 15·3. Direct contact heaters.

(2) *Drain Pump Method.* In this method condensate is extracted by the drain pump and discharges into feed pipe after the heaters as shown in Fig. 15·4. It suffers from the same disadvantages as above except that pumps have to deal with condensed steam only.

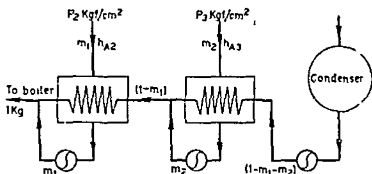


Fig 15·4. Bled steam condensate in feed pipe by drain pump method.

(3) *All Drains in Hot Well.* In this method the condensate of all heaters are directly fed into hot well as shown in Fig. 15·5. This arrangement may be used in case of break down of other complicated arrangements.

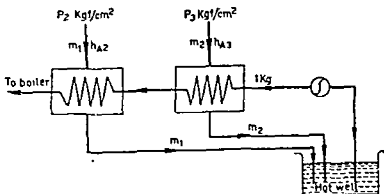
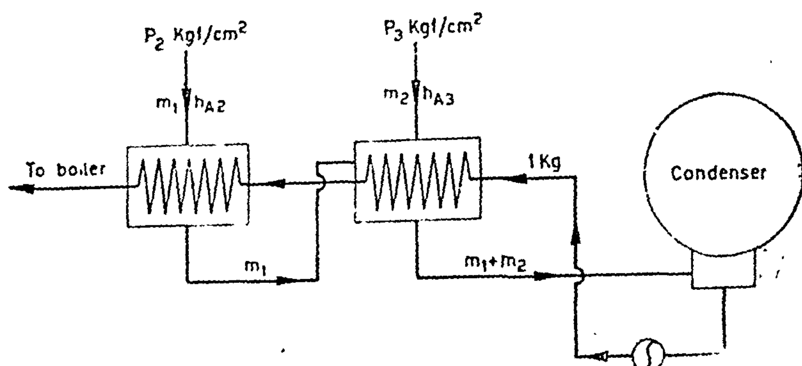


Fig 15·5. All drains in hot well.

(4) *Cascade Method.* In this method the condensate may be cascaded into the next lower heater as shown in Fig. 15·6. The heat given by the steam may be its latent heat and part of the sensible heat.

The condensate from the last low pressure heater may be put into hot well or into condenser (it somewhat reduces the efficiency) or



15·6. Cascade Method.

better it may be put back into the feed discharge from the extraction pump. A drain cooler may be placed between the last low pressure heater and condenser to recover some heat, but this increases the cost.

15·4. Efficiency of Regenerative Cycle. It can be proved that the thermal efficiency of the regenerative cycle is always greater than that of the straight Rankine cycle regardless of where the steam is tapped off.

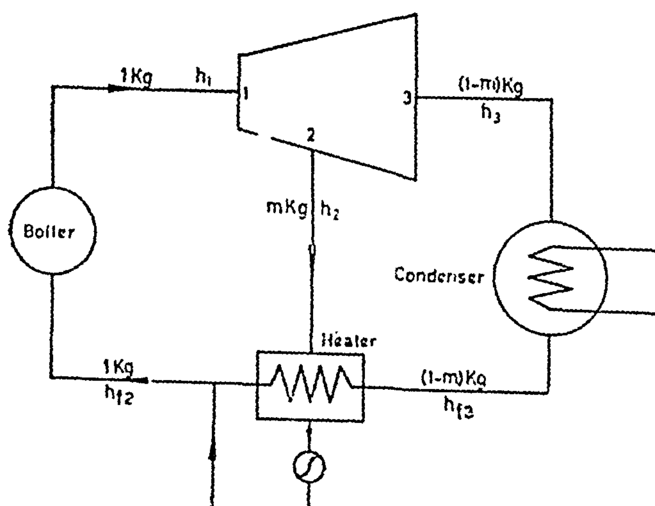


Fig. 15·7. Regeneration with one heater.

reduced size of boiler and auxiliaries for the same output. However at unnecessarily low reheat pressures, the reheat efficiency may be less than the Rankine efficiency. The main disadvantages of reheating is that the cost of the plant is increased due to the reheater and its long connections. It also increases the condenser capacity due to increased dryness fraction (*see* Fig. 15·8 and 15·9).

15·6. Regenerative Water Extraction Cycle. An alter*native method of increasing the dryness fraction at exit is to* extract moisture at certain stages of the turbine, partly or fully. The water so separated may be used for feed heating. Such a cycle may be termed regenerative water extraction cycle.

15·7. Binary Vapour Cycle. The efficiency of an engine working on Carnot cycle can be increased by increasing the initial temperature or by decreasing the exit temperature. With steam as the working fluid the temperature rise is accompanied by rise in pressure and at critical temperature of 374·15 °C the pressure is 225·65 kgf/cm². High pressure creates many complications in design and operation. The aim of high efficiency cannot be achieved by merely superheating the steam as the efficiency depends on the mean temperature at which the heat is supplied. In a superheated steam most of the heat supplied is latent heat at saturation temperature, whereas in high pressure steam, as the pressures rises, less and less heat is given at the saturation temperature and at critical temperature latent heat supplied is zero. Therefore in high temperature range we should use a substance other than steam which has low saturation pressure at high temperature. Mercury is the only vapour which has been successfully used. It has a saturation pressure 12·6 kgf/cm² at 538°C and 0·028 kgf/cm² at 204°C. Due to excessively low pressures at low temperatures mercury alone cannot be used for *the complete cycle and hence binary* vapour *cycle* has been used where mercury is used for high pressures in conjunctio steam at low pressures.

Mercury also is not the best fluid for binary vapour it is *poisonous, expensive* and does not wet the surface of th it contacts. Due to the last property serious difficulty is intr in heat transfer from the furnace without allowing burnin

ness fraction should not be below 0·88 in a steam turbine. Reheating is generally employed when the pressures are high, say above 100 kgf/cm². The other advantages of reheating are increase in thermal efficiency (because additional heat is supplied at higher mean temperature) and increase in work done per kg of steam which results in

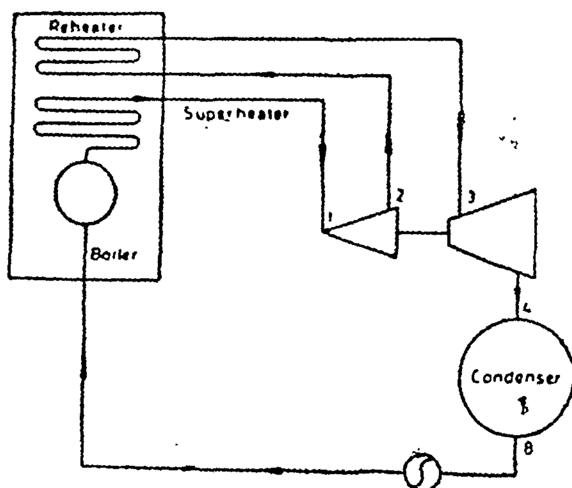
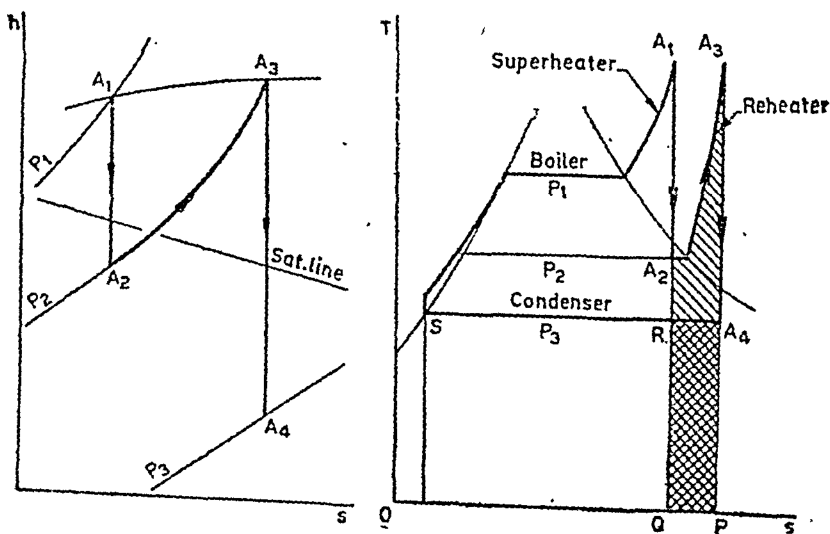


Fig. 15·8. Reheat cycle.



Heat supplied in reheat $= A_2\, A_3\, PQA_2$

Available energy for W.D. $= A_2\, A_3\, A_4\, RA_2$

Fig. 15·9. Reheat cycle on *h*-s and *T*-*s* diagrams.

reduced size of boiler and auxiliaries for the same output. However at unnecessarily low reheat pressures, the reheat efficiency may be less than the Rankine efficiency. The main disadvantages of reheating is that the cost of the plant is increased due to the reheater and its long connections. It also increases the condenser capacity due to increased dryness fraction (*see* Fig. 15·8 and 15·9).

15·6. Regenerative Water Extraction Cycle. An alternative method of increasing the dryness fraction at exit is to extract moisture at certain stages of the turbine, partly or fully. The water so separated may be used for feed heating. Such a cycle may be termed regenerative water extraction cycle.

15·7. Binary Vapour Cycle. The efficiency of an engine working on Carnot cycle can be increased by increasing the initial temperature or by decreasing the exit temperature. With steam as the working fluid the temperature rise is accompanied by rise in pressure and at critical temperature of 374·15 °C the pressure is 225·65 kgf/cm². High pressure creates many complications in design and operation The aim of high efficiency cannot be achieved by merely superheating the steam as the efficiency depends on the mean temperature at which the heat is supplied. In a superheated steam most of the heat supplied is latent heat at saturation temperature, whereas in high pressure steam, as the pressures rises, less and less heat is given at the saturation temperature and at critical temperature latent heat supplied is zero. Therefore in high temperature range we should use a substance other than steam which has low saturation pressure at high temperature. Mercury is the only vapour which has been successfully used. It has a saturation pressure 12·6 kgf/cm² at 538°C and 0 028 kgf/cm² at 204°C. Due to excessively low pressures at low temperatures mercury alone cannot be used for the complete cycle and hence binary vapour cycle has been used where mercury is used for high pressures in conjunction with steam at low pressures.

Mercury also is not the best fluid for binary vapour plants as it is poisonous, expensive and does not wet the surface of the metal it contacts. Due to the last property serious difficulty is introduced in heat transfer from the furnace without allowing burning of

tubes and the shell. Addition of 0·002 per cent of solution of magnesium and titanium to mercury gives it the property of wetting **steel.**

Fig. 15·10 shows the schematic diagram of binary vapour cycle. Fig. 15·11 shows the cycle on T-s diagram. Note that the steam is generated by the condensation of mercury. This steam cannot be

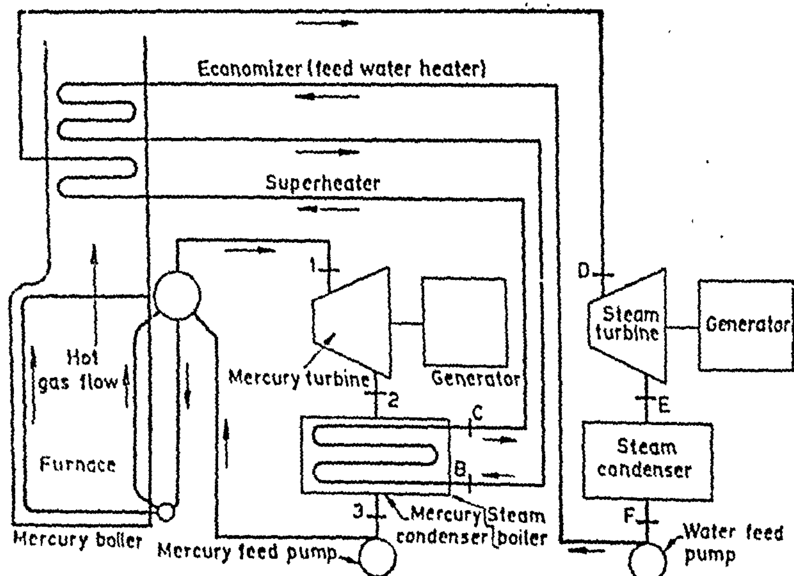


Fig. 15·10. Schematic diagram of binary vapour cycle.

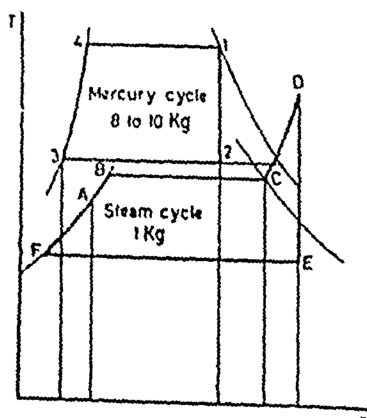


Fig. 15·11. Binary vapour cycle on T-s diagram.

superheated in the mercury condenser/steam boiler as some temperature gradient is required. Therefore superheating is done in a separate boiler.

If cycles of both mercury and steam for 1 kg are represented on same T-s diagram, the mercury cycle will be small in comparison to that of steam. Hence T-s diagram for mercury is for mass of mercury required per kg of steam which is about 8 to 10 kg.

IMPORTANT POINTS

1. The bled steam gives up only its superheat (if any) and latent heat. The heat of the condensed steam leaving is thus its liquid heat h_f, which is obtained from Steam Tables.

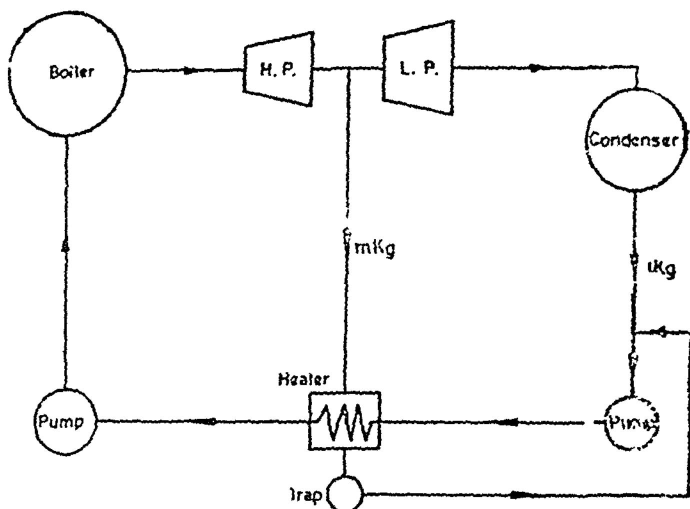
2. There is perfect heat transmission in the heater, so that the feed water is heated to the saturation temperature at the pressure of the bled steam. In actual practice feed temperature is 5 to 10°C lower.

3. Perfect heat transmission is assumed in drain cooler, if any, which means temperature of the condensed heating steam is reduced to the temperature of the feed water entering the drain cooler. In actual practice the temperature of the condensed steam is 10 to 15°C higher.

ILLUSTRATIVE EXAMPLES

15·1. Single bleeding : steam tapped ; steam generated ; overall efficiency, given efficiency ratios.

In the steam plant indicated in Fig. the two sections of the turbine are coupled and drive an alternator. When the output from the alternator is 27 000 kW, the input to the turbine is at 60 kgf/cm² and 450°C the intermediate pressure is 3 kgf/cm², the condenser pressure 0·07 kgf/cm², and the feed heater raises the condensate temperature to 110°C. The pumps absorb 9 per cent of the alternator output, the boiler efficiency is 87 per cent, the efficiency ratio of each section of the turbine is 85 per cent and the alternator efficiency is 97 per cent.



Ignoring all other losses, calculate :—

(*a*) *the value of m, the tapped steam flow to the feed heater per kg of steam to the condenser* ;

(*b*) *the steam to be generated per hour* ;

(c) *the overall thermal efficiency of the plant, neglecting the boiler feed pump work in calculating input to the boiler.*

The schematic diagram is shown in Fig. 15·12 and the sketch of the cycle on Mollier chart is shown in Fig. 15·13.

From Steam Tables, $h_{A1} = 788{\cdot}5$ kcal

From Mollier chart, $\Delta h_{T1} = h_{A1} - h_{B_1} = 165$ kcal

$\therefore \quad \Delta h_{U_1} = 0{\cdot}85 \times 165 = 140$ kcal

$\therefore \quad h_{A2} = h_{A1} - \Delta h_{U_1} = 788{\cdot}5 - 140 = 648{\cdot}5$ kcal

$\Delta h_{T2} = h_{A2} - h_{B2} = 131 \qquad \therefore \quad \Delta h_{U_2} = 0{\cdot}85 \times 131 = 111{\cdot}4$ kcal

Let $(1+m)$ kg of steam be flowing from boiler out of which m kg is tapped for heater and remaining 1 kg passes to condenser through L.P. turbine. Just after condenser m kg of water from heater and 1 kg of water from condenser are mixed. Let final heat per kg of water be h kcal.

Mixing equation is, $m \times 133{\cdot}5 + 38{\cdot}7 \times 1 = (1+m)h \qquad (1)$

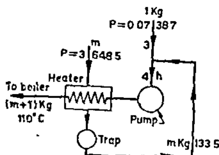


Fig. 15 12.

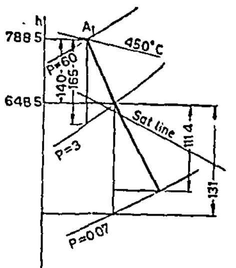


Fig. 15·13.

The heat balance for the heater is

$$648\ 5\ m+(1+m)h=m\times 133{\cdot}5+(110)(1+m) \qquad (2)$$

From equations (1) and (2) steam tapped per kg of steam to condenser,

$$\underline{m=0{\cdot}1322 \text{ kg}} \qquad \text{Ans.}$$

(*b*) W.D. by $(1+m)$ kg of steam generated

$$=(1+0\ 1322)140+111{\cdot}4=269\ 9 \text{ kcal}$$

$\therefore$ W.D. per kg. of steam generated $=\dfrac{269\ 9}{1\ 1322}$

$$\text{Power produced}=\frac{27\ 000\times 856{\cdot}7}{0{\cdot}97}=2\ 38\ 00\ 000 \text{ kcal/hr}$$

$$\therefore\ \underline{\text{Steam generated}}=\frac{2\ 38\ 50\ 000\times 1{\cdot}1322}{269\ 9}=\underline{1\ 02\ 000 \text{ kg/hr}}$$

Ans.

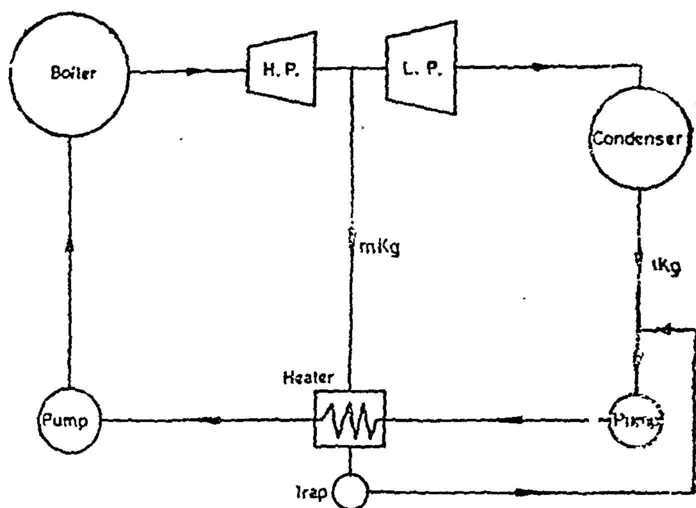
(*c*) Net power available

$$=27\ 000\times 856{\cdot}7\times 0\ 91=2\ 10\ 50\ 000 \text{ kcal/hr}$$

$$\text{Heat supplied in boiler}=\frac{1\ 02\ 000\ [788\ 5-110]}{0\ 87}$$

$$=7\ 82\ 00\ 000 \text{ kcal/hr}$$

$$\therefore\ \underline{\text{Overall thermal } \eta}=\frac{2\ 10\ 50\ 000}{7\ 82\ 00\ 000}=\underline{26\ 95 \text{ per cent}}$$



Ignoring all other losses, calculate :—

(*a*) *the value of m, the tapped steam flow to the feed heater per kg of steam to the condenser ;*

(*b*) *the steam to be generated per hour ;*

(*c*) *the overall thermal efficiency of the plant, neglecting the boiler feed pump work in calculating input to the boiler.*

The schematic diagram is shown in Fig. 15·12 and the sketch of the cycle on Mollier chart is shown in Fig. 15·13.

From Steam Tables, $h_{A1} = 788{\cdot}5$ kcal

From Mollier chart, $\Delta h_{T1} = h_{A1} - h_{B1} = 165$ kcal

$\therefore \quad \Delta h_{U_1} = 0{\cdot}85 \times 165 = 140$ kcal

$\therefore \quad h_{A2} = h_{A1} - \Delta h_{U_1} = 788{\cdot}5 - 140 = 648{\cdot}5$ kcal

$\Delta h_{T_2} = h_{A2} - h_{B2} = 131 \qquad \therefore \quad \Delta h_{U_2} = 0{\cdot}85 \times 131 = 111{\cdot}4$ kcal

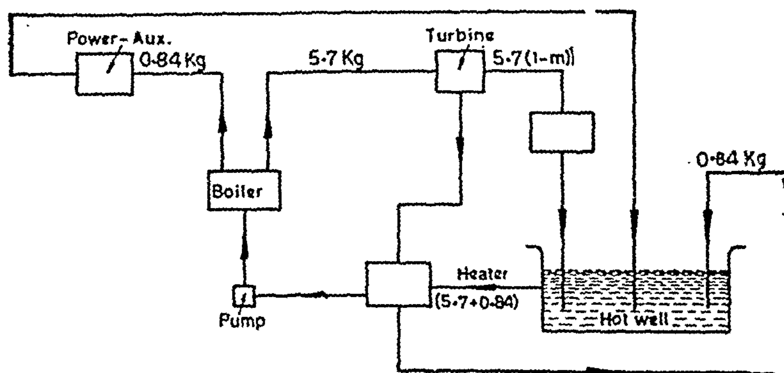
Let $(1+m)$ kg of steam be flowing from boiler out of which m kg is tapped for heater and remaining 1 kg passes to condenser through L.P. turbine. Just after condenser m kg of water from heater and 1 kg of water from condenser are mixed. Let final heat per kg of water be h kcal.

Mixing equation is, $m \times 133{\cdot}5 + 38{\cdot}7 \times 1 = (1+m)h$ (1)

15·2. Single-stage bleeding, auxiliaries : output ; η.

Distinguish between impulse and reaction turbines and indicate by curves the variation of pressure and velocity in a reaction expansion and in the nozzles and blading of a velocity compounded impulse stage.

A steam turbine receives 5·7 kg steam per second at 18 kgf/cm² and 150°C temperature. The power auxiliaries for the installations take 0·84 kg per second dry steam at 18 kgf/cm² and exhaust to the main condenser.



At a point in the turbine expansion, where the steam conditions are 2 kgf/cm², 2 per cent wet, sufficient steam is extracted to heat the total feed from 32°C to 115°C in a surface heater from which the condensate at the saturation temperature corresponding to the pressure passes to the condenser. The remaining steam expands to the exhaust pressure of 0·07 kgf/cm². Assume 32 per cent loss of heat drop due to friction throughout the entire expansion. Determine the power output and the thermal efficiency.

From steam Tables,

En halpy at 18 kgf/cm² and 300°C, $h_{A_1} = 722 \cdot 8$ kcal

Enthalpy at 2 kgf/cm² and 2 per cent wet

$$h_{A_2} = 119 \cdot 9 + 0 \cdot 98 \times 526 \cdot 4 = 635 \cdot 8 \text{ kcal}$$

Let m_1 be the total mass of steam bled. Heat balance for heate

$$635 \cdot 8\ m_1 + (5 \cdot 7 + 0 \cdot 84)(32) = (5 \cdot 7 + 0 \cdot 84)(115) + m_1 \times 119 \cdot 9$$

or $$m_1 = 1 \cdot 05 \text{ kg}$$

From Mollier chart, $\Delta h_T = \Delta h_{A_1} - h_D = 217$ kcal

$\Delta h_U = h_{A_1} - h_{A_3} = 0 \cdot 68 \times 217 = 147 \cdot 6$ kcal

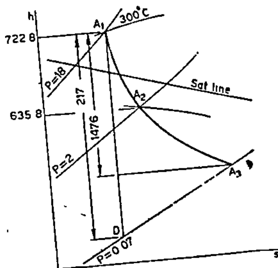


Fig. 15·14

$h_{A3}=h_{A1}-147{\cdot}6=722{\cdot}8-147{\cdot}6=575{\cdot}2$ kcal

$\text{W.D.}=5{\cdot}7(h_{A1}-h_{A2})+(5{\cdot}7-m_1)(h_{A2}-h_{A3})$

$=5\,7(722{\cdot}8-635{\cdot}8)+(5{\cdot}7-1{\cdot}05)(635{\cdot}8-575{\cdot}2)$

$=777{\cdot}7$ kcal/s

$\therefore$ Power output $=\dfrac{777{\cdot}7\times 60}{75}=\underline{4\,430}$

Heat supplied in turbine

$=5\,7[722{\cdot}8-115]+0{\cdot}84[667\,8-115]$

$=3\,929$ kcal

$\therefore$ Thermal efficiency $=\dfrac{7\,77{\cdot}7}{3\,929}=\underline{19{\cdot}78 \text{ per cent}}$

15·3. Two-stage bleeding : pressure for 2nd stage [illegible] steam tapped ; gain in η ; increase in consumption rate.

A multi-stage steam turbine has to be [illegible] bleed heating. The temperature of the f[illegible] between the two heaters. Initial steam [illegible] temperature 320°C. Exhaust pressure is [illegible] of condensate is 37°C. The first tapping [illegible] Determine the pressure for second tapping [illegible] tapped at each point. Determine the gain in [illegible] ing and the increase in consumption rate. [illegible] 70 per cent.

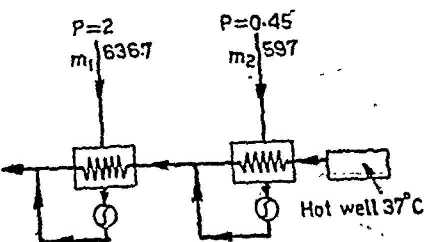


Fig. 15·15.

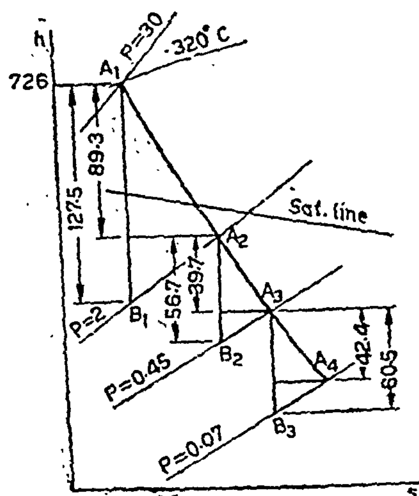


Fig. 15·16

The schematic diagram is shown in Fig. 15·15 and thermodynamic cycle on Mollier chart is shown in Fig. 15·16.

Let temperature at second tapping be t_B

Since temperature of feed is equally divided

$$t_B - 37 = 119{\cdot}6 - t_B \qquad \therefore \quad t_B = 78{\cdot}3°C$$

$\therefore$ Corresponding pressure for second tapping,

$$\underline{P = 0{\cdot}45 \text{ kgf/cm}^2} \qquad \text{Ans.}$$

From Mollier chart, $h_{A1} = 726$ kcal, $\Delta h_T = 127{\cdot}5$ kcal

$\therefore \quad \Delta h_U = 0{\cdot}7 \times 127{\cdot}5 = 89{\cdot}3$ kcal

$\therefore \quad h_{A_2} = 726 - 89{\cdot}3 = 636{\cdot}7$ kcal

$\Delta h_{T_2} = 56{\cdot}7$ kcal, $\Delta h_{U_2} = 0{\cdot}7 \times 56{\cdot}7 = 39{\cdot}7$ kcal

$\therefore \quad h_{A_3} = 636{\cdot}7 - 39{\cdot}7 = 597$ kcal

$\Delta h_{T_3} = 60{\cdot}5$ kcal $\qquad \therefore \quad \Delta h_{U_3} = 0{\cdot}7 \times 60{\cdot}5 = 42{\cdot}4$ kcal

Heat balance for 1st heater is

$$m_1(636{\cdot}7 - 119{\cdot}9) = (1 - m_1)(119{\cdot}9 - 78{\cdot}3)$$

$$\therefore \quad \underline{m_1 = 0{\cdot}0746 \text{ kg}} \qquad \text{Ans.}$$

Heat balance for 2nd heater is

$$m_2(597 - 78{\cdot}3) = (1 - 0{\cdot}0746 - m_2)(78{\cdot}3 - 37)$$

$\therefore \quad m_2 = 0.0682$ kg Ans.

W.D. per kg of main flow

$= 89.3 + (1 - 0.0746)39.7 + (1 - 0.0746 - 0.0682)42.4 = 162.4$ kcal

Heat supplied per kg of flow $= 726 - 119.9 = 606.1$ kcal

$\therefore$ Efficiency with bleed heating $= \frac{162.4}{606.1} = 26.8$ per cent

W.D. without bleed heating $= 89.3 + 39.7 + 42.4 = 171.4$ kcal

Heat supplied per kg $= 726 - 37 = 689$ kcal

$\therefore$ Efficiency without bleed heating

$$= \frac{171.4}{689} = 24.9 \text{ per cent}$$

Gain in efficiency $= \frac{26.8 - 24.9}{24.9} = 7.65$ per cent **Ans.**

Consumption rate with bleed heating

$$= \frac{0.17569 \times 3\,600}{162.4} = 3.89 \text{ kg/hp-hr}$$

Consumption rate without bleed heating

$$= \frac{0.17569 \times 3\,600}{171.4} = 3.69 \text{ kg/hp-hr}$$

$\therefore$ Increase in consumption rate

$= 0.20$ kg/hp-hr Ans

15·4. Two-stage bleeding : pump capacity for drains.

A turbine operate under the following conditions, steam being bled off from two stages for feed-heating purposes—

Initial steam pressure	*30 kgf/cm²*
Initial steam temperature	*400°C*
Steam pressure in first heater	*6 kgf/cm²*
Steam pressure in second heater	*1 kgf/cm²*
Exhaust pressure	*0·07 kgf/cm²*
Condensate temperature	*38·7°C*
Feed temperature after second heater	*95°C*
Feed temperature after first heater	*154°C.*

The bled steam is condensed in the feed heaters and there is no cooling of the condensate. The drains from the first heater are passed through a steam trap into the second heater and the combined drains

from the second heater are pumped by a drain pump into the feed pipe after the second heater.

If the internal power developed by the turbine is 12 000 kW, find the required capacity of the heater drain pump.

The efficiency ratio of the turbine is 0·8.

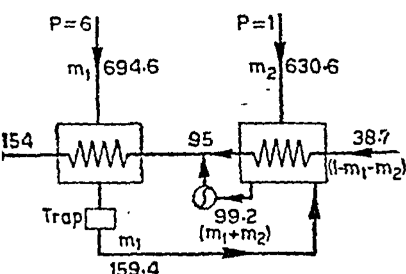


Fig. 15·17.

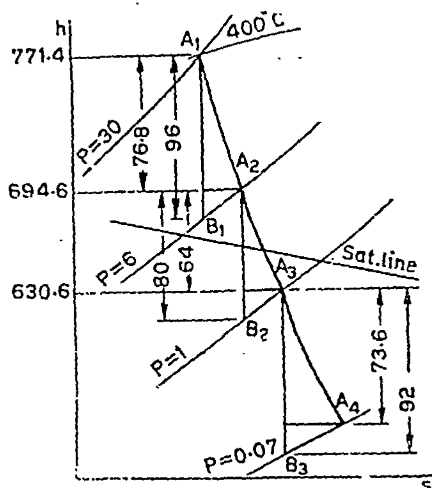


Fig. 15·18.

The schematic diagram is shown in Fig. 15·17 and the thermodynamic cycle on Mollier chart is shown in Fig. 15·18.

From Steam tables, $ha_1 = 771{\cdot}4$ kcal

From Mollier chart

$\Delta h_{T1} = 96$ kcal $\qquad \therefore\ \Delta h_{U_1} = 0{\cdot}8 \times 96 = 76{\cdot}8$ kcal

$\therefore\ h_{A2} = h_{A1} - \Delta h_{U_1} = 771{\cdot}4 - 76{\cdot}8 = 694{\cdot}6$ kcal

$\Delta h_{T_2} = h_{A2} - h_{B2} = 80$ kcal $\qquad \therefore\ \Delta h_{U_2} = 0{\cdot}8 \times 80 = 64$ kcal

$\therefore\ h_{A3} = h_{A2} - \Delta h_{U_2} = 694{\cdot}6 - 64 = 630{\cdot}6$ kcal

$\Delta h_{T3} = h_{A3} - h_{B3} = 92$ kcal $\qquad \therefore\ \Delta h_{U_3} = 0{\cdot}8 \times 92 = 73{\cdot}6$ kcal

Heat balance for 1st heater is

$$m_1(694{\cdot}6 - 159{\cdot}4) = 154 - h \qquad \therefore\ h = 154 - 535{\cdot}2\, m_1 \qquad (1)$$

Heat balance for 2nd heater is

$$630{\cdot}6\, m_2 + 159{\cdot}4\, m_1 + 38{\cdot}7\,(1 - m_1 - m_2)$$
$$= 95\,(1 - m_1 - m_2) + 99{\cdot}2\,(m_1 + m_2) \qquad (2)$$

Heat balance for mixing point is

$$h=95(1-m_1-m_2)+99\cdot2(m_1+m_2) \qquad (3)$$

From equations (1), (2) and (3), we have

$$m_1=0\cdot1086 \text{ kg and } m_2=0\cdot1003 \text{ kg}$$

Total steam bled per kg of boiler steam

$$=m_1+m_2=0\cdot1086+0\cdot1003=0\cdot2089 \text{ kg}$$

W.D. per kg of boiler steam

$$76\cdot8+(1-0\cdot1086)64+(1-0\cdot2089)73\cdot6=192\cdot1 \text{ kcal}$$

Let m kg be the steam generated per minute.

$$12\ 000=\frac{m\times192\cdot1\times60}{856\cdot7}$$

or $\qquad m=891$ kg per minute

$\therefore$ Capacity of heater drain pump

$$=891\times0\cdot2089=186\cdot1 \text{ kg/min} \qquad \textbf{Ans.}$$

15·5. Two-stage turbine with reheating, given efficiencies : comparison with simple cycle.

In a proposed steam turbine plant the steam supply is at 100 kgf/cm² and 400°C. The steam expands in a high pressure turbine to 10 kgf/cm², and then in a low pressure turbine to 0·045 kgf/cm². The efficiency of the H P. turbine is 78 per cent and that of the L.P. turbine 74 per cent.

(a) Estimate the cycle efficiency and state the final dryness fraction of the steam when reheating is employed, the steam being reheated at a constant pressure of 10 kgf/cm² to 350°C.

Compare these values with those obtained when reheating is not employed.

(b) Comment on the values obtained in the two cases and discuss the effects of either raising the reheat temperature or using the cycle without reheat but increasing the initial temperature.

The sketch of the cycle on Mollier chart is shown in Fig 15 19.

From steam tables $h_{A_1}=739\cdot8$ kcal ; $h_{A_3}=753\cdot8$ kcal

$$\Delta h_{T_1}=114 \text{ kcal} \qquad \therefore\ \Delta h_{U_1}=0\cdot78\times114=89 \text{ kcal}$$

$$\therefore\ h_{A_2}=739\cdot8-89=650\cdot8 \text{ kcal}$$

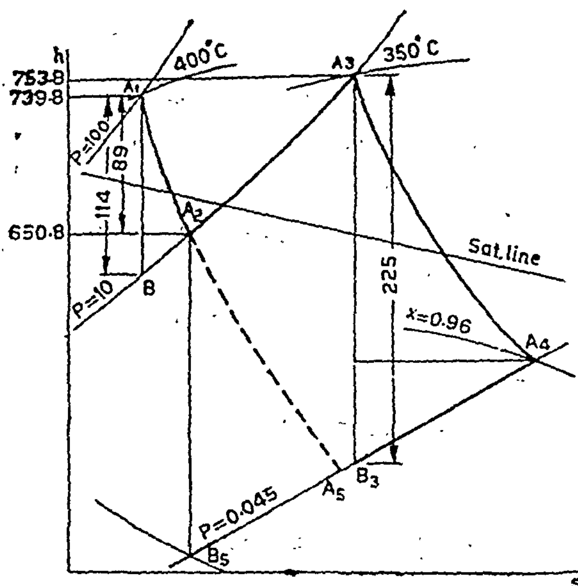


Fig. 15·19.

$\Delta h_{T_2}=225$ kcal $\qquad \therefore \ \Delta h_{U_2}=0{\cdot}74\times225=166{\cdot}5$ kcal

Final dryness fraction, $x_{A_4}=0{\cdot}96$ **Ans.**

$$\text{W.D.}=89+166{\cdot}5=255{\cdot}5 \text{ kcal}$$

$$\text{Heat supplied}=(h_{A1}-h_{fA1})+(h_{A3}-h_{A2})$$
$$=(739{\cdot}8-30{\cdot}7)+(753{\cdot}8-650{\cdot}8)=812{\cdot}1 \text{ kcal}$$

$$\therefore \quad \text{Cycle } \eta=\frac{255{\cdot}5}{812{\cdot}1}=31{\cdot}45 \text{ per cent}$$ **Ans.**

When there is no reheating

From Mollier chart $\Delta h_{T_2}=182$ kcal ;

$$\Delta h_{U_2}=0{\cdot}74\times182=134{\cdot}7 \text{ kcal}$$

$$\text{W.D.}=89+134{\cdot}7=223{\cdot}7 \text{ kcal}$$

$$\text{Heat supplied}=h_{A1}-h_{fB5}=739{\cdot}8-30{\cdot}7=709{\cdot}1 \text{ kcal}$$

$$\therefore \quad \text{Cycle } \eta=\frac{223{\cdot}7}{709{\cdot}1}=31{\cdot}5 \text{ per cent.}$$ **Ans.**

Note. Reheating increases final dryness fraction of steam.

15·6. Two-stage bleeding with drain cooler : % increase in η ; boiler capacity and reduction in condenser capacity.

Steam at 20 kgf/cm² and 300°C temperature is supplied to a three-stage pressure compounded turbine. The condenser pressure is 0·06 kgf/cm² and the efficiency ratio of each stage is 0·8. The inter-

mediate pressures are 2 and 0·4 kgf/cm². Steam is bled at the end of each stage and is being cascaded from heater to heater until it discharges through a drain cooler to the condenser. In drain cooler the temperature of the combined heater condensate is raised to condenser temperature. Calculate, (a) the percentage increase in thermal efficiency due to bleeding, (b) the percentage increase in boiler capacity for given output, and (c) the percentage reduction in the condenser capacity.

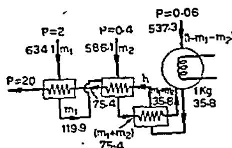


Fig. 15·20.

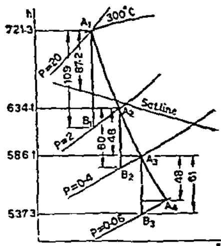


Fig 15·21.

(*a*) The schematic diagram is shown in Fig. 15·20 and the sketch of the cycle on Mollier chart is shown in the Fig. 15·21.

From steam tables $h_{A_1}=721{\cdot}3$ kcal

From Mollier chart

$\Delta h_{T_1}=109$ kcal $\quad\quad \therefore\ \Delta h_{U_1}=0{\cdot}8\times 109=87{\cdot}2$ kcal

$$h_{A_2}=h_{A_1}-\Delta h_{U_1}=721{\cdot}3-87{\cdot}2=634{\cdot}1 \text{ kcal}$$

$\Delta h_{T_2}=60$ kcal $\quad \Delta h_{U_2}=0{\cdot}8\times 60=48$ kcal

$$\therefore \quad h_{A_3}=h_{A_2}-\Delta h_{U_2}=634{\cdot}1-48=586{\cdot}1 \text{ kcal}$$

$\Delta h_{T_3}=61$ kcal $\quad\quad \therefore\ \Delta h_{U_3}=0{\cdot}8\times 61=48{\cdot}8$ kcal

$$\therefore \quad h_{A_4}=h_{A_3}-\Delta h_{U_3}=586{\cdot}1-48{\cdot}8=537{\cdot}3 \text{ kcal}$$

Heat balance for 1st heater is

$$634{\cdot}1m_1+119{\cdot}9m_1=(119{\cdot}9-75{\cdot}4) \quad (1)$$

$\therefore$ Mass of steam bled from 1st heater, $m_1 = 0{\cdot}0865$ kg

Heat balance for 2nd heater is

$$586{\cdot}1m_2 + h + 119{\cdot}9m_1 = 75{\cdot}4 + 75{\cdot}4(m_1 + m_2)$$

or
$$510{\cdot}7m_2 + h = 71{\cdot}5 \quad (2)$$

Heat balance for drain cooler is

$$75{\cdot}4(m_1 + m_2) + 1 \times 35{\cdot}8 = 35{\cdot}8(m_1 + m_2) + h$$

or
$$h = 39{\cdot}6m_2 + 39{\cdot}2 \quad (3)$$

From equation (2) and (3), $m_2 = 0{\cdot}0587$ kg

W.D. per kg of steam from boiler

$$= 87{\cdot}2 + (1 - 0{\cdot}0865)48 + (1 - 0{\cdot}0865 - 0{\cdot}0587)48{\cdot}8 = 172{\cdot}8 \text{ kcal}$$

Heat supplied per kg of steam $= 721{\cdot}3 - 119{\cdot}9 = 601{\cdot}4$ kcal

$\therefore$ Thermal efficiency with bleeding $= \dfrac{172{\cdot}8}{601{\cdot}4} = 28{\cdot}7$ per cent

W.D. per kg of steam from boiler without bleeding

$$= 87{\cdot}2 + 48 + 48{\cdot}8 = 184 \text{ kcal}$$

Heat supplied per kg of steam $= 721{\cdot}3 - 35{\cdot}8 = 685{\cdot}5$ kcal

$\therefore$ Thermal efficiency without bleeding $= \dfrac{184}{685{\cdot}5} = 26{\cdot}8$ per cent

Increase in thermal efficiency

$$= \frac{28{\cdot}7 - 26{\cdot}8}{26{\cdot}8} = 7{\cdot}1 \text{ per cent} \quad \text{Ans.}$$

(b) Let, when there is no bled, steam flow be 1 kg

Steam required, for same amount of work, with bleeding

$$= \frac{184}{172{\cdot}8} = 1{\cdot}065 \text{ kg}$$

$\therefore$ Increase in boiler capacity $= 1{\cdot}065 - 1 = 6{\cdot}5$ per cent   Ans.

(c) With heaters, heat to condenser

$$= 1{\cdot}065(1 - 0{\cdot}0865 - 0{\cdot}587)(537{\cdot}3 - 35{\cdot}8) = 456 \text{ kcal}$$

Without heaters, heat to condenser for same power

$$= 537{\cdot}3 - 35{\cdot}8 = 501{\cdot}5 \text{ kcal}$$

$\therefore$ Reduction in condenser capacity

$$= \frac{501{\cdot}5 - 546}{501{\cdot}5} = 9{\cdot}08 \text{ per cent} \quad \text{Ans.}$$

15·7. Combined reheating and bleeding : efficiency.

In a steam power plant the inlet pressure and temperature are 70 kgf/cm², 450°C. The steam is extracted after expansion in H.P. cylinder to 25 kgf/cm² and reheated to 420°C. The steam then expands to 0·07 kgf/cm² in the L.P. cylinder. There is one stage of feed heating and the feed is heated to 179°C.

The H.P. turbine efficiency is 78·5 per cent and the I.P. and L.P. turbine efficiency is 83 per cent. Using the h-s chart determine the cycle efficiency.

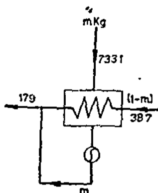


Fig. 15·22.

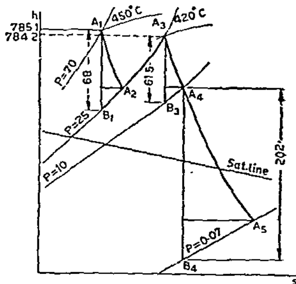


Fig. 15·23.

Pressure at the stage of bleeding, assuming it corresponds to saturation temperature of 119°C, is 10 kgf/cm².

The schematic diagram is shown in Fig. 15·22 and the sketch of the cycle on Mollier chart is shown in Fig. 15.23.

From steam tables $h_{A_1}=785{\cdot}1$ kcal ; $h_{A_3}=784{\cdot}2$ kcal

From Mollier chart $\Delta h_{T_1}=68$ kcal , $\Delta h_{U_1}=53\ 5$ kcal

$$h_{A_2}=h_{A_1}-\Delta h_{U_1}=785{\cdot}1-53{:}5=731{\cdot}6 \text{ kcal}$$

$$\Delta h_{T_2}=61{\cdot}5 \text{ kcal} ; \ \Delta h_{U_2}=51{\cdot}1 \text{ kcal}$$

$$\therefore \quad \Delta h_{T_3}=202 \text{ kcal} , \ \Delta h_{U_3}=167{\cdot}7 \text{ kcal}$$

$$\therefore \quad h_{A_4}=h_{A_3}-\Delta h_{U_2}=784{\cdot}2-51{\cdot}1=733{\cdot}1 \text{ kcal}$$

Let m be the mass of steam bled per kg of total flow

Heat balance for heater, $733{\cdot}1\ m+38\ 7(1-m)=179$

$$\therefore \qquad m=0\ 2052 \text{ kg}$$

W.D. per kg of steam

$$= \Delta h_{U_1} + \Delta h_{U_2} + (1-m)\ \Delta h_{U_3}$$

$$= 53{\cdot}5 + 51{\cdot}1 + (1-0{\cdot}2052) \times 167{\cdot}7 = 238 \text{ kcal}$$

Heat supplied per kg of steam $= (h_{A_1} - h_f) + (h_{A_3} - h_{A_2})$

$$= (785{\cdot}1 - 179) + (784{\cdot}2 - 731{\cdot}6) = 658{\cdot}7 \text{ kcal}$$

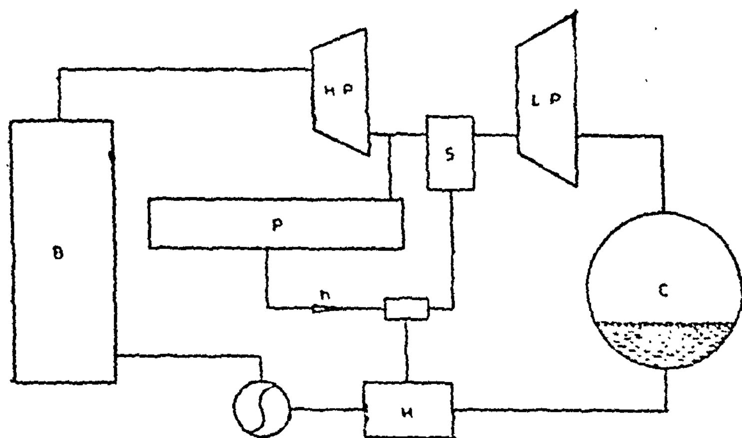
$\therefore$ Cycle efficiency $= \dfrac{238}{658{\cdot}7} = 36{\cdot}2$ per cent Ans.

Note.—It is assumed that the condensate of the heater is fed back into the line by a drain pump.

15·8. Power and process cycle, regenerative water extraction : hot well temp. ; heat in process and turbine work.

Fig. shows the arrangement of a steam turbine plant in which steam is also required for an industrial heating process. The steam leaves the boiler B at 30 kgf/cm² and 300°C and expands in the high pressure (H.P.) turbine to 2 kgf/cm², the efficiency of the H.P. turbine being 75 per cent. At this point one half of the steam passes to the process heater P and the remainder enters a separator S which removes all the moisture. The dry steam enters the low pressure (L.P.) turbine at 2 kgf/cm² and expands to 0·07 kgf/cm². The efficiency of the L.P. turbine being 70 per cent.

The drainage from the separator mixes with the condensate from the process heater and the combined flow enters the hot well H at 80°C. Traps are provided at the exits from P and S. A pump (not shown) extracts the condensate from the condenser C and this enters the hot well at 38°C.



Neglecting the feed pump work and radiation loss estimate the temperature of water leaving the hot well, which is at atmospheric pressure.

Calculate, as percentages of the heat transferred in the boiler, (a) the heat transferred in the process heater, and (b) the work done in the turbine.

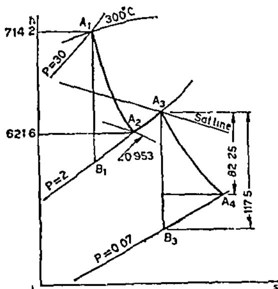


Fig. 15·24.

(*a*) The sketch of the cycle on Mollier chart is shown in Fig. 15·24.

From steam tables, $h_{A_1}=714{\cdot}2$

From Mollier chart,

$\Delta h_{T_1}=123{\cdot}5$ kcal $\qquad \therefore \quad \Delta h_{U_1}=0{\cdot}75\times 123{\cdot}5=92{\cdot}6$ kcal

$\therefore \quad h_{A_2}=714{\cdot}2-92{\cdot}6=621{\cdot}6$ kcal

Dryness fraction at A_2, $\qquad x_{A_2}=0{\cdot}953$

Mass of moisture trapped in separator $=\dfrac{1-0{\cdot}953}{2}=0{\cdot}0235$ kg

Mass of steam in L P. turbine $=0{\cdot}5-0{\cdot}0235=0{\cdot}4765$ kg

Mixing equation for hot well

$0{\cdot}765\times 38+0{\cdot}5235\times 80=t$

$\therefore$ Temperature of water leaving hot well, $t=60°C$ Ans.

Heat balance for trap

$0{\cdot}5h+0{\cdot}0235\times 119{\cdot}9=0{\cdot}5235\times 80 \qquad \therefore \quad h=78{\cdot}12$ kcal

Heat transferred in boiler $=714{\cdot}2-60=654{\cdot}2$ kcal

Heat transferred in the process heater

$$=0\cdot5(621\cdot6-78\cdot1)=271\cdot8 \text{ kcal}$$

Heat transferred in heater

$$=\frac{271\cdot8}{654\cdot2}=\underline{41\cdot5\%} \text{ of heat transferred in boiler} \qquad \text{Ans.}$$

(*a*) From Mollier chart,

$$\Delta h_{T_2}=117\cdot5 \text{ kcal} \therefore \Delta h_{U_2}=0\cdot7\times117\cdot5=82\cdot25 \text{ kcal}$$

W. D. in turbines$=92\cdot6+0\cdot4765\times82\cdot25=131\cdot8$ kcal

$$=\frac{131\cdot8}{654\cdot2}=\underline{20\cdot15\%} \text{ of heat transferred in boiler} \qquad \text{Ans.}$$

Note.—(1) The problem illustrates that the efficiency of a steam cycle is increased when power generation is combined with process work.

(2) Water extraction has been adopted between the stages to increase the final dryness of the steam.

15·9. Binary Vapour plant, superheated steam with bleeding : Hg per lb of steam : efficiency.

In a mercury steam binary vapour cycle, mercury vapour is generated at 8·45 kgf/cm² dry saturated, and expands to 0·07 kgf/cm² in the mercury turbine. Steam is generated in the condenser boiler at 40 kgf/cm² and 98 per cent quality and is superheated to 420°C in a separate superheater. The steam is expanded to 0·07 kgf/cm². Two regenerative heaters are used, one using steam at 1 kgf/cm² and heating the feed water to 98°C, the other using steam at 7 kgf/cm² and heating the feed to 160°C. The mercury turbine converts 80 per cent of the available enthalpy to work, and each section of the steam turbine (high, intermediate and low pressure) converts 80 per cent of the available enthalpy to work. The enthalpies per kg of mercury are :

Dry saturated vapour at 8·45 kgf/cm²$=83\cdot5$ kcal

After isentropic expansion to 0·07 kgf/cm²$=56\cdot1$ kcal

Saturated liquid at 0·07 kgf/cm²$=8\cdot3$ kcal

Find kg of mercury circulated per kg of steam and efficiency of the unit.

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